Vol. XLIX. MAY, 1923. No. 5.

AMERICAN SOCIETY OF CIVIL ENGINEERS

INSTITUTED 1852

REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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THEORETICAL FREQUENCY CURVES AND THEIR APPLICATION TO ENGINEERING PROBLEMS

By H. Alden Foster,* Assoc. M. Am. Soc. C. E.

Synopsis

The solution of certain types of engineering problems may be aided by the use of probability methods. The ordinary theory of probability, as developed in Least Squares, is not usually suitable for the analysis of natural phenomena. In general, an unsymmetrical probability formula is better adapted to engineering problems. The object of this paper is to develop such a formula and to show its application to practical work.

Frequency and duration curves are first defined, and their chief characteristics are described. A theoretical frequency curve is then developed, the mathematical steps being given only in outline. The method of evaluating the frequency formula is explained in greater detail. A table (Table 2) is given, from which may be obtained the ordinates of the duration curve, for any values of the coefficient of variation and coefficient of skew.

The numerical results are tested by taking numerous values from the computed curves and determining the corresponding values of the coefficient of variation and coefficient of skew.

Finally, the practical application of the theoretical curves is discussed, and several examples are presented to illustrate the usefulness of the method.

Introduction

The solution of an engineering problem may be divided, in general, into three steps: (1) The determination of the fundamental data; (2) the design of the required structures; and (3) the construction of the project. It often happens that the first step is the most difficult, requiring a high type of engineering judgment and broad experience. The selection of the live loads for a bridge or the determination of the capacity of a water supply system is a problem that, as a rule, cannot be definitely solved. The ultimate decision must be based on experience and sound judgment.

At the same time, it is quite proper that the engineer who faces this problem should take advantage of any legitimate means of assisting his judgment. If he has to estimate the future population of a city, he will study its growth in the past and try to determine whether the population is increasing from year to year according to any definite law or formula. He bases his estimate of future growth on past experience. Similarly, the bridge designer will estimate

Note.—Written discussion on this paper, which will not be presented at any meeting of the Society, will be closed with the **September, 1923,** *Proceedings.* When finally closed the paper, with discussion, will be published in *Transactions.*

* Structural Designer, Dwight P. Robinson & Co., New York City.

future load requirements in the light of past experience regarding the development of train and engine loadings.

In certain engineering problems, the fundamental data are of a progressive nature, such as bridge loads and city population, which tend to increase from year to year. This increase often follows a more or less definite law and, in such a case, the estimate of future needs becomes a relatively simple task. In other problems, however, particularly those relating to climatic phenomena, the solution is much more difficult. How can any one predict what the rainfall will be ten years hence; or what the run-off of a stream will be next year?

Climatic conditions are dependent on a large number of independent factors. If it was possible to evaluate all these factors, one could probably determine their resultant effect and thereby forecast future conditions with considerable accuracy. This has been done in the case of ocean tides, for which a number of the controlling factors have been combined into one formula by which the state of the tide at a particular place can be foretold for any future date. Even with tides, the prophecy is not always accurate, as certain controlling factors, such as the prevailing winds, cannot be accurately foretold, with the result that the actual state of the tide may be quite different from the computed values.

With rainfall and stream flow, the controlling factors become so complicated that it is impossible to evaluate them, at least with the present limited knowledge of the causes of natural phenomena. One cannot expect, therefore, to determine a law or formula by which to calculate the rainfall at any future date, but some other line of approach to this problem must be sought.

The insurance actuary, in determining the premium that shall be charged on a life insurance policy, does not know how many years the policy holder will live. He does know by experience, however, that of every 100 000 persons who reached the age of 25 years, 80 000 of them, for example, lived to be 50 years old. Therefore, if the policy holder is now 25 years old, there is an eight-tenths probability that he will live to be 50 years old, and this fact is used in the computation of the premium rate.

In the same manner, the hydrologist can examine the rainfall records and compute the probability of occurrence of a given intensity of rainfall. If the records show that, during the last 50 years, the annual rainfall at a given place has exceeded 40 in. in only 2 years, it would be assumed that the probability of the rainfall exceeding 40 in. in any future year was only one-twenty-fifth.

The theory of probability is discussed in works on Least Squares, and is familiar to most engineers. If natural phenomena followed the assumptions adopted in the theory, it would be possible to determine the probability of the occurrence of a given rainfall with considerable accuracy. The common theory of probability, or "normal law of error," is based on certain axioms, one of which states that "positive and negative variations are equally frequent." With a rainfall record, this would mean that there were just as many years with rainfall greater than the average as there were years with rainfall less than the average. Experience shows that this is not the case. In other words, rainfall records do not follow the normal law of error, and most of the natural phenomena based on climatic conditions have the same characteristic.

The object of this paper is to determine the relation that exists between actual observed phenomena and the "normal law of error." As the normal law may be called the "symmetrical probability formula", an attempt will be made to derive a formula representing "unsymmetrical probability", and to show whether it can be applied to natural phenomena. This "unsymmetrical probability" formula is also called the "skew probability formula."

The application of probability methods to engineering problems is not new. In the discussion of a paper by W. E. Fuller, M. Am. Soc. C. E., entitled "Flood Flows", G. B. Pillsbury, M. Am. Soc. C. E., considers* the application of the normal law of error to flood flows. In his closing discussion† on this paper, Mr. Fuller states:

"The values [of flood flows] follow, not the curve of normal probabilities, but what is called a skew curve. If Mr. Pillsbury will deduce the formula that most accurately represents this skew curve, and will show its application, he will give a useful solution of the problem he has attempted, and will be doing the Profession a real service."

In a paper on "Storage to Be Provided in Impounding Reservoirs for Municipal Water Supply",‡ Allen Hazen, M. Am. Soc. C. E., makes a practical application of probability methods to storage problems, using the normal law of error, with some modification.

In a paper on "The Probable Variations in Yearly Run-off as Determined from a study of California Streams",§ L. Standish Hall, Assoc. M. Am. Soc. C. E., gives an empirical method for determining the unsymmetrical probability curve, together with an excellent discussion of the relation of the theory of probability to stream-flow studies. In discussing Mr. Hall's paper, Mr. Hazen says:‖

"Data represented by skew curves are capable of being handled by mathematical analysis, but the methods are complicated. Textbooks on probabilities are available, which describe the necessary procedure; but for stream-flow data there are two reasons why the mathematical analysis is not advisable: First, the difficulty of applying the methods is too great for some engineers; and, second, and more important, such short-term records as are available are not long enough to serve as a basis for a conclusive determination of the shape of the curve."

In the same discussion,¶ Mr. Hazen states:

"For his own use, to aid in drawing smooth curves when points are few and irregular at the ends, the writer is now using a series of factors for curves with different degrees of skew. These factors correspond to those which he used in 1913, but the scheme has been developed and extended to cover varying degrees of skew. The factors used, in tabular form, are given in Table 8. * * *

"The factors in Table 8 were reached by an approximate procedure that will not be described. They are used as a means of drawing smooth curves and do not replace, but are used as an aid to, the purely graphical procedure. If

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1917), p. 670.
† *Loc. cit.*, p. 694.
‡ *Loc. cit.*, p. 1539.
§ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 191.
‖ *Loc. cit.*, p. 214.
¶ *Loc. cit.*, p. 218.

some member of the Society who enjoys mathematical problems would work out factors corresponding exactly with theoretical skew curves, it would be appreciated."

The writer's attention was brought to this subject by a study of the papers by Messrs. Hazen and Hall, and by the suggestions quoted. The general subject of probability and its application to engineering work is very broad, and the references noted previously should not be considered as complete.

The derivation of the frequency curves, as given in this paper, is taken largely from "Frequency Curves and Correlation", by W. Palin Elderton. As stated in Mr. Hall's paper, the method was developed originally by Professor Karl Pearson, of University College, London, England. The writer has attempted to present Elderton's mathematical analysis in a brief outline, only the essential formulas being discussed in detail, and has shown the application of the formulas to engineering problems.

Frequency Curves

Definition.—"If statistics are arranged so as to show the number of times, or frequency with which, an event happens in a particular way, then the arrangement is called a frequency distribution." The curve or formula used to describe such distributions is called a "frequency curve."

There are many kinds of frequency distributions. A tabulation of the number of persons in every 100 000 who die at various ages, or the number of students of various ages in a certain grade in school, would be a frequency distribution. Such a distribution of stream run-off data would show, of the total for which there were records, the number of years during which the run-off had a certain value, or the number of years during which the run-off fell between certain limits. For example, such a distribution might show the number of years of every 100 that the run-off was between 0 and 500 sec-ft., 500 and 1 000 sec-ft., 1 000 and 1 500 sec-ft., etc.

The frequency curve should be distinguished from the duration curve as ordinarily used in stream-flow study. The duration curve shows the number of years during which the run-off was greater than a certain quantity; the frequency curve shows the number of years that the run-off just equalled a certain quantity. Evidently, if all the items represented by the frequency curve are added together, the total number of items in the series will be obtained. If all the items in the frequency distribution, that have values less than a given quantity, are added together, the number of items which have the corresponding value on the duration curve will be obtained. Hence, any point on the duration curve represents the sum of all the frequencies to the corresponding point on the frequency curve. This relation will be further considered subsequently.

Typical Form.—The frequency distributions that are generally found have the same typical form. They usually start with zero frequency, then rise to a maximum value, and again decrease finally to zero. Fig. 1 (*A*) gives two examples of this typical shape. The ordinates represent frequencies, and the abscissas show values of annual run-off expressed in terms of the mean annual

run-off. The typical frequency curve is generally tangent to the base at both the lower and upper ends.

The point at which the curve reaches its maximum ordinate is called the "mode". This represents the value of run-off which occurs most frequently. For example, Curve *A* shows that there are more years in the series that have a run-off equal to 0.96 of the mean, than there are years with a run-off of any other value.

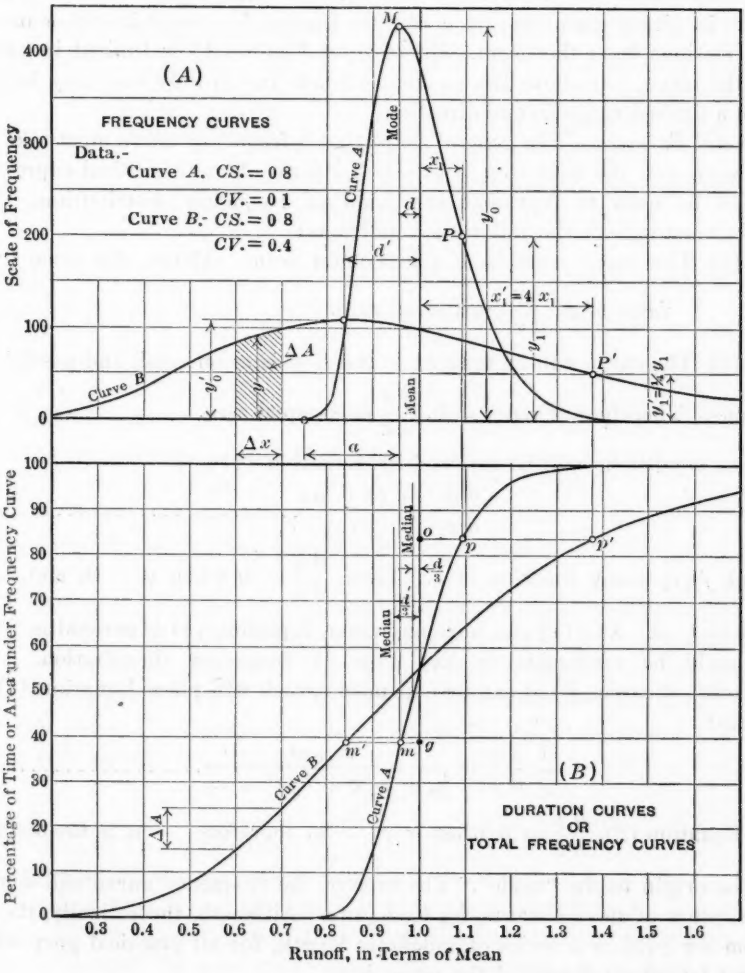

FIG. 1.

The "mean" or "average run-off" is the arithmetical average of all the run-off items. In the frequency series, the mean is found by multiplying each ordinate by its distance from a certain point (such as zero run-off), and dividing the sum of these products by the sum of the ordinates. In other

words, the mean ordinate passes through the center of gravity of the area under the frequency curve, or the mean is on the gravity axis of the curve.

The principal characteristic of most frequency curves is that they are not symmetrical, which indicates that there are more items on one side of the mean than on the other, and that the mode will not be located at the mean. When the mode and the mean coincide, the curve will be symmetrical. An unsymmetrical curve is called a "skew curve".

Another feature to be noticed is that the curve may extend to an indefinite distance in either direction; or it may be limited in extent in either one or both directions from the mean. Curve *A* of Fig. 1 (*A*) is limited in extent below the mean, but above the mean it extends indefinitely and may be said to have a limited range in one direction.

General Formula.—The general shape that a frequency curve must take has been shown and the next step is to select a form of mathematical expression that can be used to represent any kind of frequency distribution. The formula must satisfy the following conditions:

(1) The curve must have a maximum point. Hence, for some finite value of x, $\frac{d\,y}{d\,x}$ must equal zero.

(2) The curve will be tangent to the X-axis at one end, and possibly at both ends. Therefore, when $y = 0$, $\frac{d\,y}{d\,x}$ must equal zero.

These conditions will be satisfied by Equation (1):

$$\frac{d\,y}{d\,x} = \frac{y\,(x + a)}{f\,(x)} \qquad (1)$$

in which $f\,(x)$ is any function of x. Then, $\frac{d\,y}{d\,x} = 0$ when $y = 0$, and, also, when $x = -a$. As $f\,(x)$ can have any form, Equation (1) is general in form, and should be applicable to any type of frequency distribution. By Maclaurin's theorem, $f\,(x)$ can be expanded, which will place Equation (1) in the form:

$$\frac{1}{y}\frac{d\,y}{d\,x} = \frac{x + a}{b_0 + b_1\,x + b_2\,x^2 + \ldots} \qquad (2)$$

In Equation (2), $\frac{d\,y}{d\,x} = 0$ when $x = -a$, therefore, $-a$ is the distance from the origin to the "mode". The type of the frequency curve will depend on the values of the constants, b_0, b_1, b_2, etc. Although, theoretically, the expression for $f\,(x)$ is a series of indefinite length, for all practical purposes it need not be extended beyond the term, $b_2\,x^2$.

Equation (2) is a differential equation of the frequency curve. To obtain the actual curve, the expression,

$$\frac{x + a}{b_0 + b_1\,x + b_2\,x^2}$$

must be integrated. The final equation, which will be in the form, $y = f\,(x)$,

will depend on the actual values of b_0, b_1 and b_2, which are constants the values of which must be related to the shape of the frequency curve. Certain general types which the frequency curve may assume, are:

I.—Limited range in both directions, and skew
II.—Limited range in both directions, and symmetrical
III.—Limited range in one direction, and skew
IV.—Unlimited range in both directions, and skew
V.—Unlimited range in both directions, and symmetrical.

All frequency curves may be included in these five general types. In stream-flow studies, attention can be limited to Type III—limited range in one direction only, and skew—as it is obvious that the run-off cannot have a value less than zero, and no definite upper limit can be assigned.

With these conditions in mind, it can be shown that the constant, b_2, in Equation (2), must equal zero.* Equation (2) then becomes:

$$\frac{1}{y}\frac{dy}{dx} = \frac{x+a}{b_0 + b_1 x}$$

When this equation is integrated, it can be written in the form:

$$y = y_0 \cdot e^{-\gamma x} \times \left[\frac{1+x}{a}\right]^{\gamma a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which,

y = ordinate or frequency corresponding to the run-off, x;
e = base of Napierian logarithms; and
y_0, γ, and a are constants.

In Equation (3) a has a meaning different from that implied in Equation (1).

Equation (3) can be used to represent any frequency distribution which has a limited range in one direction, and is unsymmetrical. The actual shape of the curve will depend on the three constants involved. For the particular case in which the distribution is unlimited in both directions, and is symmetrical, it can be shown that, in Equation (2), $b_1 = b_2 = 0$. When it is integrated, Equation (2) then becomes:

$$y = y_0 \cdot e^{\frac{-x^2}{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which y_0 and c are constants.

Equation (4) is the equation for the "normal curve of error".

Standard Variation.—On Fig. 1 two curves plotted from Equation (3) are given. As was shown previously, the sum of all the ordinates of the frequency curve must equal the total number of items considered, or the total number of years. In Fig. 1 (A), the total number of years is considered as 100, hence, the area under each curve must equal 100. If the ordinates, y, are scaled at equal intervals, Δx, then the product, $y \Delta x$, will be the area of the vertical strip of width, Δx. This area, ΔA, will be the frequency of occurrence of run-off between the limits covered by Δx. The sum of all the separate areas, or the whole area, will be 100 years.

* Proof of this step is given by Elderton, but is omitted here for the sake of brevity.

It will be seen that, although Curve B has the same area as Curve A, the former is spread out much farther from the mean. The extent of this spreading is measured by the "standard variation", which is determined as follows: The area of each vertical strip, each equal in width to Δx, is multiplied by the square of its distance from the mean, and the sum of these products is divided by the whole area to give the (standard variation)^2. That is,

$$(\text{Standard variation})^2 = \frac{\Sigma\,(y \,.\, \Delta x)\, x^2}{\Sigma\,(y \,.\, \Delta x)} = \frac{\int_{-a}^{\alpha} y\, x^2\, d\, x}{\int_{-a}^{\alpha} y\, d\, x}$$

It will be noticed that this equation is the same as the formula for the square of the "radius of gyration" of an area. In fact, the standard variation is the radius of gyration of the area under the frequency curve taken about the gravity axis.

The "coefficient of variation", or $c\,v$, is obtained by dividing the standard variation by the mean. If, originally, all the run-off values are expressed in terms of the mean, the standard variation will equal the coefficient of variation.

To define any frequency curve, it is necessary to know not only its spread laterally from the mean (that is, its standard variation), but also to have some indication of its lack of symmetry. Having two curves with the same standard variation, but with the mode at different distances from the mean, it can be stated that the skewness is proportional to that distance. If the distance to the mode, however, is the same in two curves the standard variation of which is different, it will be evident that the curve the standard variations of which are the larger will be the more symmetrical. Hence, it will be convenient to define the skewness of the curve, as follows:

$$\text{Skewness} = \frac{\text{mean} - \text{mode}}{\text{standard variation}}.$$

Solution of Frequency Curves.—In most of the discussion that follows, the given frequency series is considered to represent stream run-off. This is for convenience only, as providing something definite on which to base the theoretical analysis. The principles and formulas that are considered will apply, however, to any kind of unsymmetrical frequency distribution.

A general formula for the unsymmetrical frequency curve has been derived. It remains to find some way in which this formula can be applied to represent any actual frequency series. A frequency diagram for a given run-off series can be plotted, but the problem is to project a curve through the given values in accordance with the theoretical formula. This is most conveniently done by the "method of moments".

If, in the given curve, each area strip is multiplied by its distance from the mean, the sum of the products will be what is called the "first area moment of the series" about the mean; that is,

$$\text{First moment} = \int_{-a}^{x} x\, y\, d\, x.$$

Likewise, if each area strip is multiplied by the square of its distance from the mean, the sum of the products will be the "second area moment of the

series", and the sum of the products of each area strip multiplied by the cube of its distance from the mean will be the "third area moment of the series".

Thus:

$$\text{Second moment} = \int_{-a}^{\alpha} x^2 y \, d x$$

$$\text{Third moment} = \int_{-a}^{\alpha} x^3 y \, d x$$

Dividing each moment by the total area under the curve, we have,

$$\frac{\text{First moment}}{\text{Total area}} = \mu_1 = \frac{\int_{-a}^{\alpha} x y \, d x}{\int_{-a}^{\alpha} y \, d x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\frac{\text{Second moment}}{\text{Total area}} = \mu_2 = \frac{\int_{-a}^{\alpha} x^2 y \, d x}{\int_{-a}^{\alpha} y \, d x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\frac{\text{Third moment}}{\text{Total area}} = \mu_3 = \frac{\int_{-a}^{\alpha} x^3 y \, d x}{\int_{-a}^{\alpha} y \, d x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

μ_1, μ_2 and μ_3 are called the first, second, and third "moment coefficients" of the curve. As the moments are taken about the mean, or gravity axis of the area, $\mu_1 = 0$, as was shown previously. Also, μ_2 equals standard variation, equals radius of gyration. There is no counterpart in Mechanics to the coefficient, μ_3, but as will be shown subsequently, μ_3 is a measure of the skewness of the curve.

In the same manner, moment coefficients can also be computed for the actual run-off series, obtaining the mean of the series, the standard variation, and μ_3 for the series. If such values are now given to the constants in Equation (3), that the resulting values of μ_1, μ_2, and μ_3, as defined by Equations (5), (6), and (7), will be equal to the corresponding values as computed for the actual series, the theoretical curve will have all the essential characteristics possessed by the series, and will represent the series in a satisfactory manner. This is the method used by Elderton in evaluating the constants in Equation (3). The details of the analysis will be omitted for the sake of brevity.

In Equation (3), let,

$p = \gamma \cdot a$

$z = \gamma\,(a + x) = p + \gamma\, x$; and

N = total area under the frequency curve.

Then, Equation (3) can be written:

$$y = y_0 \, e^{\,p-z} \cdot \frac{z^p}{p^p}$$

$$= \frac{y_0 \cdot e^p}{p^p} \cdot z^p \cdot e^{-z}$$

By letting

$$c = \frac{y_0 \cdot e^p}{p^p},$$

this equation becomes:

$$y = c \cdot z^p e^{-z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

It can be shown that,

$$y_0 = \frac{N \cdot p^{p+1}}{a \cdot e^p \Gamma(p+1)}$$

Therefore,

$$c = \frac{N p}{a \Gamma(p+1)} = \frac{N \gamma}{\Gamma(p+1)}$$

The "gamma function" of $(p + 1)$ is called $\Gamma(p + 1)$, values of which are tabulated in texts on integral calculus. $\Gamma(p + 1) = p \cdot \Gamma(p)$. As a table of this function may not be available to some engineers, Table 1, of log. $\Gamma(p)$, taken from "Frequency Curves and Correlation," by W. Palin Elderton, previously mentioned, is given.

The constants in Equation (8) are computed by the following:

$$\gamma = \frac{2 \mu_2}{\mu_3}$$

$$a = \frac{2 \mu_2^2}{\mu_3} - \frac{\mu_3}{2 \mu_2}$$

$$p = \frac{4 \mu_2^3}{\mu_3^2} - 1$$

$$\text{Maximum ordinate} = y_0 = \frac{N}{a} \frac{p^{p+1}}{e^p \cdot \Gamma(p+1)} = \frac{c \cdot p^p}{e^p}$$

N equals the total area under the frequency curve, equals 100% of time.

When $x = -a$, $y = 0$. Hence, $-a$ defines the lower limit of the curve. When $z = p$, $\frac{dy}{dx} = 0$. Hence, $z = p$ gives the position of the mode. As $z = p + \gamma x$, when $z = p$, $x = 0$. Therefore, the mode is the origin for x.

The distance from mean to mode $= d = \frac{1}{2} \frac{\mu_3}{\mu_2} = \frac{1}{\gamma}$. μ_2 and μ_3 are determined from the actual data as follows:

$$\mu_2 = \frac{\Sigma \text{ (second moments about mean)}}{N} = (c\,v)^2.$$

If the coefficient of skew is defined as $\frac{\Sigma \text{ (third moments)}}{N (c\,v)^3}$,

then,

$$\text{coefficient of skew} = c\,s = \frac{\mu_3}{(c\,v)^3}$$

and,

$$\mu_3 = (c\,s)(c\,v)^3$$

TABLE 1.—Of Log $\Gamma(p)$.

p	0		1	2	3	4	5	6	7	8	9
1.00	....	.....	9 750	9 500	9 251	9 003	8 755	8 509	8 263	8 017	7 773
1.01	1.99	7 529	7 285	7 043	6 801	6 560	6 320	6 080	5 841	5 602	5 365
1.02	1.99	5 128	4 892	4 656	4 421	4 187	3 953	3 721	3 489	3 257	3 026
1.03	1.99	2 796	2 567	2 338	2 110	1 883	1 656	1 430	1 205	0 981	0 757
1.04	1.99	0 533	0 311	0 089	9 868	9 647	9 427	9 208	8 989	8 772	8 554
1.05	1.98	8 338	8 122	7 907	7 692	7 478	7 265	7 053	6 841	6 629	6 419
1.06	1.98	6 209	6 000	5 791	5 583	5 376	5 169	4 963	4 758	4 553	4 349
1.07	1.98	4 145	3 943	3 741	3 539	3 338	3 138	2 939	2 740	2 541	2 344
1.08	1.98	2 147	1 951	1 755	1 560	1 365	1 172	0 978	0 786	0 594	0 403
1.09	1.98	0 212	0 022	9 833	9 644	9 456	9 269	9 082	8 896	8 710	8 525
1.10	1.97	8 341	8 157	7 974	7 791	7 610	7 428	7 248	7 068	6 888	6 709
1.11	1.97	6 531	6 354	6 177	6 000	5 825	5 650	5 475	5 301	5 128	4 955
1.12	1.97	4 783	4 612	4 441	4 271	4 101	3 932	3 764	3 596	3 429	3 262
1.13	1.97	3 096	2 931	2 766	2 602	2 438	2 275	2 113	1 951	1 790	1 629
1.14	1.97	1 469	1 309	1 150	0 992	0 835	0 677	0 521	0 365	0 210	0 055
1.15	1.96	9 901	9 747	9 594	9 442	9 290	9 139	8 988	8 838	8 688	8 539
1.16	1.96	8 390	8 243	8 096	7 949	7 803	7 658	7 513	7 369	7 225	7 082
1.17	1.96	6 939	6 797	6 655	6 514	6 374	6 234	6 095	5 957	5 818	5 681
1.18	1.96	5 544	5 408	5 272	5 137	5 002	4 868	4 734	4 601	4 469	4 337
1.19	1.96	4 205	4 075	3 944	3 815	3 686	3 557	3 429	3 302	3 175	3 048
1.20	1.96	2 922	2 797	2 672	2 548	2 425	2 302	2 179	2 057	1 936	1 815
1.21	1.96	1 695	1 575	1 456	1 337	1 219	1 101	0 984	0 867	0 751	0 636
1.22	1.96	0 521	0 407	0 293	0 180	0 067	9 955	9 843	9 732	9 621	9 511
1.23	1.95	9 401	9 292	9 184	9 076	8 968	8 861	8 755	8 649	8 544	8 439
1.24	1.95	8 335	8 231	8 128	8 025	7 923	7 821	7 720	7 620	7 520	7 420
1.25	1.95	7 321	7 223	7 125	7 027	6 930	6 834	6 738	6 642	6 547	6 453
1.26	1.95	6 359	6 267	6 173	6 081	5 989	5 898	5 807	5 716	5 627	5 537
1.27	1.95	5 449	5 360	5 273	5 185	5 099	5 013	4 927	4 842	4 757	4 673
1.28	1.95	4 589	4 506	4 423	4 341	4 259	4 178	4 097	4 017	3 938	3 858
1.29	1.95	3 780	3 702	3 624	3 547	3 470	3 394	3 318	3 243	3 168	3 094
1.30	1.95	3 020	2 947	2 874	2 802	2 730	2 659	2 588	2 518	2 448	2 379
1.31	1.95	2 310	2 242	2 174	2 106	2 040	1 973	1 907	1 842	1 777	1 712
1.32	1.95	1 648	1 585	1 522	1 459	1 397	1 336	1 275	1 214	1 154	1 094
1.33	1.95	1 035	0 977	0 918	0 861	0 803	0 747	0 690	0 634	0 579	0 524
1.34	1.95	0 470	0 416	0 362	0 309	0 257	0 205	0 153	0 102	0 051	0 001
1.35	1.94	9 951	9 902	9 853	9 805	9 757	9 710	9 663	9 617	9 571	9 525
1.36	1.94	9 480	9 435	9 391	9 348	9 304	9 262	9 219	9 178	9 136	9 095
1.37	1.94	9 054	9 015	8 975	8 936	8 898	8 859	8 822	8 785	8 748	8 711
1.38	1.94	8 676	8 640	8 605	8 571	8 537	8 503	8 470	8 437	8 405	8 373
1.39	1.94	8 342	8 311	8 280	8 250	8 221	8 192	8 163	8 135	8 107	8 080
1.40	1.94	8 053	8 026	8 000	7 975	7 950	7 925	7 901	7 877	7 854	7 831
1.41	1.94	7 808	7 786	7 765	7 744	7 723	7 703	7 683	7 664	7 645	7 626
1.42	1.94	7 608	7 590	7 573	7 556	7 540	7 524	7 509	7 494	7 479	7 465
1.43	1.94	7 451	7 438	7 425	7 413	7 401	7 389	7 378	7 368	7 358	7 348
1.44	1.94	7 338	7 329	7 321	7 312	7 305	7 298	7 291	7 284	7 278	7 273
1.45	1.94	7 268	7 263	7 259	7 255	7 251	7 248	7 246	7 244	7 242	7 241
1.46	1.94	7 240	7 239	7 239	7 240	7 241	7 242	7 243	7 245	7 248	7 251
1.47	1.94	7 254	7 258	7 262	7 266	7 271	7 277	7 282	7 289	7 295	7 302
1.48	1.94	7 310	7 317	7 326	7 334	7 343	7 353	7 363	7 373	7 384	7 395
1.49	1.94	7 407	7 419	7 431	7 444	7 457	7 471	7 485	7 499	7 514	7 529
1.50	1.94	7 545	7 561	7 577	7 594	7 612	7 629	7 647	7 666	7 685	7 704
1.51	1.94	7 724	7 744	7 764	7 785	7 806	7 828	7 850	7 873	7 896	7 919
1.52	1.94	7 943	7 967	7 991	8 016	8 041	8 067	8 093	8 120	8 146	8 174
1.53	1.94	8 201	8 229	8 258	8 287	8 316	8 346	8 376	8 406	8 437	8 468
1.54	1.94	8 500	8 532	8 564	8 597	8 630	8 664	8 698	8 732	8 767	8 802
1.55	1.94	8 837	8 873	8 910	8 946	8 983	9 021	9 059	9 097	9 135	9 174
1.56	1.94	9 214	9 254	9 294	9 334	9 375	9 417	9 458	9 500	9 543	9 586
1.57	1.94	9 629	9 672	9 716	9 761	9 806	9 851	9 896	9 942	9 989	0 035
1.58	1.95	0 082	0 130	0 177	0 225	0 274	0 323	0 372	0 422	0 472	0 522
1.59	1.95	0 573	0 624	0 676	0 728	0 780	0 833	0 886	0 939	0 993	1 047
1.60	1.95	1 102	1 157	1 212	1 268	1 324	1 380	1 437	1 494	1 552	1 610
1.61	1.95	1 668	1 727	1 786	1 845	1 905	1 965	2 025	2 086	2 147	2 209
1.62	1.95	2 271	2 333	2 396	2 459	2 522	2 586	2 650	2 715	2 780	2 845
1.63	1.95	2 911	2 977	3 043	3 110	3 177	3 244	3 312	3 380	3 449	3 517
1.64	1.95	3 587	3 656	3 726	3 797	3 867	3 938	4 010	4 081	4 154	4 226
1.65	1.95	4 299	4 372	4 446	4 519	4 594	4 668	4 743	4 819	4 894	4 970
1.66	1.95	5 047	5 124	5 201	5 278	5 356	5 434	5 513	5 592	5 671	5 740
1.67	1.95	5 830	5 911	5 991	6 072	6 154	6 235	6 317	6 400	6 482	6 566
1.68	1.95	6 649	6 733	6 817	6 901	6 986	7 072	7 157	7 243	7 322	7 416

Note.—The characteristic of each log is negative. Thus, log $\Gamma(1.50) = 1.947545 = 9.947545 - 10$.

TABLE 1.—(*Continued*).

p	0		1	2	3	4	5	6	7	8	9
1.69	1.95	7 503	7 590	7 678	7 766	7 854	7 943	8 032	8 122	8 211	8 301
1.70	1.95	8 391	8 482	8 573	8 664	8 756	8 848	8 941	9 034	9 127	9 220
1.71	1.95	9 314	9 409	9 502	9 598	9 693	9 788	9 884	9 980	0 077	0 174
1.72	1.96	0 271	0 369	0 467	0 565	0 664	0 763	0 862	0 961	1 061	1 162
1.73	1.96	1 262	1 363	1 464	1 566	1 668	1 770	1 873	1 976	2 079	2 183
1.74	1.96	2 287	2 391	2 496	2 601	2 706	2 812	2 918	3 024	3 131	3 238
1.75	1.96	3 345	3 453	3 561	3 669	3 778	3 887	3 996	4 105	4 215	4 326
1.76	1.96	4 436	4 547	4 659	4 770	4 882	4 994	5 107	5 220	5 333	5 447
1.77	1.96	5 561	5 675	5 789	5 904	6 019	6 135	6 251	6 367	6 484	6 600
1.78	1.96	6 718	6 835	6 953	7 071	7 189	7 308	7 427	7 547	7 666	7 787
1.79	1.96	7 907	8 028	8 149	8 270	8 392	8 514	8 636	8 759	8 882	9 005
1.80	1.96	9 129	9 253	9 377	9 501	9 626	9 751	9 877	0 003	0 129	0 255
1.81	1.97	0 383	0 509	0 637	0 765	0 893	1 021	1 150	1 279	1 408	1 538
1.82	1.97	1 668	1 798	1 929	2 060	2 191	2 322	2 454	2 586	2 719	2 852
1.83	1.97	2 985	3 118	3 252	3 386	3 520	3 655	3 790	3 925	4 061	4 197
1.84	1.97	4 333	4 470	4 606	4 744	4 881	5 019	5 157	5 295	5 434	5 573
1.85	1.97	5 712	5 852	5 992	6 132	6 273	6 414	6 555	6 697	6 838	6 980
1.86	1.97	7 123	7 266	7 408	7 552	7 696	7 840	7 984	8 128	8 273	8 419
1.87	1.97	8 564	8 710	8 856	9 002	9 149	9 296	9 443	9 591	9 739	9 887
1.88	1.98	0 036	0 184	0 333	0 483	0 633	0 783	0 933	1 084	1 234	1 386
1.89	1.98	1 537	1 689	1 841	1 994	2 147	2 299	2 453	2 607	2 761	2 915
1.90	1.98	3 069	3 224	3 379	3 535	3 690	3 846	4 003	4 159	4 316	4 474
1.91	1.98	4 631	4 789	4 947	5 105	5 264	5 423	5 582	5 742	5 902	6 062
1.92	1.98	6 223	6 383	6 544	6 706	6 867	7 029	7 192	7 354	7 517	7 680
1.93	1.98	7 844	8 007	8 171	8 336	8 500	8 665	8 830	8 996	9 161	9 327
1.94	1.98	9 494	9 660	9 827	9 995	0 162	0 330	0 498	0 666	0 835	1 004
1.95	1.99	1 173	1 343	1 512	1 683	1 853	2 024	2 195	2 366	2 537	2 709
1.96	1.99	2 881	3 054	3 227	3 399	3 573	3 746	3 920	4 094	4 269	4 443
1.97	1.99	4 618	4 794	4 969	5 145	5 321	5 498	5 674	5 851	6 029	6 206
1.98	1.99	6 384	6 562	6 740	6 919	7 098	7 277	7 457	7 637	7 817	7 997
1.99	1.99	8 178	8 359	8 540	8 722	8 903	9 085	9 268	9 450	9 633	9 816

Now, substitute these values in the formulas for the constants in Equation (8). Then,

$$\gamma = \frac{2\,\mu_2}{\mu_3} = \frac{2}{(c\,s)\,(c\,v)}$$

$$d = \frac{1}{\gamma} = \frac{c\,s\,.\,c\,v}{2}$$

$$\text{"Skewness"} = \frac{d}{c\,v} = \frac{1}{2}\ \text{(coefficient of skew)}.$$

$$p = \frac{4\,\mu_2^{\,3}}{\mu_3^{\,2}} - 1 = \frac{4\,(c\,v)^6}{(c\,s)^2\,(c\,v)^6} - 1 = \frac{4}{(c\,s)^2} - 1$$

$$a = [4 - (c\,s)^2]\,\frac{c\,v}{2\,c\,s} = \frac{p}{\gamma}$$

The distance from the lower limit to the mean equals $a + d$:

$$(a + d) = \frac{p+1}{\gamma} = \frac{4}{(c\,s)^2}\,.\,\frac{c\,s\,.\,c\,v}{2} = \frac{2\,c\,v}{c\,s}$$

These formulas enable one to compute all the constants involved in Equation (8), as soon as the coefficient of variation and the coefficient of skew which the curve is to possess, are known.

Curve A of Fig. 1 (A) was computed from Equation (3), or Equation (8), taking $c\,v = 0.10$ and $c\,s = 0.80$. The constants for this curve are computed as follows:

$$\gamma = \frac{2}{c\,s\,.\,c\,v} = \frac{2}{0.08} = \frac{1}{0.04} = 25.0$$

$$d = \frac{1}{\gamma} = 0.04$$

$$p = \frac{4}{(c\,s)^2} - 1 = \frac{4}{0.64} - 1 = 6.25 - 1 = 5.25$$

$$a = \frac{p}{\gamma} = 0.21 \qquad a + d = \frac{2\,c\,v}{c\,s} = \frac{0.20}{0.80} = 0.25$$

$$N = 100\ \% \qquad c = \frac{N\,.\,\gamma}{\Gamma\,(p+1)} \qquad p + 1 = 6.25$$

Now,

$$\Gamma\,(p+1) = p\,\Gamma\,(p) = \Gamma\,(6.25) = 5.25\,.\,\Gamma\,(5.25)$$
$$= 5.25 \times 4.25\,.\,\Gamma\,(4.25)$$
$$= 5.25 \times 4.25 \times 3.25 \times 2.25 \times 1.25\,.\,\Gamma\,(1.25)$$

log. 5.25	= 0.720159		
" 4.25	= 0.628389	log. N	= 2.000000
" 3.25	= 0.511883	" γ	= 1.397940
" 2.25	= 0.352182		3.397940
" 1.25	= 0.096910		
log. $\Gamma\,(1.25)$	= 9.957321 − 10	log. $\Gamma\,(p+1)$	= 2.266844
log. $\Gamma\,(p+1)$	= 2.266844	log. c	= 1.131096

By Equation (8),

$$y = c\,z^p\,.\,e^{-z},$$

in which

$$z = \gamma\,(a + x) = \gamma\,[a + d + (x - d)]$$

or

$$z = 25\,[0.25 + (x - d)] = 25\,[0.25 + x_1],$$

in which

$$x_1 = x - d.$$

Hence, by assuming values for x_1, the corresponding values of y can readily be computed.

In plotting the curve, the following points should be noted: The origin for x is the mode and not the mean. Also, the run-off must be expressed in terms of the mean, in order to apply the formulas involving $c\,s$ and $c\,v$ given previously. As d is distance from mean to mode, it is more convenient to express abscissas in terms of $x - d$, which can be called x_1. In this manner, the formula for z can be expressed as

$$z = \gamma\,[(a + d) + (x - d)],$$

as shown previously, or

$$z = \gamma\,[(a + d) + x_1]$$

The curve is limited in its extent below the mean by the distance, $(a + d)$, as $y = 0$ when $x = -a$. The curve is unlimited in extent above the mean, but beyond some definite point the ordinates become so small that it is practically tangent to the X-axis. This matter is considered subsequently, in the practical application of the formulas.

Influence of c v and c s on Frequency Curve.—It has been shown how a frequency curve can be computed for any value of $c\,v$ and $c\,s$. Consider, now, what effect these two coefficients have on the shape of the curve:

$$\gamma = \frac{2}{c\,s \,.\, c\,v} \qquad d = \frac{1}{\gamma} = \frac{c\,s \,.\, c\,v}{2}$$

$$p = \frac{4}{(c\,s)^2} - 1 \qquad a + d = \frac{2\,c\,v}{c\,s}. \qquad a = \frac{p}{\gamma}.$$

First, assume that $c\,s$ is a constant, and consider the effect of changing the value of $c\,v$. Then $p =$ constant; γ varies inversely with $c\,v$; a varies directly which $c\,v$; d varies directly with $c\,v$; and the lower range, $(= a + d)$, varies directly with $c\,v$.

As

$$c = \frac{N\,\gamma}{\Gamma\,(p+1)}$$

c varies inversely with $c\,v$; and as

$$y_0 = \text{maximum ordinate} = \frac{c\,p^p}{e^p};$$

y_0 varies inversely with $c\,v$.

$$z = \gamma\,[(a + d) + (x - d)].$$

Hence, if $x - d$ is varied directly as $c\,v$, z will remain constant, and y will vary inversely with $c\,v$. That is, keeping $c\,s$ constant, y will vary inversely with $c\,v$ when $(x - d)$, or x_1, is varied directly with $c\,v$. This characteristic is illustrated in Fig. 1 (*A*). The two curves, *A* and *B*, have the same $c\,s$ $(= 0.80)$; but the $c\,v$ for Curve *B* is four times that for Curve *A*. Therefore, the ordinate, y_1, on Curve *A*, is four times the ordinate, y'_1, on Curve *B* when the abscissa on Curve *B* is four times the abscissa on Curve *A*. Notice, also, that $y_0 = 4\,y'_0$ and $d' = 4\,d$.

When the coefficient of variation is held constant, d varies directly with $c\,s$, and p decreases as $c\,s$ increases. Also, $(a + d)$ varies inversely with $c\,s$. The maximum ordinate, y_0, will increase when $c\,s$ increases. These features are illustrated in Fig. 2 on which six curves with the same $c\,v$ are plotted, having different values for $c\,s$.

Theoretically, the curve cannot extend farther below the mean than zero run-off; or the limiting value of $a + d$ is 1.0. This means that $c\,s > 2\,c\,v$, or, $c\,v < \frac{c\,s}{2}$. In other words, when $c\,v$ increases to larger values, $c\,s$ must also increase, as $c\,s$ must always be larger than $2\,c\,v$.

In the method of computing $c\,v$ and $c\,s$, which was explained previously, it was assumed that the data were arranged as a frequency distribution. In most run-off computations, it is more convenient to arrange the data as a duration series, and the area moments must be computed in a slightly different manner. Instead of dividing the area under the frequency curve into vertical strips of equal width, vertical strips of equal area, but of variable width, may be used. By multiplying each of these equal areas by its distance from the

mean, the first area moment can be obtained, and, similarly, for the other moments. The final result will be the same as if the moments had been computed by Equations (5), (6), and (7).

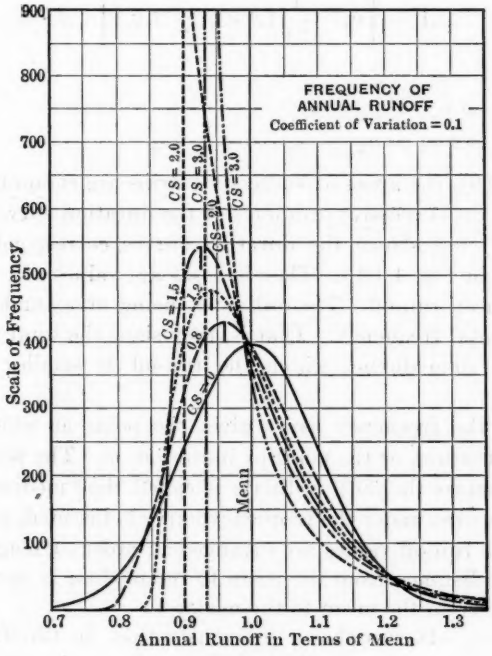

FIG. 2.

DURATION CURVES

The distinction between frequency and duration curves has been noted, and it has been shown that any point on the duration curve represents the sum of all the frequencies to the corresponding point on the frequency curve. Also, the area, ΔA, of any vertical strip under the frequency curve with width, Δx, shows the frequency of occurrence of run-off between the limits covered by Δx. Hence, any point on the duration curve indicates the summation of the areas under the frequency curve to the corresponding ordinate. The duration curve, therefore, is represented by the integral of the frequency, and its equation is:

$$y = \int y_0 e^{-\gamma x} \left(1 + \frac{x}{a}\right)^{\gamma a} d x$$

This equation, in its general form, cannot be solved directly. In order, therefore, to obtain values of different ordinates of the duration curve, some approximate method of computing areas under the frequency curve must be adopted. Elderton gives a method which is very convenient for this purpose.

Computation of Areas.—A series of ordinates is computed for the frequency curve, these ordinates being spaced at equal intervals, Δx. Let three consecu-

tive ordinates be y_{-1}, y_0, and y_{+1}. Then the area of the vertical strip extending to the distance, $\frac{1}{2} \Delta x$, on each side of y_0, is given as,

$$\Delta A = \left[y_0 - \frac{1}{24} (\Delta y_{-1} - \Delta y_0) \right] . \Delta x \ldots\ldots\ldots\ldots\ldots (9)$$

in which,

$$\Delta y_{-1} = y_0 - y_{-1}$$
$$\Delta y_0 = y_{+1} - y_0$$

By Equation (9), the areas of successive strips are computed, and by summing these areas, the successive ordinates of the duration curve are obtained.

In Fig. 1 (*B*) are shown the duration curves corresponding to the two frequency curves in Fig. 1 (*A*). The abscissas are values of run-off, expressed in terms of the mean run-off. The ordinates, being areas under the frequency curve, represent total frequency. That is, they show the total number of years or percentage of time during which the run-off is smaller than a certain quantity.

The mode of the frequency curve gives the point at which the duration curve changes curvature, or the point of inflection, *m*. The point at which the duration curve crosses the 50% ordinate is called the "median". This is the point at which the area under the frequency curve is bisected, and is the middle point when all the run-off values are arranged in order of magnitude. It may be noted that the distance from the mean to the median is approximately one-third the distance from the mean to the mode.

Influence of c v.—It was shown previously that, in two frequency curves which have the same *c s*, the ordinates will be inversely proportional to the *c v* when the abscissas, x_1, are directly proportional to the *c v*, the abscissas being measured from the mean. From this, it follows that if Point p' on Curve *B* (Fig. 1 (*A*)), has an abscissa four times the abscissa to Point *p* on Curve *A*, then the area under Curve *B* from the mean to p' must be equal to the area under Curve *A* from the mean to *p*, as the *c v* of Curve *B* is four times that of Curve *A*. Hence, the abscissas of the corresponding points, p' and *p*, of the duration curves, (Fig. 1 (*B*)), must bear the same relation.

Therefore, the distance from the mean to any point on the duration curve is directly proportional to the coefficient of variation. This fact is very important, because it enables the duration curves for all streams having the same coefficient of skew to be computed as soon as the curve for one particular coefficient of variation has been found. In Fig. 1 (*B*),

$$\frac{o\ p'}{o\ p} = \frac{g\ m'}{g\ m} = \frac{4}{1}$$

Influence of c s.—In Fig. 3 are shown the duration curves corresponding to the frequency curves in Fig. 2. All these curves have the same *c v*, but different values of *c s*. It is interesting to note that the points at which these duration curves cross the mean are spaced on the percentage-of-time scale proportionately to the difference in *c s*.

Computation of Duration Curves.—By Equation (3), the formula for the frequency curve is:

$$y = y_0 \, e^{-\gamma x} \left(1 + \frac{x}{a}\right)^{\gamma a}$$

or, by Equation (8),

$$y = c \, z^p \; e^{-z}$$

when

$$p = \gamma a;$$
$$z = \gamma a + \gamma x; \text{ and,}$$
$$d z = \gamma \, d x.$$

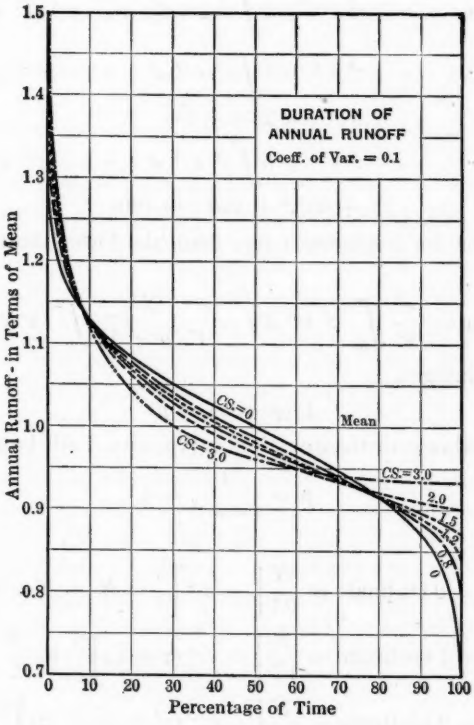

FIG. 3.

As shown previously, the ordinates to the duration curve represent the areas under the frequency curve, and are given by the formula:

$$\text{Area} = \int y_0 \, e^{-\gamma x} \left(1 + \frac{x}{a}\right)^{\gamma a} d \, x$$

Substituting the values given previously, this equation becomes:

$$\text{Area} = \frac{1}{\gamma} \int c \, z^p \, e^{-z} \, d \, z = \frac{c}{\gamma} \int z^p \, e^{-z} \, d \, z$$

But,

$$\frac{c}{\gamma} = \frac{N}{\Gamma (p+1)} \; \therefore \; \text{Area} = \frac{N}{\Gamma (p+1)} \int z^p \, e^{-z} \, d \, z$$

Now,

$$\int z^p\, e^{-z}\, dz = -z^p\, e^{-z} + p \int z^{p-1}\, e^{-z}\, dz$$

When

$p = 0, \int z^p\, e^{-z}\, dz = -e^{-z}$

$p = 1, \quad " \quad = -z\, e^{-z} + \int e^{-z}\, dz = -e^{-z}\,(z+1)$

$p = 2, \quad " \quad = -z^2\, e^{-z} + 2\int z\, e^{-z}\, dz = -e^{-z}\,(z^2 + 2z + 2)$

$p = 3, \quad " \quad = -z^3\, e^{-z} + 3\int z^2\, e^{-z}\, dz = -e^{-z}\,(z^3 + 3z^2 + 6z + 6)$

$p = 4, \quad " \quad = -z^4\, e^{-z} + 4\int z^3\, e^{-z}\, dz = -e^{-z}\,(z^4 + 4z^3 + 12z^2 + 24z + 24)$

$p = 5, \quad " \quad = -z^5\, e^{-z} + 5\int z^4\, e^{-z}\, dz = -e^{-z}\,(z^5 + 5z^4 + 20z^3 + 60z^2 + 120z + 120)$

The area under the frequency curve from the lower limit, where $z = 0$, to $z = z_1$ is

$$\text{Area} = \frac{c}{\gamma}\int_0^{z_1} z^p\, e^{-z}\, dz = \frac{N}{\Gamma(p+1)}\int_0^{z_1} z^p\, e^{-z}\, dz$$

When p is an integer,

$$\Gamma(p+1) = p!$$

Therefore, the ordinate to the duration curve, at z_1, will be as follows:

$$p = \frac{4}{(c\,s)^2} - 1$$

When

$C\,S = 2.0 \quad p = 0$ Ordinate $= \frac{N}{0!}\left[-e^{-z}\right]_0^{z_1} = N - N\,e^{-z_1}.$

$C\,S = \sqrt{2}\,p \quad = 1$ Ordinate $= \frac{N}{1!}\left[-e^{-z}(z+1)\right]_0^{z_1} = \frac{N}{1} - \frac{N}{1}e^{-z_1}(z_1+1)$

$C\,S = \frac{2}{3}\sqrt{3}\,p = 2$ Ordinate $= \frac{N}{2!}\left[-e^{-z}(z^2+2z+2)\right]_0^{z_1}$

$$= N - \frac{N}{2}\left(z_1^2 + 2z_1 + 2\right)$$

$C\,S = 1.0 \quad p = 3$ Ordinate $= \frac{N}{3!}\left[-e^{-z}(z^3+3z^2+6z+6)\right]$

$$= N - \frac{N}{6}\left[e^{-z_1}(z_1^3 + 3z_1^2 + 6z_1 + 6)\right]$$

$C\,S = \frac{2}{5}\sqrt{5}\,p = 4$ Ordinate $= \frac{N}{4!}\left[-e^{-z}(z^4+4z^3+12z^2+24z+24)\right]_0^{z_1}$

$$= N - \frac{N}{24}e^{-z_1}(z_1^4 + 4z_1^3 + 12z_1^2 + 24z_1 + 24)$$

$$C S = \frac{\sqrt{6}}{3} \quad p = 5 \text{ Ordinate} = \frac{N}{5!}\Big[- e^{-z} (z^5 + 5 z^4 + 20 z^3 + 60 z^2 + 120 z + 120)\Big]$$

$$= N - \frac{N}{120} e^{-z_1} (z_1^5 + 5 z_1^4 + 20 z_1^3 + 60 z_1^2 + 120 z_1 + 120)$$

It is evident, therefore, that when p is an integral number, the ordinates to the duration curve can be computed directly, and it will not be necessary to resort to approximate methods for computing areas under the frequency curve.

Using the formulas just given, ordinates to the duration curve were computed for values of p equal to 0, 1, 3, and 5, and for $c\,v = 0.1$. These values were plotted, giving duration curves for $c\,s$ of 2.0, $\sqrt{2}$, 1.0, and $\frac{\sqrt{6}}{3}$.

It has been shown that, for any given $c\,s$, the distance from the mean to any point on the duration curve is directly proportional to the $c\,v$. Therefore, from these curves, the $c\,v$ of which is 0.1, can be obtained corresponding curves for any other $c\,v$.

By the method of approximate integration, the duration curve for $c\,v = 0.1$ and $c\,s = 3.0$ was computed. Curves for other values of $c\,s$ were then interpolated between the curves already computed. The results are given in Table 2. In this table are given, opposite each value of $c\,s$, first, the number of terms greater than the mean, or the percentage of time at which the duration curve crosses the mean. Then, for a particular percentage of time, is given the distance from the mean to the corresponding point on the duration curve, which may be called the variation from the mean. This variation is given in terms of the $c\,v$, or for $c\,v = 1.0$. For any other value of $c\,v$, the given variation should be multiplied by the $c\,v$. Negative variations are measured down from the mean and positive variations are measured above the mean. By means of Table 2, the duration curve for any given $c\,v$ and $c\,s$ can readily be plotted.

The factors in Table 2 correspond to those given by Mr. Hazen in his Table 8.*

Symmetrical Frequency Curve

In Equation (4) is given the formula for the symmetrical curve, or "normal curve of error":

$$y = y_0 e^{\frac{-x^2}{c}}$$

In this equation, the constants are determined as follows:

$$c = 2\,\mu_2 = 2\,(c\,v)^2$$

$$y_0 = \frac{N}{\sqrt{2\,\pi\,\mu_2}} = \frac{100}{(c\,v)\,\sqrt{2\,\pi}}$$

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 219.

TABLE 2.—Run-off Variation in Terms of Coefficient of Variation.

Skew Curve Factors to be Multiplied by Coefficient of Variation and Added to or Subtracted from Mean.

Coefficient of skew.	Terms above mean, in percent.	Variation from Mean, in Terms of c v Percentage of Time =												
		0.1	1.0	5	10	20	30	40	50	60	70	80	90	95	99.0	99.9
0.0	50.0	+3.09	+2.33	+1.65	+1.28	+0.84	+0.52	+0.25	0.00	−0.25	−0.52	−0.84	−1.28	−1.65	−2.33	−3.09
0.2	48.7	3.38	2.48	1.69	1.30	0.83	0.51	0.32	−0.03	0.28	0.55	0.85	1.25	1.58	2.18	2.81
0.4	47.3	3.67	2.62	1.74	1.32	0.82	0.48	0.19	0.06	0.31	0.57	0.85	1.22	1.51	2.03	2.54
0.6	46.0	3.96	2.77	1.79	1.33	0.80	0.45	0.15	0.09	0.34	0.58	0.86	1.19	1.45	1.88	2.28
0.8	44.7	4.25	2.90	1.83	1.34	0.78	0.42	0.12	0.13	0.37	0.60	0.86	1.16	1.38	1.74	2.03
1.0	43.3	4.54	3.03	1.87	1.34	0.76	0.38	0.08	0.16	0.40	0.61	0.86	1.12	1.31	1.59	1.80
1.2	42.0	4.82	3.15	1.90	1.35	0.74	0.35	0.05	0.19	0.42	0.62	0.85	1.08	1.25	1.45	1.59
1.4	40.7	5.11	3.28	1.93	1.34	0.71	0.32	+0.02	0.22	0.44	0.63	0.84	1.05	1.18	1.32	1.40
1.6	39.4	5.39	3.40	1.96	1.33	0.68	0.28	−0.01	0.25	0.46	0.64	0.83	1.00	1.11	1.19	1.24
1.8	38.1	5.66	3.50	1.98	1.32	0.64	0.24	0.05	0.28	0.48	0.64	0.80	0.95	1.04	1.08	1.11
2.0	36.8	5.93	3.60	2.00	1.30	0.61	0.21	0.08	0.30	0.49	0.64	0.78	0.90	0.97	0.99	1.00
2.2	35.5	6.20	3.70	2.01	1.28	0.58	0.17	0.11	0.33	0.49	0.63	0.75	0.85	0.90	0.90	0.91
2.4	34.3	6.47	3.78	2.01	1.25	0.54	0.13	0.14	0.35	0.50	0.62	0.71	0.79	0.82	0.83	0.83
2.6	33.0	6.73	3.87	2.01	1.23	0.51	0.10	0.17	0.37	0.50	0.60	0.68	0.74	0.76	0.77	0.77
2.8	31.9	6.99	3.95	2.02	1.20	0.47	0.06	0.20	0.38	0.50	0.59	0.65	0.70	0.71	0.71	0.71
3.0	30.8	+7.25	+4.02	+2.02	+1.18	+0.42	+0.03	−0.23	−0.40	−0.50	−0.57	−0.62	−0.65	−0.66	−0.67	−0.67

or, using the same expressions as in Least Squares, in which $\epsilon = c\,v$:

$$c = 2\,\varepsilon^2\,;\; y_0 = \frac{100}{\varepsilon\,\sqrt{2\,\pi}}\,;\text{ and } y = \frac{100}{\varepsilon\,\sqrt{2\,\pi}}\,e^{\frac{-x^2}{2\,\varepsilon^2}}$$

The coefficient of variation is the same as the "mean square error". These equations can be used directly in computing ordinates for the frequency curve and the duration curve, as previously described for the unsymmetrical curve.

Equation (4) can be simplified by substituting:

$$t^2 = \frac{x^2}{2\,\varepsilon^2}\,;\; t = \frac{x}{\varepsilon\,\sqrt{2}}$$

Then,

$$y = \frac{100}{\varepsilon\,\sqrt{2\,\pi}}\,e^{-t^2}$$

and the equation for the duration curve is,

$$p_t = \int y\,d\,x = \frac{100}{\sqrt{\pi}}\int_0^t e^{-t^2}\,d\,t$$

in which, p_t equals probability or percentage of time that the run-off will differ from the mean by any value lying between zero and t.

In Least Squares, it is shown that the "probable error," or r, equals $0.4769\left[\varepsilon\,\sqrt{2}\right]$, therefore,

$$r = 0.4769.\,\frac{x}{t}$$

and

$$\frac{t}{0.4769} = \frac{x}{r}.$$

In Merriman's "Method of Least Squares" is given a table which shows the values of the integral, $\frac{1}{\sqrt{\pi}}\int_{-t}^{+t} e^{-t^2}\,.\,d\,t$, for various values of $\frac{t}{0.4769}$, or $\frac{x}{r}$. The values in this table give the area under the frequency curve between the points corresponding to $-t$ and $+t$. If the values in the table are divided by 2, the area from zero to $+t$ will be obtained.

Ordinates for the duration curve can be computed from Professor Merriman's table. Thus, the 90% point on the percentage-of-time scale is 40% from the mean. The area under the frequency curve to this point is then 40%, or 0.4, if the whole area is considered as 1.0. The corresponding value of $\frac{x}{r}$ will be found in the table for the Area 0.8, which gives $\frac{x}{r} = 1.9$; but,

$$r = 0.4769\,.\,\varepsilon\,.\,\sqrt{2} = 0.675\,\varepsilon$$

therefore,

$$x_{90} = 1.9\,r = 1.9 \times 0.675\,\varepsilon = 1.282\,\varepsilon$$

Similarly, for $p_t = 99\%$; area $= 0.49$. From the table, under the area, 2×0.49, or 0.98, we get,

$$\frac{x}{r} = 3.450$$

and

$$x_{99} = 3.45 \times 0.675\ \varepsilon = 2.33\ \varepsilon.$$

Other values of x can be computed in the same manner, giving the duration curve in terms of the coefficient of variation.

Probability Paper

The probability paper* used by Mr. Hazen is constructed in such a manner that when a series of values, which vary in accordance with the normal law of error, are plotted as functions of the probability of their occurrence, the resulting points will lie on a straight line. The X-axis is graduated in terms of the probability, or percentage of time; but the spacing of the successive graduations is varied in accordance with the values of t corresponding to the values of $(p)_0{}^t$ in the formula,

$$(p)_0{}^t = \frac{1}{\sqrt{\pi}} \int_0^t e^{-t^2} \cdot d\,t.$$

The values of $(p)_0{}^t$ are given in Professor Merriman's table for values of $\frac{x}{r}$, as noted previously. It is these values of $\frac{x}{r}$ which Mr. Hazen uses in plotting his probability paper. With this paper, a series which is distributed exactly according to the normal law of error will plot as a straight line passing through the point at which the mean crosses the 50% line. The inclination of the resulting line to the X-axis will be directly proportional to the coefficient of variation.

The duration curves already mentioned have been plotted on probability paper in Fig. 4. The curve with $c\,s = 0$, being a symmetrical curve, is a straight line in this diagram. The other curves are not straight; and their curvature increases with the coefficient of skew.

Application to Run-off Study

Thus far, this study has been conducted from a theoretical standpoint. The development has been indicated of a formula by which a frequency curve, and its corresponding duration curve, can be computed for any coefficient of skew and coefficient of variation. The question now is to decide whether the use of these formulas is justified to represent the run-off of a stream, the $c\,v$ and $c\,s$ of which have been computed from a series of actual run-off observations

If the run-off observations are plotted as a frequency or duration diagram, they will not fall on a smooth curve. The problem, therefore, is to determine the form of curve which will give the most logical representation of the original series. The method of moments, as used in the derivation of the

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1666.

formulas, seems to give a satisfactory solution of this problem. The resulting curve will have the same characteristics in regard to distribution as the original series. The area of the frequency curve will equal the number of original observations, and the coefficient of variation and skewness of the observations will be repeated in the curve.

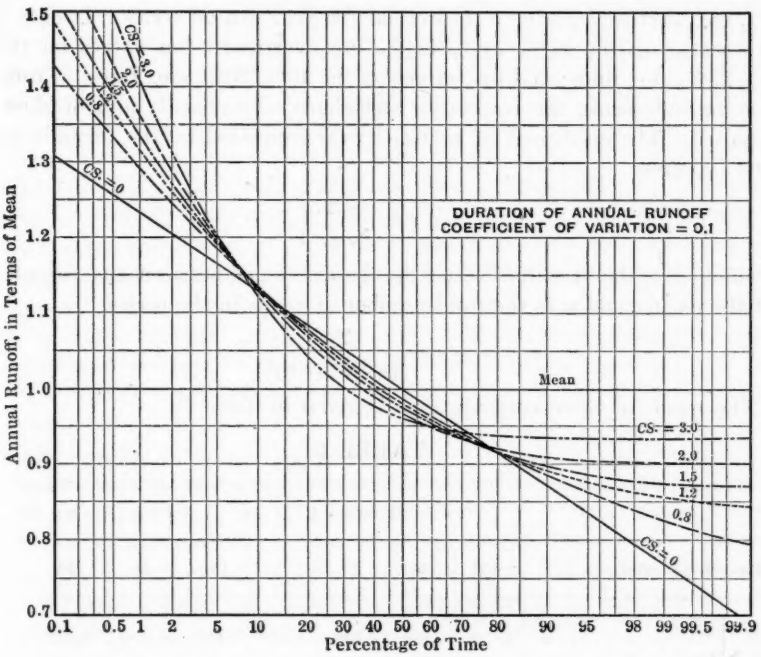

FIG. 4.

One of the important uses of the theoretical curve is to extend a limited number of observations so as to cover a longer period of time. A ten-year record of stream flow is too short for many purposes of study. The theoretical curve provides a method of making such an extension. The shape of the curve is determined by the values of $c\,s$ and $c\,v$, which are computed from the ten-year record. The theoretical curve will have a definite lower limit, which is required by practical considerations. Theoretically, there is no upper limit to the curve. In any actual case, however, it will be found that, above a certain point, the ordinates to the frequency curve become so infinitesimal that they may be considered equal to zero. For example, in the computation of the duration curve for $c\,s = 1.0$ and $c\,v = 0.1$, it was found that 99.9997% of the area under the frequency curve was comprised between the lower limit of 0.80 times the mean and 1.80 times the mean; that is, a run-off of 1.80 would be exceeded only once in about 167 000 years. Although, theoretically, the curve has no upper limit, there is a practical limit in any case, and the curve also satisfies the essential requirement that there can be no definite upper limit to any run-off series.

Accuracy of Coefficients.—Before expressing any conclusions regarding the applicability and limitations of these methods, it will be proper to check the accuracy of the curves that have been computed. For this purpose the curves for $c\,s = 0.60$ and $c\,v = 0.10$ and for $c\,s = 1.20$ and $c\,v = 0.10$ were selected. These duration curves were plotted on probability paper and four sets of values were scaled from them. One hundred values were scaled, on the ½%, 1½%, 2½%, etc., verticals, giving a theoretical 100-year run-off series. Twenty values were scaled on the 2½%, 7½%, 12½%, etc., verticals; ten values on the 5%, 15%, 25%, etc., lines; and five values on the 10%, 30%, etc., lines. From these three run-off series, the respective coefficients of variation and of skew were computed. The coefficient of variation was computed by the formula used in Least Squares:

$$c\,v = \sqrt{\frac{\Sigma\, v^2}{n-1}},$$

in which, v is the "residual" (that is, the difference between any run-off value and the mean) and n is the total number of years in the series,

$$c\,s = \frac{\Sigma\, v^3}{(n-1)\,(c\,v)^3}.$$

The result of these computations is given in Table 3.

TABLE 3.

	$c\,s = 0.6$; $c\,v = 0.1$.				$c\,s = 1.2$; $c\,v = 0.1$.			
Length of series = n =	100	20	10	5	100	20	10	5
$c\,v = \sqrt{\frac{\Sigma\, v^2}{n-1}}$ ..... =	0.0996	0.0988	0.0980	0.0950	0.0997	0.0980	0.0965	0.0916
Error in $c\,v$ ..... =	0.0004	0.0012	0.0020	0.0050	0.0003	0.0020	0.0035	0.0084
$c\,s = \frac{\Sigma\, v^3}{(n-1)\,(c\,v)^3}$ ..... =	0.552	0.425	0.308	0.102	1.065	0.813	0.583	0.365
Error in $c\,s$ ..... =	0.048	0.175	0.292	0.498	0.135	0.387	0.617	0.835

These results illustrate the important fact that the accuracy of the coefficients that are computed from any run-off series will increase when the length of the series is increased. The computations given in Table 3 assume, of course, that the run-off record follows the theoretical curve exactly. An actual record will not correspond precisely to the theoretical curve; and the coefficients, as computed from an actual record, therefore, would probably show greater variations than are indicated in Table 3.

Assuming the theoretical series to be correct, it is reasonable to expect that if a series of very great length was taken, the computed values of $c\,v$ and $c\,s$ would equal the values assumed in the computation of the duration curve.

An examination of Table 3 will show the following to be true:

(1) The coefficient of variation is only slightly affected by the length of the series; thus, in the 5-year period, the error in $c\,v$ is only 5% for $c\,s = 0.6$ and only 8% for $c\,s = 1.2$.

(2) The coefficient of skew is greatly affected by the length of the series, and only a very long record would give a value for $c\,s$, which could be considered very accurate.

(3) The error in the coefficients increases as $c\,s$ increases; and the error in either $c\,s$ or $c\,v$ is, approximately, proportional to $c\,s$.

It may be concluded, therefore, that the coefficient of variation of a stream could be computed with considerable accuracy from a record of moderate length, and a good approximation might even be obtained from a short record. The coefficient of skew, however, cannot be determined with any degree of precision except from a very long record.

There is one fact, however, that tends to counteract this difficulty. An examination of Fig. 3 will show that, between 2% and 98% of the time, the duration curve is not very greatly affected by changes in the $c\,s$. Therefore, if certain limits can be determined between which the $c\,s$ for the streams under consideration will probably lie, an average $c\,s$ can be assumed as applied to all the streams, and the duration curve adopted for this average $c\,s$ as the typical curve for all the streams. Although any particular stream might prove to have a $c\,s$ somewhat different from the adopted value, the duration curve that would be drawn for that stream would be sufficiently accurate for any practical work.

Relation of $c\,v$ to $c\,s$ in Actual Records.—It has been shown that, with a particular $c\,s$, the $c\,v$ cannot have a larger value than $\frac{1}{2}\,c\,s$. This suggests that, in actual records, there may be some relation between $c\,v$ and $c\,s$. In Mr. Hall's paper,* previously referred to, is given a table of corresponding values of $c\,v$ and $c\,s$, as averaged for groups of five streams. These values are plotted in Fig. 5 as open circles. In Fig. 13,† of his paper, Mr. Hall gives a series of duration curves for various coefficients of variation, as determined empirically from his study of the records of a large number of streams. The percentage of time at which one of these curves crosses the mean is an index of the $c\,s$ which will apply to this curve, and the proper value of $c\,s$ may be determined from the percentages given in the second column of Table 2 in this paper. These values of $c\,s$ are plotted as solid circles in Fig. 5, in which is also drawn the line representing the equation, $c\,s = 2\,c\,v$. It will be seen that the curves in Mr. Hall's Fig. 13 satisfy this relation for $c\,s$ less than 2.0. The points plotted from Mr. Hall's Table 19 are more scattered, as might be expected because of the fact that each point represents an average for only five streams.

Allowing for the fact that the $c\,s$ as computed from a record of moderate length may not be very accurate, it seems reasonable to conclude that the $c\,s$ for any particular stream will be about twice the $c\,v$.

In applying these methods to the study of run-off in the Adirondack Section of New York State, it was decided to adopt an average $c\,s$ as applying throughout that region. As a guide in selecting this average $c\,s$, seven streams, with fairly long records and having a drainage area of more than 100 sq. miles, were selected. The computation of a weighted average $c\,s$ is shown in Table 4 ‡

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 249, Table 19.

† *Loc. cit.*, p. 244.

‡ The values for the Hudson River are from data of the New York Water Power Investigation. Other data are taken from Mr. Hazen's Table 4, *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 217.

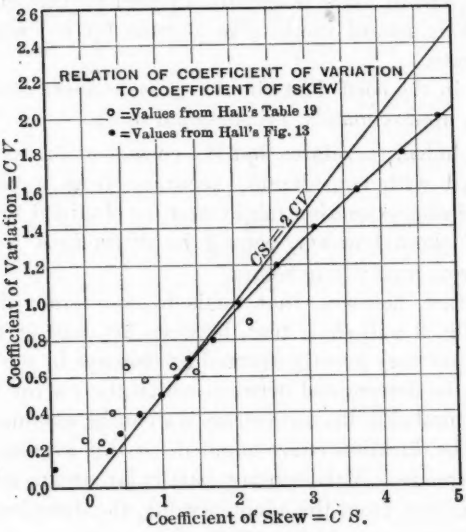

FIG. 5.

TABLE 4.

Stream.	Drainage area, in square miles.	$c\,s$.	Years in record = n.	$c\,s \times n$.
Hudson River	4 500	0.75	32	24.00
Croton River	375	0.41	45	18.45
Neshaminy Creek	139	0.62	28	17.35
Perkiomen Creek	152	0.56	28	15.68
Tohickon Creek	102	0.51	29	14.79
West Branch of Susquehanna River	5 670	0.47	21	9.87
Wachusett River	109	0.63	22	13.86
	.....	....	205	114.00

Weighted average, $c\,s = \frac{114\ 00}{205} = 0.556$.

As the $c\,s$, computed from a short record, has a tendency to be too small, a value of $c\,s = 0.60$ was adopted as typical of the streams of New York State.

One point should be noted regarding the computation of $c\,s$ and $c\,v$. In the derivation of the theoretical curves, $c\,v$ was determined by the formula,

$$(c\,v)^2 = \frac{\Sigma\, v^2}{n}.$$

In the formulas of least squares $c\,v$ is computed as equal to $\sqrt{\frac{\Sigma\, v^2}{n-1}}$. The reason for using n in one case and $n-1$ in the other is as follows: The coefficient of variation (or "mean square error") is computed by taking the squares of differences between the observed quantities and their true mean.

Now, the true mean is not known, and the observed mean is assumed to differ from the true mean by an amount measured by the "mean square error". By applying this correction to the observed differences, or residuals, the formula becomes,

$$c\,v = \sqrt{\frac{\Sigma\, v^2}{n-1}},$$

in which, v is the difference between an observed quantity and the observed mean.

Practical Examples.—In Figs. 6 to 10, inclusive, are given some examples of the practical application of the methods that have been presented. Figs. 6, 7, and 8 show the annual run-off values computed from the records of certain streams. The smooth curves drawn on Figs. 6 and 7 are obtained from the factors of Table 2, using a $c\,s$ of 0.6 and the proper $c\,v$ computed for the given stream. In these examples, the longer records show a closer agreement with the theoretical curve. The run-off record for the Murray River (Fig. 8), indicated that a $c\,s$ of 1.5 or 2.0 should be used, and the corresponding curves are drawn on the diagram. In this case, the $c\,s$ of 2.0 seems to be more satisfactory.

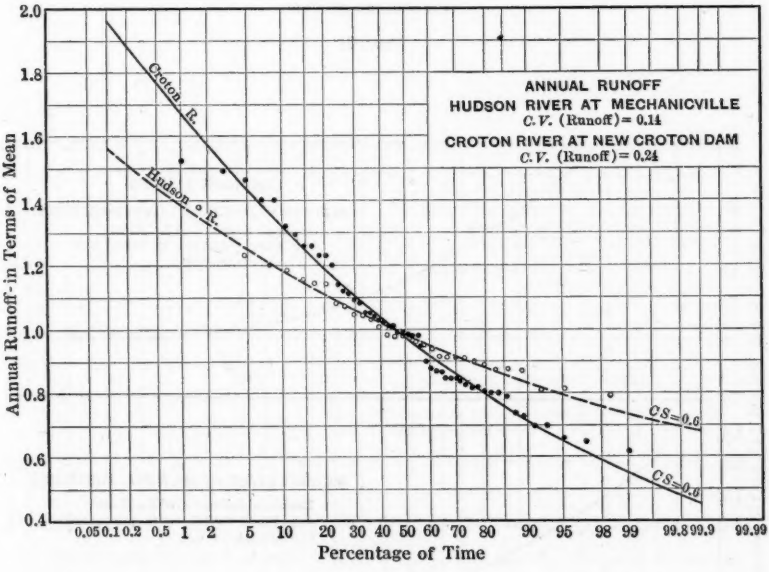

FIG. 6.

Fig. 9 gives two examples of the use of the curves with the records of a 24-hour flood flow, and Fig. 10 gives two examples of a 24-hour minimum flow. In Figs. 9 and 10, a $c\,s$ of 1.5 was adopted as giving the best results.

Numerical Example.—To illustrate the use of Table 2 more clearly, a numerical example will be given. The annual run-off record of West Canada Creek, for ten consecutive years, at Twin Rock Bridge, N. Y., is used. The

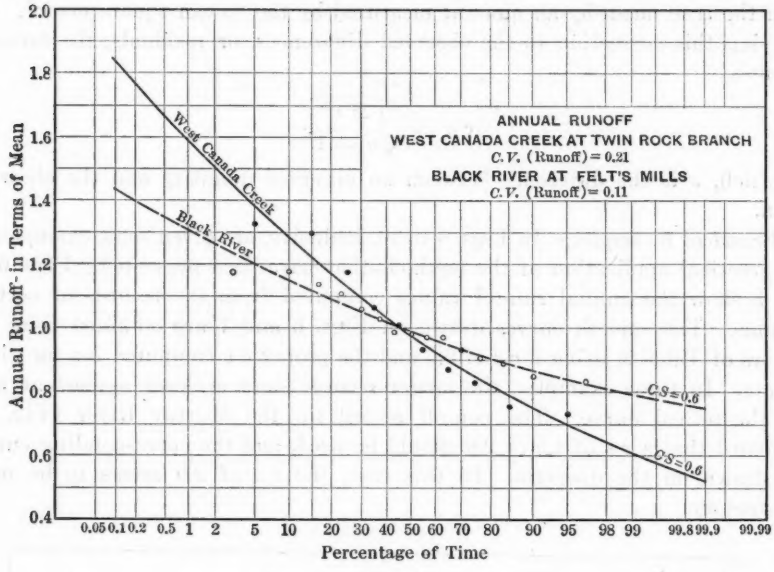
ANNUAL RUNOFF
WEST CANADA CREEK AT TWIN ROCK BRANCH
C.V. (Runoff) = 0.21
BLACK RIVER AT FELT'S MILLS
C.V. (Runoff) = 0.11
West Canada Creek
Black River
CS=0.6
CS=0.6
Annual Runoff in Terms of Mean
Percentage of Time
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99

FIG. 7.

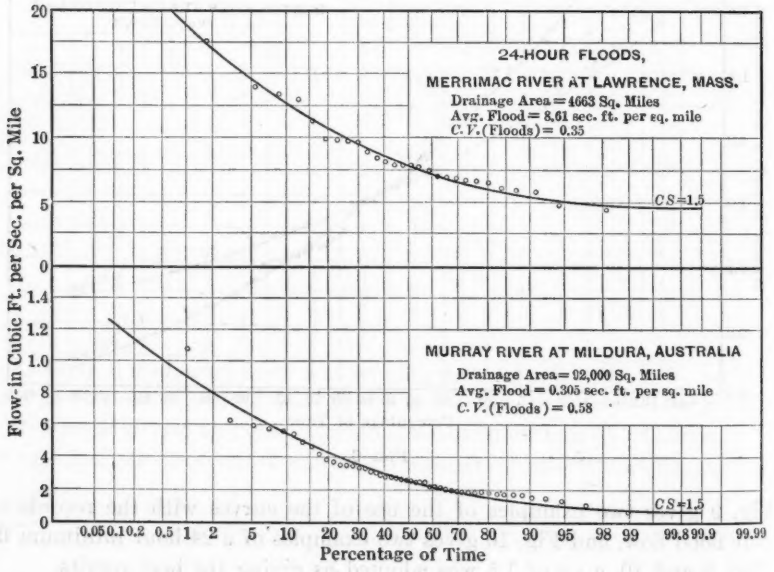
24-HOUR FLOODS,
MERRIMAC RIVER AT LAWRENCE, MASS.
Drainage Area = 4663 Sq. Miles
Avg. Flood = 8.61 sec. ft. per sq. mile
C.V. (Floods) = 0.35
CS=1.5
MURRAY RIVER AT MILDURA, AUSTRALIA
Drainage Area = 92,000 Sq. Miles
Avg. Flood = 0.305 sec. ft. per sq. mile
C.V. (Floods) = 0.58
CS=1.5
Flow in Cubic Ft. per Sec. per Sq. Mile
Percentage of Time
20
15
10
5
0
1.4
1.2
1.0
8
6
4
2
0
0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99

FIG. 8.

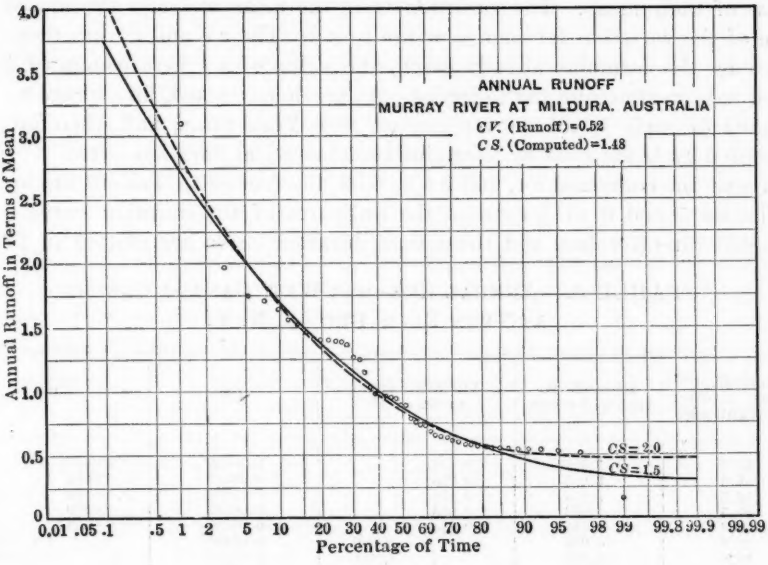
ANNUAL RUNOFF
MURRAY RIVER AT MILDURA, AUSTRALIA
C V. (Runoff) = 0.52
C S. (Computed) = 1.48
CS = 2.0
CS = 1.5
Annual Runoff in Terms of Mean
Percentage of Time
0.01 .05 .1 .5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0

FIG. 9.

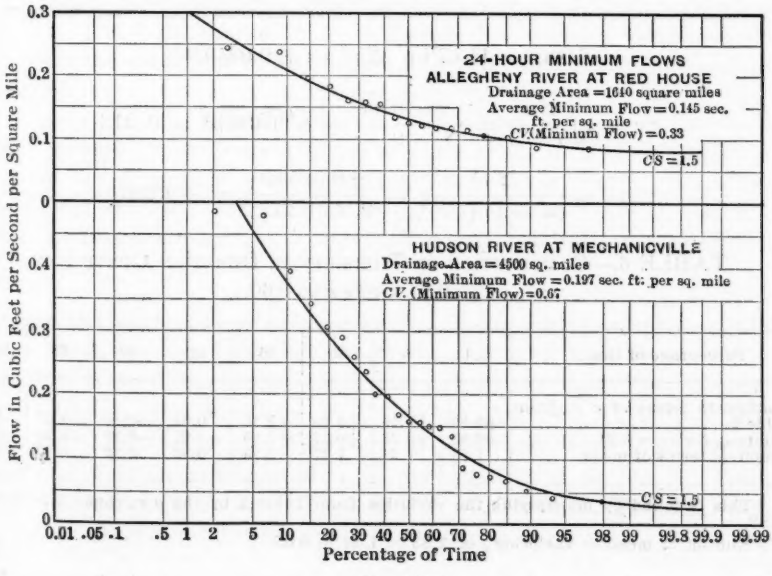
24-HOUR MINIMUM FLOWS
ALLEGHENY RIVER AT RED HOUSE
Drainage Area = 1640 square miles
Average Minimum Flow = 0.145 sec. ft. per sq. mile
CV (Minimum Flow) = 0.33
CS = 1.5
HUDSON RIVER AT MECHANICVILLE
Drainage Area = 4500 sq. miles
Average Minimum Flow = 0.197 sec. ft. per sq. mile
CV (Minimum Flow) = 0.67
CS = 1.5
Flow in Cubic Feet per Second per Square Mile
Percentage of Time
0.01 .05 .1 .5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
0 0.1 0.2 0.3 0.4 0 0.1 0.2 0.3

FIG. 10.

run-off values are arranged in order of magnitude, and are then expressed in terms of their mean. The variation from the mean and also the square and cube of the variation for each year are found. The $c\,v$ and $c\,s$ are then computed by the formulas already given, the value of $c\,s$ here computed being given only to illustrate the formulas. As previously noted, an average $c\,s$ was adopted to apply to all the streams of New York State, and, therefore, the computed $c\,s$ is not used to determine the theoretical duration curve.

Using the computed $c\,v$, and $c\,s = 0.60$, the factors in Table 2 are applied in Tables 5 and 6 to determine the ordinates of the duration curve. The recorded run-off values and theoretical duration curve are plotted in Fig. 7.

TABLE 5.—ANNUAL RUN-OFF; WEST CANADA CREEK AT TWIN ROCK BRIDGE, N. Y.

Run-off, in inches, in order of magnitude.	Run-off, in terms of mean.	Variation from mean = V.		V^2	V^3	
59.2	1.326	+0.326		0.10628	+0.034646	
57.9	1.296	0.296		0.08762	0.025984	
52.5	1.175	0.175		0.03062	0.005359	
47.5	1.063	0.063		0.00397	0.000250	
45.3	1.014	0.014		0.00020	0.000003	
41.7	0.934		−0.066	0.00436		−0.000287
38.8	0.869		0.131	0.01716		0.002248
37.1	0.830		0.170	0.02890		0.004913
33.9	0.759		0.241	0.05808		0.013997
32.8	0.734		0.266	0.07076		0.018821
446.7	10.000	+0.874	−0.874	0.40795	+0.066192	−0.040266

$$\text{Mean} = 44.67 \text{ in.} \quad \Sigma\, v^3 = +\,0.025926$$

$$c\,v = \sqrt{\frac{\Sigma\, v^2}{n-1}} = \sqrt{\frac{0.40795}{9}} = \sqrt{0.04533} = 0.213$$

$$c\,s = \frac{\Sigma\, v^3}{(n-1)\,(c\,v)^3} = \frac{+0.025926}{9 \times (0.213)^3} = +0.296$$

TABLE 6.—COMPUTATION OF THEORETICAL DURATION CURVE FOR $c\,v = 0.21$ AND $c\,s = 0.60$.

Percentage of time.	0.1	5	20	40	60	80	95	99.9
Variation, in terms of $c\,v$, from Table 2	+3.96	+1.79	+0.60	+0.15	−0.34	−0.86	−1.45	−2.28
*Variation, for $c\,v = 0.21$	+0.83	+0.38	+0.17	+0.03	−0.07	−0.18	−0.30	−0.48
†Run-off, in terms of mean	1.83	1.38	1.17	1.03	0.93	0.82	0.70	0.52

* This is found by multiplying the variation from Table 2 by the $c\,v$, thus: $3.96 \times 0.21 = 0.83$.

† Run-off = mean + variation; or $1.83 = 1.00 + 0.83$.

These examples may help to demonstrate that the methods just described are applicable to a variety of problems. In fact, any data that can be treated as a frequency series may be studied by these methods. The equation of the

frequency curve can be readily solved for the constants in Equation (8). The most important use of the methods, however, to engineers at least, will probably be in the plotting of duration curves. In many problems that confront the hydraulic engineer, the duration curve becomes an important factor. By the use of Table 2, as illustrated in the numerical example just given, the duration curve that should apply to any given data can be plotted with little effort, and without the necessity of solving any complicated formulas.

Conclusions

The mathematical analysis of skew frequency curves can be applied in the determination of the proper duration curve for stream run-off, or in the study of other similar data. To apply the formulas to any stream, the coefficients of variation and of skew for that stream must be computed, or assumed. The run-off duration curve can then be plotted by the factors given in Table 2. Although the mathematics involved may be rather difficult for some engineers, as Mr. Hazen suggests, the application of the methods by means of Table 2 is very simple, and should not result in any difficulty.

It is possible to compute the $c\,v$ with sufficient accuracy from a record of moderate length. Although the $c\,s$ cannot be determined accurately, an approximate value will generally be sufficient. If an average value of $c\,s$ can be assumed as applying to a group of streams, all the duration curves for these streams may be derived from the same set of factors; and the ordinates to the run-off curve measured from the mean will be directly proportional to the $c\,v$. The main limitation to be placed on the use of these curves is the fact that the coefficient of skew must be greater than twice the coefficient of variation.

Finally, due to the difficulties that have been mentioned, the theoretical curves should be used only as a guide in the study of a given record. If the data do not correspond even approximately with the theory, then, obviously, the theory cannot be applied to the data. Common sense is always a prerequisite to the application of any mathematical theory to practical work.

Acknowledgment

The subject-matter of this paper was developed in connection with the study of storage requirements on the streams of New York State. This study has been made in connection with the work of the New York Water Power Investigation, of which John P. Hogan, M. Am. Soc. C. E., is Consulting Engineer. The research in hydrology, of which this paper is a part, has been under the direct supervision of C. E. Sudler, M. Am. Soc. C. E. The writer is also indebted to E. E. Halmos and E. W. Maloney, Members, Am. Soc. C. E., for their helpful criticism of the paper.

THE DESIGN OF EARTH DAMS

By Joel D. Justin,* M. Am. Soc. C. E.

To Be Presented May 2, 1923.

Synopsis

The object of this paper is to aid in establishing more scientific methods of designing earth dams. Six criteria for the design of such dams are presented. These criteria are practically axiomatic, and most engineers will agree that an earth dam which satisfies each and all of them will be safe.

The design has been examined under each of the six criteria. The laws of the flow of water through soils, sands, and gravels are discussed, and their application to the problems of designing earth dams are explained. Approximate methods of determining, in advance of construction, the position of the line of saturation in the cross-section of an earth dam, are derived. Methods of computing the approximate seepage that will take place through and under earth dams under certain conditions are discussed and illustrative examples given. Methods for determining the necessary thickness of a given material for safety against blow-outs or piping are considered. It is believed that the methods of design herein presented should lead to safer and more economical earth dams, and furnish greater assurance to the engineer in regard to the actual margin of safety he is using.

Earth Dam Design

Theory has been little used in the past, in the design of earth dams. The successful designer has been governed by the lessons gained through experience. Earth dams that have stood the test of time have been widely copied. As regards safety, this procedure is good practice if all the conditions that existed at the site of the dam copied, also exist at the site where a dam is about to be built. This, however, is rarely the case. The soil at the proposed dam site may be greatly different from that of the foundation of the established dam. Indeed, a dam which has been in use many years and which has a high factor of safety, might prove to be unsafe if transported to another site, if such a thing was possible.

Experimental Research.—All scientific design has an experimental as well as a mathematical basis. Experimental data on the factors affecting the design of earth dams, although not as extensive as might be desired, are sufficient to serve as a sound, although not precise, basis of procedure.

The work of Hazen and Slichter in establishing and elucidating the laws of flow through sands, gravels, and soils will be referred to subsequently.

Note.—These papers are issued before the date set for presentation and discussion. Correspondence is invited, and may be sent by mail to the Secretary. Discussion on the paper will be closed in **September, 1923,** and, when finally closed, the paper, with discussion in full, will be published in *Transactions*.

* With Day & Zimmermann, Philadelphia, Pa.

J. B. T. Colman,* Assoc. M. Am. Soc. C. E., gives interesting data on the pressures of water under a model dam under various conditions, with a previous foundation. He also showed the effect of using sheet-piling in such a foundation.

In 1901, a board of consulting engineers made a series of tests of the dams of the Croton water-shed and located the lines of saturation in those structures.

James B. Hays,† Assoc. M. Am. Soc. C. E., gives some interesting and thorough investigations on the seepage and loss of head for certain materials. For the design and construction of the Wachusett dikes, some of the most thorough research work was conducted, which included extensive experiments on seepage through various materials. The work was done under the general direction of the late Frederic P. Stearns,‡ Past-President, Am. Soc. C. E.

J. Albert Holmes,§ M. Am. Soc. C. E., made extensive tests and experiments in connection with the design and construction of the Somerset Dam, in Vermont, and the Bridgewater Dams, in North Carolina.

C. M. Saville,‖ M. Am. Soc. C. E., gives the detailed results of extensive experiments made in connection with the design and construction of the Gatun Dam.

D. C. Henny,¶ M. Am. Soc. C. E., describes seepage experiments showing reduction in seepage due to the inclusion of vegetable matter in the soil.

Allen Hazen,** M. Am. Soc. C. E., describes tests made at the San Pablo Dam, California, to determine the coefficient of friction of various materials.

F. W. Scheidenhelm,†† M. Am. Soc. C. E., gives tests made to determine the coefficients of friction of various materials at the Stony River Dam.

Mr. Allen Hazen,‡‡, also gives the results of an extensive investigation at the Calaveras Dam after the slip, together with other experimental data dealing particularly with the problem of building earth dams by the hydraulic and semi-hydraulic-fill method.

Charles H. Paul,§§ M. Am. Soc. C. E., gives the results of tests and investigations in connection with the construction of the dams of the Miami Conservancy District, all of which were built by the hydraulic-fill method.

Scientific Methods of Design.—Of recent years engineers have brought to bear the scientific application of all pertinent data on the design of earth dams of magnitude and importance. To-day, an engineer of experience would not attempt to design and build an earth dam of magnitude without

* "The Action of Water Under Dams", *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 421.

† "Designing an Earth Dam Having a Gravel Foundation, with the Results Obtained in Tests on a Model", *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1.

‡ *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), p. 267.

§ "Some Investigations and Studies in Hydraulic-Fill Dam Construction", *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1922), p. 331.

‖ Report, Isthmian Canal Commission, 1908.

¶ *Engineering News*, Vol. 57, p. 251.

** *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1728.

†† *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 907.

‡‡ "Hydraulic-Fill Dams", *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1713.

§§ "Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1181.

first making a thorough study and examination of the surface and sub-surface conditions at the site and the materials available for constructing the dam.

Line of Saturation and Hydraulic Gradient.—The line of saturation may be defined as the uppermost line of flow of the water through the dam and subsoil. The hydraulic gradient, as applied to earth dams, may be defined as the gradient of a line joining the highest points to which water would rise in a series of pipes placed in the cross-section. This line is generally, but not necessarily, coincident with the line of saturation.

Materials in Earth Dams.—It is quite generally believed among those unfamiliar with the subject that to be safe an earth dam must be built of impervious material. Many of the textbooks still state that a dam should not be built unless an impervious material is available. That such is not the case, will be evident to any one who cares to examine the list of successful earth dams. Earth dams have been built successfully of loose rock, of sand of all degrees of fineness and coarseness, of gravel and silt, as well as of rock flour, soil, and clay. Assuming a thorough knowledge of all the available materials, it is possible to design and build a dam using such materials with safety.

Safety Against Sliding and Overturning.—Earth dams as well as masonry dams must be safe against sliding and overturning. It is difficult to conceive of an earth dam overturning as a single cohesive mass, but it is not so difficult to conceive of an earth dam sliding. Sliding has played a part in the failure of many earth dams. However, an earth dam which meets the other necessary requirements of design, will always be safe against overturning and, in almost all cases, will be safe against sliding. Accordingly, it is not generally necessary to investigate closely the safety of earth dams against sliding and overturning, as the factor of safety against failure from these causes is generally extremely high.

Criteria for the Design of Earth Dams

The practical criteria for the design of earth dams may be stated briefly as follows: An earth dam should be designed so that:

(1) The spillway capacity is so great that there is no danger of overtopping;

(2) The line of saturation is well within the down-stream toe;

(3) The up-stream and down-stream slopes must be such that, with the materials used in the construction, they will be stable under all conditions;

(4) There must be no opportunity for the free passage of water from the up-stream to the down-stream face;

(5) Water which passes through and under the dam, must, when it rises to the surface below the toe, have a velocity so small that it is incapable of moving any of the material of which the dam or its foundation is composed;

(6) The freeboard must be such that there is no danger of overtopping by wave action.

All rules have their exceptions, and it cannot be stated positively that any dam which does not meet fully all the foregoing criteria will be unsafe. It will be found, however, that with the few safe dams which do not fully meet all these criteria, some special means have been adopted to overcome the difficulty. For instance, the writer knows of one earth dam, 60 ft. high, founded on ledge rock with practically no soil cover. The engineer recognized that the down-stream toe would be saturated, under these conditions, and placed a heavy rock fill against the toe. A small quantity of water issues from the toe of the rock fill, but the velocity is so low that the material of which the dam is composed is not disturbed.

An earth dam, designed and built to meet these criteria, will prove as permanent as any of the works of man if proper attention is given to important details during construction.

Criterion 1

An earth dam should be designed with the spillway capacity so great that there is no danger of overtopping. One frequent cause of the failure of earth dams is the use of spillways of insufficient capacity. A masonry dam with an insufficient spillway will generally stand overtopping to a considerable depth without serious damage, but with an earth dam, overtopping means practically certain failure. Many earth dams are in use, that have spillways of insufficient capacity to care for floods which are certain to arrive sooner or later.

Weston E. Fuller,* M. Am. Soc. C. E., presents methods for determining the greatest probable flood. After determining the greatest flood to be expected in a long term of years, the spillway should be designed to take such a flood with a fair factor of safety.

Criterion 2

An earth dam should be designed so that the line of saturation is well within the down-stream toe. In Fig. 1, $A\,B\,C$, $A'\,B'\,C$, and $A''\,B''\,C$ are the lines of saturation. The line of saturation, $A\,B\,C$, is well inside the toe and is also well below the surface of the original ground below the dam. There is no question that such a dam is amply safe as regards Criterion 2.

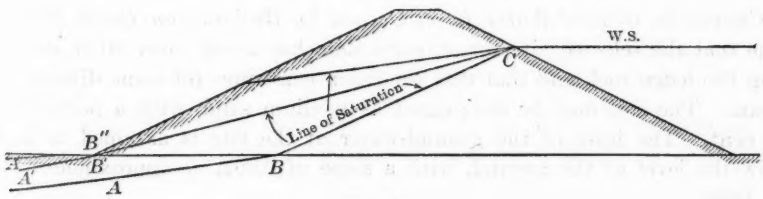

Fig. 1.

The line of saturation, $A'\,B'\,C$, intersects the down-stream face at or near the toe and is near the surface of the ground down stream from the dam. The danger of this condition would depend largely on the material composing

* "Flood Flows", *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.

the down-stream part of the dam and on the velocity of the water issuing from the down-stream toe. If the down-stream part of the dam was composed of fine sand or loam, boils at the toe might result, and cause the failure of the structure by piping. If the material in the down-stream part of the dam were composed of clay, the sloughing of the down-stream face would probably result and cause a partial or complete failure of the dam. If, however, the down-stream part of the dam was composed of coarse gravel and stones, and the velocity of the water issuing from the toe was insufficient to carry with it any of the material of which the dam was composed, the structure might be safe.

The line, *A″ B″ C*, Fig. 1, shows an extreme case. In an earth dam, built of any material except heavy rock, having a line of saturation as shown by *A″ B″ C*, failure would be impending by the sloughing of the down-stream face or by piping or by both. Piping as here used means the passage of water under or through an earth dam along a well-defined passageway and at a sufficient velocity to move the particles of which the dam is composed. When piping takes place at or down stream from the toe of the dam, the action is much like that of a boiling spring.

Factors Affecting the Position of the Line of Saturation.—The position and slope of the line of saturation in the cross-section of an earth dam is affected by many factors, such as:

(1) The porosity of the soil composing the dam and its foundation;
(2) The effective size of the particles composing the material of the dam and its foundation;
(3) The distribution of particles of various effective sizes throughout the cross-section;
(4) The depth of soil below the base of the dam;
(5) The flow of ground-water and its depth below the original surface at the site;
(6) The nature and depth of soil down stream from the dam;
(7) The existence of a core-wall, puddle-wall, or apron within the dam itself;
(8) The use of drains for collecting seepage in the down-stream part of the dam.

Change in Ground-Water Level Caused by Building an Earth Dam.—Assume that the selected site for an earth dam has a soil cover 80 ft. deep overlying the ledge rock and that this condition continues for some distance down stream. The soil may be designated as medium sand, with a porosity of 32 per cent. The level of the ground-water at the site is assumed to be 6 ft. below the level of the ground, with a slope of 0.002, or approximately 10 ft. per mile.

Fig. 2 is a cross-section at the site after the dam has been built and placed in service. The dam is assumed to be built of homogeneous material, practically the same as that at the site. As was to be expected, Fig. 2 shows that, due to the placing of the dam in service, there has been a general rise of the ground-water level in the vicinity. The slope of the ground-water

plane is also much increased, due to the increased head on the ground-water. The line of saturation, as shown, intersects the base well up stream from the down-stream toe, thence follows a gradual curve and finally reaches a tangent some distance down stream from the toe. The terms "ground-water level", and "line of saturation" are in reality synonymous when used in this connection. Owing to the increased head, the quantity of water discharged down stream through the interstices of the soil is greatly increased over that prior to the building of the dam.

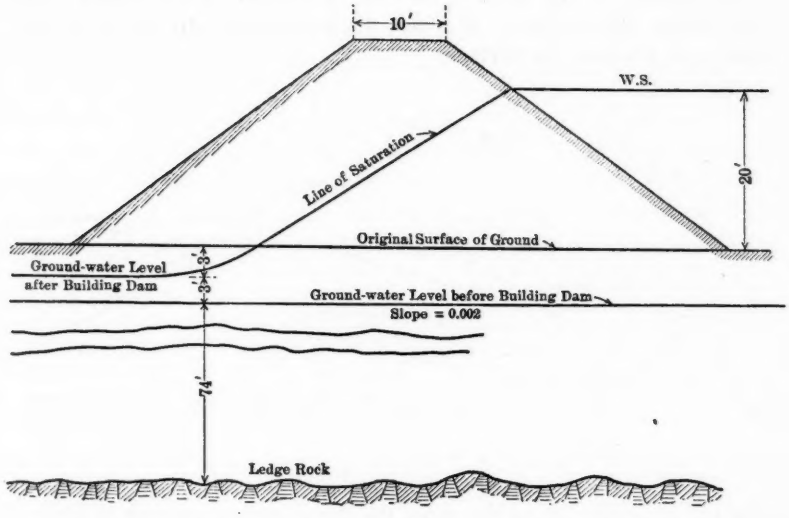

FIG. 2.

Effect on Line of Saturation of Ledge Rock Near the Surface.—Fig. 3 shows the cross-section at another dam site where the conditions were identical with those shown on Fig. 2, except that the ledge rock is near the surface.

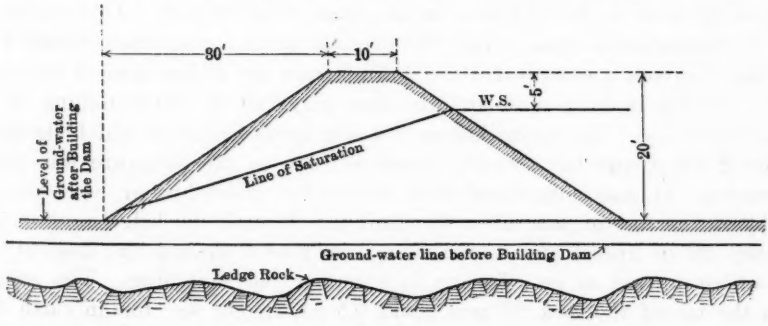

FIG. 3.

Prior to the construction of the dam at this site, the ground-water level was only a slight distance below the surface, and during certain seasons of the year the site was swampy in places. After water had been stored behind the dam, the ground-water line rose to the surface of the ground just below

the dam and the line of saturation intersected the down-stream face above the toe. In this case, the dangerous condition is due to the shallow depth to ledge rock. The underground discharge channel, consisting of the interstices in the foundation material, is greatly restricted over that in Fig. 2, and the pressure forces the water to the surface down stream from the dam.

Effect on Line of Saturation of Rise of Ledge Rock, Down Stream from Site.—Fig. 4 is a cross-section at the site of a proposed earth dam. The ledge rock at the center line of the proposed dam is at a considerable depth below the surface of the ground, and the elevation of the ground-water is also well below the surface. A short distance down stream from the site the ledge rock rises to the surface

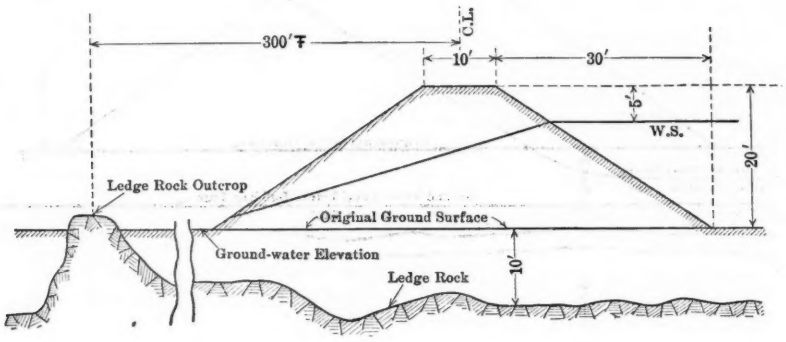

FIG. 4.

After the construction of the dam, the down-stream outcrop of ledge rock creates an effective barrier to the flow of the increased quantity of ground-water through the soil. The increased head on the ground-water due to raising the water behind the dam causes the ground-water to rise to such a height that the increased quantity flowing through the soil can only escape by passing over the lower places in the ridge of ledge rock. This causes the line of saturation to rise, so that the original surface just down stream from the dam becomes a swamp and the down-stream toe of the dam is saturated. The condition is dangerous, and the dam may fail by the sloughing of the down-stream toe. The writer knows of a case quite similar in which sloughing of the down-stream toe actually began as soon as the structure was placed in service. It was impracticable to reduce the ground-water level and the height of the line of saturation by drainage through the ledge outcrop, and a heavy fill of gravel and coarse stone was placed against the down-stream toe, which proved to be effective in stopping the sloughing. The seepage from the toe of the rock fill was about 0.5 cu. ft. per sec. for an earth dam 1 000 ft. long, and the velocity was insufficient to move the finest sand. The structure has been in service for 5 years, and is considered entirely safe, although it would not be safe without the heavy rock fill at the toe. If the impervious strata were clay or hardpan the results would be exactly the same.

Effect on Line of Saturation of Making Down-Stream Part of Dam More Pervious Than the Up-Stream Part.—Only homogeneous structures have been considered in the previous illustrations. There is, however, an advantage in selecting and segregating the materials so that the up-stream part of the dam will be less pervious than the down-stream part. In Fig. 5 the material composing the foundation is assumed to be a fairly tight soil, and the up-stream part of the dam to be built of very impervious clay, whereas the down-stream part of the dam is assumed to be of quite pervious material. The effect of this arrangement is to bring the line of saturation much farther within the down-stream toe than would be the case if the structure were composed entirely of such impervious material as that in the up-stream part. It will be noted that the line of saturation is much steeper in the pervious material. If the dam were to be built entirely of the same impervious material, the slope of the line of saturation would be uniform. In this arrangement, the function of the up-stream part is to keep as much of the water as possible out of the dam, and the function of the down-stream part is to drain away as rapidly as possible such water as passes through the up-stream part, thus greatly increasing the stability of the structure over what it would have been had impervious material been used throughout the entire cross-section.

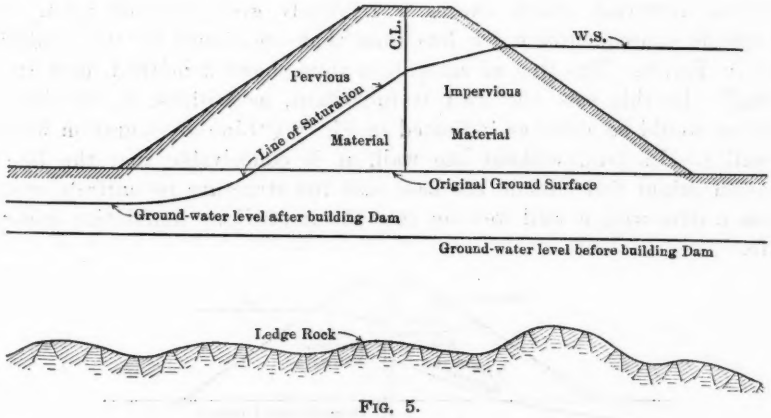

FIG. 5.

Effect of Core-Wall in Dam of Fairly Pervious Material on a Foundation of Like Material.—In Fig. 6 the dam is assumed to be built of fairly pervious material which is practically identical with the foundation soil at the site. The core-wall, whether of masonry or puddle, is assumed to be impervious and to extend only a slight distance below the original surface of the ground. Such a core-wall does little good. Although the water does not pass through the wall, it does pass under it, and its head will be only slightly decreased by the core-wall, and the line of saturation will be little if any different from what it would have been without a core-wall. In some instances the drop in the line of saturation at the core-wall is only 2 or 3 ft. In most such cases the money spent on a core-wall is wasted.

Effect of Core-Wall in Dam of Pervious Material on a Foundation of Impervious Material.—In Fig. 7 conditions are the same as those described for Fig. 6, except that the foundation material is quite impervious. The core-wall, which is assumed to be impervious, is extended down only sufficient

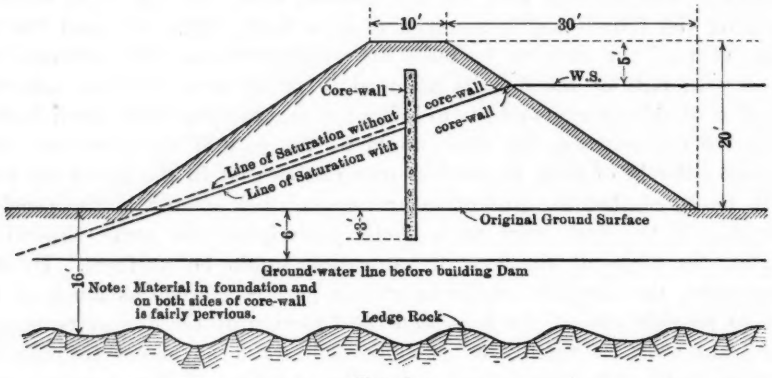

FIG. 6.

to make a good seal with this material. In order to enter the down-stream part of the dam the water must pass under the core-wall and through quite impervious material, which causes a relatively great loss of head. The discharge is consequently much less than that occasioned by the conditions shown in Fig. 6. The line of saturation thus shows a marked drop at the core-wall. In this case the wall is important, as without it, the line of saturation would be about as indicated in Fig. 6 ("Line of saturation without core-wall"). In fact, without the wall, it is conceivable that the line of saturation might fall outside the base and the structure be entirely unsafe, whereas a dam with a wall well set into the impervious foundation material is safe.

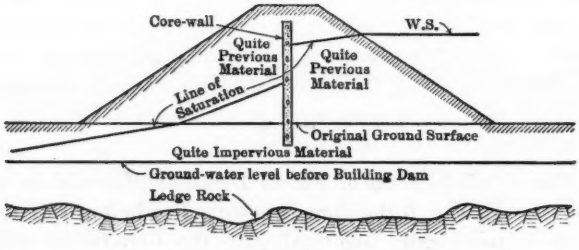

FIG. 7.

Effect of Extending Core-Wall into Pervious Foundation.—Fig. 8 is a cross-section of an earth dam at a site where conditions are identical with those shown on Fig. 6, except that the core-wall has been extended much farther into the pervious foundation. The core-wall is assumed to be impervious, and the water, in passing to the down-stream side of the core-wall, must traverse a greater distance than in Fig. 6. The frictional resistance is proportionately greater, and the water rises on the down-stream side of the

core-wall to a lesser height than was the case in Fig. 6. The line of saturation is lowered, and the stability of the structure increased. If the core-wall had been extended into the foundation by the driving of a wall of sheet-piling the effect would have been the same except that such a wall would not be so likely to be tight.

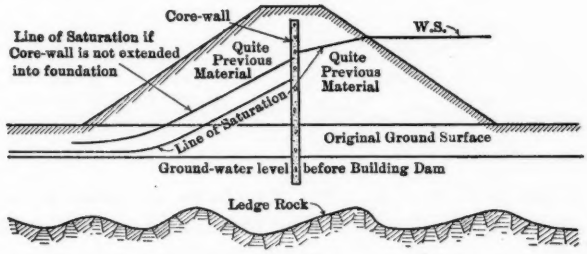

FIG. 8.

It is often assumed that the water in passing under a wall flows down the up-stream side and then up the down-stream side, thus making the distance traversed by the water twice the length of the sheet-piling or wall. Mr. J. B. T. Colman has shown in his paper on "The Action of Water Under Dams,"* that this theory is incorrect, and that the loss of head, due to the insertion of sheet-piling at the heel of a dam, takes place almost entirely on the up-stream side of the sheet-piling.

What actually happens is about as follows: The water flowing under pressure through the soil of the foundation comes in contact with the extended core-wall, which restricts the discharge area through the soil. As the water must flow through this restricted area the velocity and the loss of head accordingly increases. After passing the wall, some of the water rises, which causes additional loss of head. Most of the water, however, travels in a generally horizontal direction, as illustrated in Fig. 9.

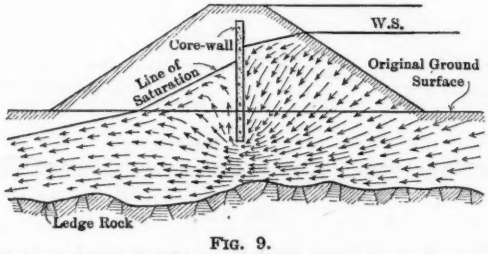

FIG. 9.

Effect of Extending Core-Wall to Ledge Rock.—If the core-wall had been properly bonded to ledge rock the effect on the lowering of the line of saturation on the down-stream side of the core-wall would have been much more marked, and, theoretically, there would be no water on the down-stream side. However, as there are usually some seams in the rock and the concrete or puddled core-wall is never entirely impervious, there is, consequently, some

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 421.

saturation on the down-stream side. Whether or not it is desirable to extend the core-wall to rock can be determined only after a study of all the conditions.

Effect of Core-Wall in a Dam Founded on Ledge Rock.—Fig. 10 is a cross-section of an earth dam founded on ledge rock. With a fairly impervious material in the dam, the line of saturation would intersect the down-stream face at a point considerably above the down-stream toe. The dam illustrated

FIG. 10.

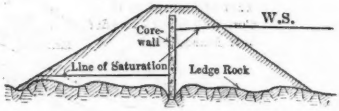

FIG. 11.

in Fig. 10 would probably fail. Fig. 11 shows the same dam at the same site with a core-wall added. Note that the line of saturation intersects the down-stream face at the ledge. A small quantity of seepage will take place at the toe, under these conditions, but may be cared for by proper drainage. The structure shown in Fig. 10 was made safe by the addition of a core-wall. Some earth dams that have been founded directly on ledge rock have been made safe by an elaborate system of drainage, but, ordinarily, it is not good practice to build such a dam without a core-wall.

Effect on Line of Saturation of Placing Core-Wall Up Stream from Center of Dam.—The position of the core-wall in the cross-section of the dam is of great importance. It is frequently placed up stream from, instead of directly on, the center line, as in Fig. 8. The conditions for the dam shown on Fig. 12 are identical with those for Fig. 8, except that the core-wall intersects the up-stream face of the dam at about the water line. It will be noted that although the line of saturation has the same slope as that in Fig. 8, it is much lower. As the proportion of the cross-section which is saturated is much less in

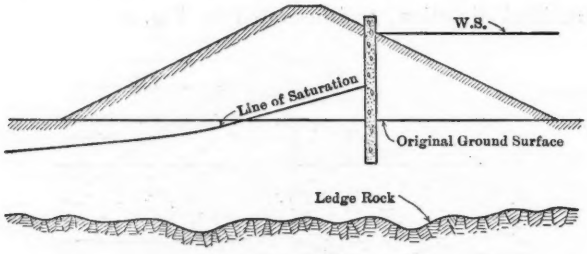

FIG. 12.

Fig. 12 than in Fig. 8, it follows that the former section is much more stable. An additional advantage claimed for the arrangement shown in Fig. 12 is that the part of the core-wall above the intersection with the up-stream face may be used, in many cases, in lieu of rip-rap, to break the force of waves and protect the slope. Wave action, of course, will erode the bank on the up-stream side of the core-wall to a very flat slope, but the effect of this is limited to a few feet below the water surface. The arrangement is not so effective as a protection against wave action where there is a great variation

in the elevation of the water surface. As a core-wall of this kind is generally reinforced so that for a short distance it will act as a cantilever, it is unimportant if the earth on the up-stream side of the core-wall is worn down for several feet.

The Movement of Underground Water and Its Relation to the Design of Earth Dams

The foregoing cases serve to illustrate certain general principles in the design of earth dams. They particularly emphasize the fact that the position of the line of saturation is an important indication of the relative safety of the structure. It is not essential for the engineer to be able to predict the exact position of the line of saturation in a dam designed for some particular site; but it is desirable that he be able to predict, with reasonable accuracy, the limiting positions between which the line of saturation will be found. In order to be able to do this, it is essential that he have some knowledge of the movement of underground waters.

When the height of the water back of an earth dam is raised, the velocity of flow of the ground-water is increased. In some instances the ground-water flow has been increased so much in volume and velocity that it emerged from the surface of the ground below the down-stream toe of the dam. Failures have been caused by piping or by sloughing of the down-stream bank, owing to the resulting saturation.

Earth's Crust Filled with Water.—The earth, for a depth of 5 or 6 miles below the surface, as estimated by Van Hise, is filled with water. Even the igneous rocks have some pores which are filled with water, and there are some sandstones which have a porosity of 30 per cent. The pores or voids of soil, sand, and gravel are generally greater, running as high as 50% in some cases.

Total Quantity of Ground-Water.—The total ground-water in the earth's crust is estimated by Slichter as equivalent to one-third of the total of all the oceanic waters of the globe, thus vastly exceeding the quantity flowing in the rivers.

The elevation of the ground-water varies and the contour of the water-table follows roughly that of the ground surface, but is much less rugged. In places it coincides with the ground surface and produces swamps; in other places it is hundreds of feet below the surface. A cross-section of a typical water-table is shown in Fig. 13.*

Ground-Water in Motion.—All ground-water above the natural ledge rock and much of the ground-water in the ledge rock is in motion. The general direction of the flow of underground water is toward the stream draining the valley. In country with steep slopes and narrow valleys, the predominating movement is toward the stream; at the lower end of great river valleys, where the slopes are flat, the movement is nearly parallel to the general direction of the stream.

* From *Water Supply Paper No. 67*, U. S. Geological Survey.

The rate of movement of underground waters is not great, 10 ft. per day being a high velocity. At very great depths the velocity is doubtless much less than that near the surface.

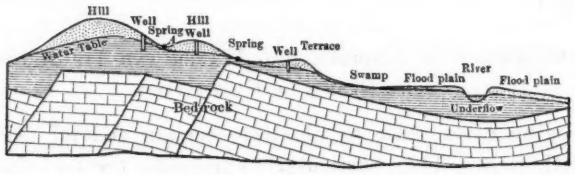

FIG. 13.

FORMULAS FOR THE FLOW OF WATER THROUGH SOILS

Darcy Formula.—Several formulas have been deduced to express the flow of water through soils. That of H. Darcy, published at Paris in 1858, and containing the experimental data, is probably the earliest. Darcy expressed the relation by the formula:

$$V = K \frac{p}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which,

V = velocity of moving water,
p = the difference in pressure,
h = length of column of soil,
K = a constant depending largely on porosity and determined experimentally.

Other investigators have derived similar expressions, covering the same relation.

Hazen Formula.—Mr. Allen Hazen* conducted exhaustive experiments on the flow of water through filter sands, and derived the following formula:

$$V = c\, d^2 \frac{h}{l} (0.70 + 0.03\, t) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which,

V = velocity of water, in meters daily, in a solid column of the same area as that of the sand;
c = a constant factor (generally 1 000);
d = the effective size of particles, in millimeters;
l = the thickness of sand through which water passes;
t = the temperature of the water, in degrees, centigrade;
h = the loss of head in passing through the sand.

Effective Size.—"The effective size is that size such that 10% by weight of the particles are smaller than it."

"The diameter of a sand grain is defined as that of a sphere of equal volume. This must be rigorously insisted on. Taking the diameter of grains as equal to the measured mesh of sieve or some such arbitrary basis invariably leads to errors which may be large.

* Report, Massachusetts State Board of Health, 1892.

"The method of rating the sieves * * * consisted in shaking sieves with a sample of representative material precisely as in the ordinary course of analysis, then taking the sieves apart, giving each sieve separately a slight further amount of shaking, when the particles passing are all of nearly the same size, which size is larger than nearly all the particles that have passed and is smaller than nearly all the particles that remain on the sieve. Some of the particles resulting from this further shaking are weighed and counted, and the average diameter is calculated from the average weight: the size thus found is the size of separation of the sieve. This process is repeated several times, with representative sands of different types and the average taken. The shaking is not to be continued until no more particles will pass, as often stated, but only until the bulk of that sand that will pass that sieve with a reasonable amount of shaking has passed, as, for example, to the point where doubling the number of shakes will not make considerable difference. There are always some holes larger than the average, and if shaking were continued until no more particles would pass, the process would be a very slow one and the result would be to bring the size of separation up to the size of the largest holes.

"The sieves must be of regularly woven wire cloth, and must be carefully rated. It has not been found possible to buy ready-made sieves which can be relied on without rating.

"The process of mechanical sand analysis, as a basis for estimating the flow of water in sands and gravels, is emphatically not one that can be taken up in a spare hour, in a way to yield reliable results; but, a careful manipulator, by following a perfectly simple procedure, may reach results which are trustworthy if not precise."*

Slichter's Formula.—The most extensive studies and experiments thus far conducted on the flow of water through soils are probably those of Professor Charles S. Slichter. He derives the formula:

$$q = 11.3 \frac{p\, d^2\, s}{h\, k} (1 + 0.0187\ (t - 32°)) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which

q = discharge, in cubic feet per minute;
p = difference in pressure-loss of head, in feet;
h = length of column of soil, in feet;
s = the area of cross-section of the soil column, in square feet;
d = the effective size of the grain particles composing the soil, as defined by Hazen;
k = a constant which depends on the porosity of the soil;
t = temperature of flowing water, in degrees, Fahrenheit.

With regard to this formula, Professor Slichter states:

"It is seen * * * that the quantity of water transmitted by a column of sand not only depends upon the length of the column and the head of water as expressed by Darcy's law, but varies in a most remarkable way with the effective size of the soil grain, with the temperature of the water, and with the porosity. Since the flow varies as the square of the size of the soil grain this element in the formula has a most important effect, as doubling the size of the soil grain will quadruple the flow of water. Thus, the flow through a sand whose effective size of grain is 1 mm. is 10 000 times the flow through a soil whose effective size of grain is 0.01 mm. The variation of flow with temperature is also important, as the flow at 70° F. is about double

* Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), p. 201.

that at 32° F. The variation in porosity is quite as important as the variation in temperature. * * * If the two samples of the same sand are packed so that their porosities are 30 per cent. and 40 per cent., the flow through the latter sample will be about 2.6 times the flow through the former sample."*

Professor Slichter further simplifies his formula by grouping the factors which depend only on the character of the soil, and represented them by a single letter, K, thus,

$$K = 11.3\,\frac{d^2}{k}\,(1 + 0.0187\,(t - 32°)) \qquad (4)$$

and the formula becomes:

$$q = K\frac{p\,s}{h} \qquad (5)$$

Transmission Constant.—It is evident that K will be different for different soils, and different for the same soils at different temperatures. The factor, K, has been termed by Professor Slichter "the transmission constant", and is the "quantity of water that is transmitted in unit time through a cylinder of the soil of unit length and unit cross-section under unit difference in head."

"It should be especially noted that the velocity of flow through a soil for the pressure gradients and size of grain that commonly occur is exceedingly slow, and much less than might at first be supposed. Darton states that the rate of flow in the sands of the Dakota formation, from which the remarkable artesian wells of South Dakota draw their supply, does not exceed a mile or two a year. Mr. E. L. Rogers reported to the Denver society of civil engineers that American estimates agree with careful and exhaustive studies of French engineers, which show the average velocity in sands to be about a mile a year, or about an eighth of an inch a minute. In Arizona the rate has been figured out as between one-fourth and one-third of an inch per minute, while on the Arkansas River above Dodge, Kans., a ditch a mile long and 5 feet below the water table in the sand developed a flow of about three-eighths inch per minute."†

The foregoing, of course, refers to the flow of water through soils under natural conditions. The discharge through earth dams would ordinarily be much greater, as the slope or gradient would ordinarily be so much steeper than that found under natural conditions.

Table 1‡ gives the transmission constants for soils of various effective sizes and degrees of porosity, at 60° Fahr., as computed by Professor Slichter. Table 2 is a correction table which gives the multipliers for use with the transmission constants derived from Table 1, in order to obtain the transmission constant for the various temperatures stated in Table 2.

The Use of the Slichter Formula and Tables.—In order to illustrate the use of Tables 1 and 2, for the solution of the Slichter formula and similar problems, a concrete case will be assumed. Assume that the ground-water level at some point is Elevation 122, and at another point 1 200 ft. nearer the stream, it is Elevation 112. The soil is found to have an effective size of

* *Water Supply Paper No. 67*, U. S. Geological Survey, p. 25.
† *Loc. cit.*, p. 26.
‡ From *Water Supply Paper No. 140*, U. S. Geological Survey.

0.30 mm.; the temperature of the ground-water is 70° Fahr. The porosity of the soil is 40%, and it is desired to determine the velocity of the water flowing through the soil.

TABLE 1.—TRANSMISSION CONSTANTS FROM WHICH THE VELOCITY OF WATER IN SANDS OF VARIOUS EFFECTIVE SIZES OF GRAIN CAN BE OBTAINED.

Table computed for temperature of 60° Fahr.; results for other temperatures can be found by the use of Table 2.

Diameter of soil grains, in millimeters.	Porosity.						Kind of soil.
	30%.	32%.	34%.	36%.	38%.	40%.	
0.01	0.000033	0.000040	0.000050	0.000060	0.000072	0.000085	
0.02	0.000131	0.000162	0.000198	0.000239	0.000286	0.000339	Silt.
0.03	0.000296	0.000364	0·0004460	0.000538	0.000645	0.000763	
0.04	0.000527	0.000648	0.0007940	0.000958	0.001145	0.001355	
0.05	0.000822	0.001012	0.001240	0.001495	0.001790	0.002120	
0.06	0.001182	0.001458	0.001784	0.002150	0.002580	0.003050	
0.07	0.001610	0.001983	0.002430	0.002930	0.003510	0.004155	Very fine sand.
0.08	0.002105	0.002590	0.003175	0.003825	0.004585	0.005425	
0.09	0.002660	0.003280	0.004018	0.004845	0.005800	0.006860	
0.10	0.003282	0.004050	0.004960	0.005980	0.007170	0.008480	
0.12	0.004725	0.005830	0.007130	0.008620	0.01032	0.01220	
0.14	0.006430	0.007940	0.009720	0.01172	0.01404	0.01662	
0.15	0.007390	0.009120	0.01115	0.01345	0.01611	0.01910	Fine sand.
0.16	0.008410	0.01036	0.01268	0.01531	0.01835	0.02170	
0.18	0.01064	0.01311	0.01605	0.01940	0.02320	0.02745	
0.20	0.01315	0.01620	0.01983	0.02390	0.02865	0.03390	
0.25	0.02050	0.02530	0.03100	0.03740	0.04480	0.05300	
0.30	0.02960	0.03640	0.04460	0.05380	0.06450	0.07630	
0.35	0.04025	0.04960	0.06075	0.07330	0.08790	0.01039	Medium sand.
0.40	0.05270	0.06480	0.07940	0.09575	0.1145	0.1355	
0.45	0.06650	0.08200	0.1005	0.1211	0.1450	0.1718	
0.50	0.08220	0.1012	0.1240	0.1495	0.1780	0.2120	
0.55	0.09940	0.1225	0.1500	0.1810	0.2165	0.2565	
0.60	0.1182	0.1458	0.1784	0.2150	0.2580	0.3050	
0.65	0.1390	0.1710	0.2095	0.2530	0.3030	0.3580	
0.70	0.1610	0.1983	0.2430	0.2930	0.3510	0.4155	Coarse sand.
0.75	0.1850	0.2278	0.2785	0.3365	0.4030	0.4770	
0.80	0.2105	0.2590	0.3175	0.3825	0.4585	0.5425	
0.85	0.2375	0.2925	0.3580	0.4325	0.5175	0.6125	
0.90	0.2660	0.3280	0.4018	0.4845	0.5800	0.6860	
0.95	0.2965	0.3650	0.4470	0.5400	0.6460	0.7650	
1.00	0.3282	0.4050	0.4960	0.5980	0.7170	0.8480	
2.00	1.315	1.620	1.983	2.390	2.865	3.390	
3.00	2.960	3.640	4.460	5.380	6.450	7.630	Fine gravel.
4.00	5.270	6.480	7.940	9.575	11.45	13.55	
5.00	8.220	10.12	12.40	14.95	17.90	21.20	

TABLE 2.—VARIATION, WITH THE TEMPERATURE, OF THE FLOW OF WATER OF VARIOUS TEMPERATURES THROUGH A SAND, 60° FAHR. BEING TAKEN AS THE STANDARD TEMPERATURE.

Temperature, in degrees Fahrenheit.	Percentage of relative flow.*	Temperature, in degrees Fahrenheit.	Percentage of relative flow.*
32	0.64	70	1.15
35	0.67	75	1.23
40	0.73	80	1.30
45	0.80	85	1.39
50	0.86	90	1.47
55	0.93	95	1.55
60	1.00	100	1.64
65	1.08	...	....

*"Relative flow" means flow at given temperature compared with flow at 60° Fahr. It is expressed as a percentage.

Table 1 shows that, for an effective size of 0.30 mm. and 40% porosity, the soil will have a transmission constant of 0.07630 at a temperature of 60° Fahr. Table 2 shows that, for a temperature of 70°, the transmission constant obtained from Table 1 must be multiplied by 1.15.

The transmission constant, therefore, is $0.07630 \times 1.15 = 0.08774$.

Thus, in the formula,

$$q = K\frac{p\,s}{h} \qquad (5)$$

we have,

$K = 0.08774.$
$p = 10,$
$s = 1,$
$h = 1\,200,$

$$q = 0.08774\,\frac{10 \times 1}{1\,200} = 0.000736 \text{ cu. ft. per min.}$$

In this case, q is the discharge per unit area (s being taken equal to 1). If the entire cross-section were water, this would also be the velocity of the water, but as only 40% of this area is water, the velocity of the ground-water is $\frac{0.000736}{0.40} = 0.00184$ ft. per min.

Difficulty of Applying Formulas.—The difficulty in applying these formulas to the practical problems of earth dam design lies in the great variation in the effective size of the grains of any natural soil, either down stream from the dam, in the foundation of the dam, or in the dam itself. This is illustrated by Fig. 6 of the paper by Mr. J. Albert Holmes.*

Safe Values for Effective Size.—In applying the formula, a safe value for the effective size should be selected. If the cross-section is found to be quite homogeneous, a mean value for the effective size should be used. If, however, there is a great variation in the effective size of the particles, it will be wise to choose an effective size considerably greater than the mean. If there is a marked variation in the effective size from one cross-section to another, the cross-section having the coarser material should be selected for special investigation.

Thus, the material in an old stream bed which is to be closed by an earth dam may be composed of much coarser material than that in the foundation of the remainder of the dam. It is conceivable that such a layer of coarse material in the base of an earth dam might be a source of considerable weakness, and would probably lead to concentrated leakage along this layer. By a thorough examination and study, it may be determined whether it would be safe to build the dam on this material, or whether it would be necessary to extend a core-wall to a great depth in order to increase the resistance to the flow of water under the dam, or to increase the width of the base for the same purpose. In problems of this kind Professor Slichter's formula is a great aid.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 341.

Slichter's Electrical Device.—Professor Slichter has also developed an electrical device* for determining the velocity and direction of the movement of underground water. This instrument might be used with good results in the study of some earth dam foundations. By its use the transmission constant for the material at the site could be determined experimentally.

Position of the Line of Saturation.—From what has been stated about the importance of having the line of saturation well within the down-stream toe of earth dams, it will be seen that it would be desirable, if, with a given material of which to build the dam and a given site, one could predict, within reasonable limits, where the line of saturation will cut the base. Judgment and experience are sometimes sufficient to indicate that the line of saturation will fall far enough inside the down-stream toe to be amply safe.

If analysis, however, shows that the line of saturation would probably intersect the down-stream face, possibly causing piping or sloughing and consequent failure, the section could be increased until it would contain the line of saturation. This would be much wiser than guessing at the dimensions and erecting the structure only to find that the line of saturation intersected the down-stream face some distance above the ground.

There are many difficulties in trying to predict the position and slope of the line of saturation, but with trustworthy field data and by the proper application of Professor Slichter's formula by the methods herein set forth, it is possible to do this with sufficient precision for all practical purposes.

Essential Features.—The essential features to be determined in the field before the method can be applied successfully are:

1.—Elevations and contours of original ground surface;
2.—Elevations and contours of rock surface or other impervious strata below the ground surface;
3.—Elevation, slope, and direction of flow of ground-water;
4.—Effective size, porosity, and nature of material to be used in building the dam;
5.—Effective size, porosity, and nature of material composing the foundation soil at the site and down stream from the site.

The possession of these data presupposes a thorough sub-surface investigation at the site.

Point Where the Line of Saturation Intersects the Base.—With the Slichter formula as a basis, a formula may be derived which will give the point where the line of saturation intersects the base of the dam. Fig. 14 shows a typical earth dam. Some point, O, at a distance, C, down stream from the intersection of the head-water surface with the up-stream face of the dam, has been chosen, such that, at O, the slope of the ground-water surface suddenly increases. In most cases there will also be a break in the slope of the surface of the ground above O. It is not essential that the distance, C, should be in a straight line, but it should represent the distance along the path followed by the ground-water. It may be intended to place the earth dam on a side hill, the bottom of the valley being occupied by a masonry structure,

* Described in *Water Supply Paper No. 140*, U. S. Geological Survey.

in which case the direction of flow of the ground-water after passing the dam would probably be perpendicular to the center line of the earth dam for a short distance below the dam, where it would turn and flow toward the bottom of the valley.

In the subsequent discussion, the point, O, is first assumed to be fixed, that is, it is assumed that, after the dam is built and the discharge of ground-water greatly increased, there is no increase in the elevation of the ground-water at the point, O. If the distance, C, is great and the slope of the ground-water becomes much steeper below O, this assumption does not involve great error. It will be shown later that, by selecting other points, farther down stream than O, a series of approximations (of the same nature as those used in many other hydraulic calculations) may be made, and the rise in the elevation of the ground-water at O computed, and the effect of this rise on the position and slope of the line of saturation in the dam also determined. In Fig. 14:

P_3 = the difference in elevation, in feet, between the base of the earth dam and the head-water level;

h_3 = the horizontal distance, in feet, from the point where the head-water level intersects the up-stream face to the point where the line of saturation intersects the base of the dam;

P_1 = the difference in elevation, in feet, between the fixed point, O, and the base of the earth dam;

h_1 = the horizontal distance, in feet, from the point where the line of saturation intersects the base of the earth dam to the fixed point, O;

P_2 = the vertical distance, and h_2 the horizontal distance, taken at any convenient points for the determination of the slope of the natural ground-water surface, before the construction of the dam;

S_1 = the area of flow, per linear foot of cross-section, of the water flowing through the soil below the dam, and is equal to the difference in elevation, in feet, of the ledge rock or other impervious strata and the base of the dam;

S_2 = the area of flow, per linear foot of cross-section, of the natural ground-water, before building the dam, and is equal to the difference in elevation between the ledge rock or other impervious strata under the dam and the surface of the ground-water;

S_3 = the effective area of flow, per linear foot of cross-section, of the water flowing through the dam itself, and is equal to $\frac{1}{2} P_3$, in feet, or is equal to one-half the difference in elevation between the base of the dam and the surface of head-water;

K_1 = Slichter's transmission constant for the soil forming the foundation of the dam and the soil below the dam site;

K_3 = Slichter's transmission constant for the material composing the dam itself;

q_1 = the total discharge, per linear foot of cross-section, through the foundation soil just below the dam, in cubic feet per minute, after placing the dam in service;

q_2 = the total discharge, per linear foot of cross-section, through the foundation soil just below the dam, of the natural ground-water before building the dam;

q_3 = the total discharge, per linear foot of cross-section, of the water passing through the saturated part of the earth dam.

Then:

$\frac{P_3}{h_3}$ = slope of line of saturation;

$\frac{P_2}{h_2}$ = slope of ground-water level before building dam;

$\frac{P_1}{h_1}$ = slope of ground-water surface below the dam after the dam is placed in service;

$$q_3 = K_3 \frac{P_3}{h_3} S_3 \quad \ldots\ldots (6)$$

$$q_2 = K_1 \frac{P_2}{h_2} S_2 \quad \ldots\ldots (7)$$

$$q_1 = K_1 \frac{P_1}{h_1} S_1 \quad \ldots\ldots (8)$$

The foregoing are simply restatements of Slichter's formula. It should be stated here that, in this discussion, only the discharge per linear foot of dam or cross-section is being considered. Hence the discharges determined should be multiplied by the length of the dam or the distance perpendicular to the cross-section being considered, in order to determine the total discharge through the soil or the quantity of water lost by seepage.

Let, $q_1 = q_2 + q_3$ (Equations (6), (7), and (8)), on the assumption that the quantity of water which actually passes beneath the dam, due to seepage from the bottom of the reservoir, is relatively insignificant. This assumption is reasonably correct, except where the dam is built of very impervious material and the foundation material is very pervious, in which case another method—given later—should be used to determine whether the base is sufficiently thick to cause such a reduction in the velocity of any water that might pass under it that there would be no danger of piping below the downstream toe. A method of checking the relative accuracy of the assumption involved in this formula will be given subsequently.

All the foregoing quantities are determinable before the construction of a dam, except h_1 and h_3, which, therefore, are the unknowns in the following equations:

Substituting in Equation (8),

$$K_1 \frac{P_1}{h_1} S_1 = q_2 + K_3 \frac{P_3}{h_3} S_3 \quad \ldots\ldots (9)$$

$$h_1 = C - h_3 \text{ (see Fig. 14)} \quad \ldots\ldots (10)$$

Substituting and transposing,

$$h_3 = \frac{K_3 P_3 S_3 C}{K_1 P_1 S_1 + q_2 + K_3 P_3 S_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

q_2 being determined from Equation (7). Equation (11) determines the position and slope of the line of saturation in the dam.

As $h_1 = C - h_3$, q_1 may be determined from Equation (10).

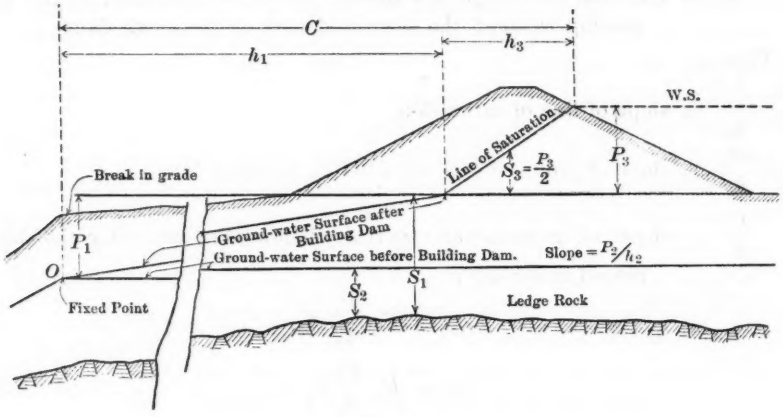

FIG. 14.

As to the reasonableness of the assumption involved in Equation (8), one may, as a rough check, place $C = h$ and $P =$ the entire difference in elevation between head-water and the fixed point, O.

$$q = K_1 \frac{P}{h} S_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

It will generally be found that this value of q does not differ greatly from q_1. The quantity, q, will not give the true discharge, as Equation (12) assumes that the line of saturation and the ground-water surface below the dam are one continuous line having the same slope; which condition will not generally be true. However, it serves as a rough check on the previous assumptions, using Equations (7) and (8), and should always be made.

The point O, of course, does not actually remain fixed, but will rise after the dam is placed in service. O should be chosen at the first marked break in the slope of the ground surface below the dam, as, for instance, on the bank of a gully. If the "get-away" below O is good, there will be little rise in the ground-water at this point after placing the dam in service, and there will be only a small error in the computed slope of the line of saturation due to this assumption.

The rise in the ground-water elevation at O may be estimated and its effect on the line of saturation computed as follows: Choose some point down stream from O, say, O_1, at the next break in grade. Determine in the field the ground-water elevation, depth to rock or other impervious strata, and the distance from O to O_1. Consider O_1 as fixed. In the equation, $q = K \frac{P}{h} S$,

all factors are known except P. Solving for P will thus determine the rise in ground-water elevation at O after the construction of the dam. Taking this new elevation for O, again apply Equations (11) and (7). If very different results from those already recorded are obtained, further approximations should be made, using the last computed value of q_1 each time. Another point at the next break in grade down stream is then selected and considered as a fixed point, O_2, and the rise in the ground-water elevation at O_1, due to the construction of the dam, is computed and the computations again carried back to the base of the dam. Still further approximations may be made if desirable, but, as a rule, one or two approximations are all that are necessary. Problems illustrating the principles involved will be presented subsequently. However, any degree of refinement in computations of this kind is unnecessary.

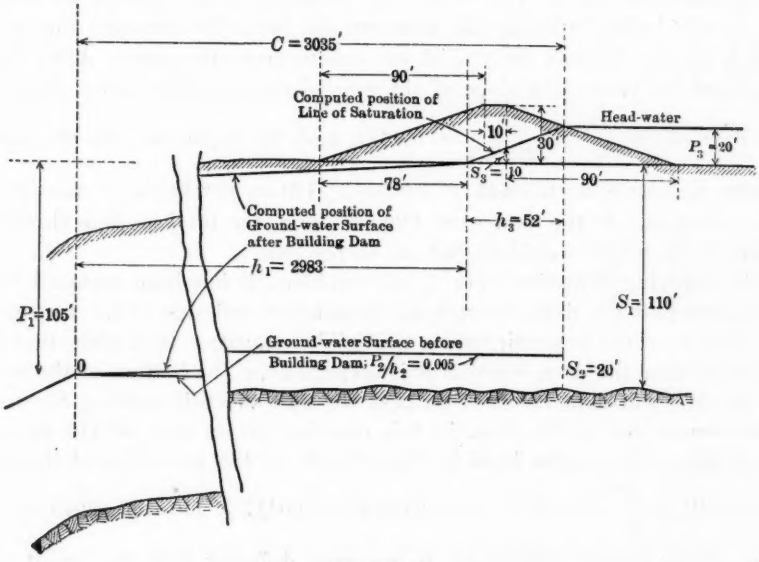

FIG. 15.

Position of the Line of Saturation in Fig. 15.—The earth dam illustrated in cross-section in Fig. 15 is taken as a specific case, and the position of the line of saturation is computed under the given conditions. The ground-water surface at the site is 90 ft. and ledge rock is 110 ft. below the surface of the ground. The slope of the natural ground-water at the site is 5 ft. per 1 000. Down stream from the site the ground surface has a fairly uniform slope for 3 000 ft., and there the surface elevation is 180, at which point it breaks rapidly to the valley of a tributary. At this point the elevation of the ground-water has been determined as 105 and beyond it the slope of the ground-water becomes very much steeper. The soil at the site has an average effective size of 0.14 mm., and the voids in the material are 30 per cent.

Table 1 shows that, for an effective size of 0.14 mm. and a porosity of 30%, the transmission constant will be 0.00643, at a temperature of 60° Fahr.

Applying Equation (7), q_2 is found to be 0.000643 cu. ft. per min. Substituting in Equation (11),

$$h_3 = \frac{0.00643 \times 20 \times 10 \times 3\,035}{0.00643 \times 105 \times 110 + 0.000643 + 0.00643 \times 20 \times 10} = 51.7 \text{ ft.}$$

As the distance from the point where the head-water intersects the up-stream face to the down-stream toe is 130 ft., the line of saturation will intersect the base at a point 78.3 ft. inside the down-stream toe.

The resulting position of the line of saturation in the cross-section of the dam is shown in Fig. 15. The slope of the line of saturation is $\frac{P_3}{h_3} = \frac{20}{51.7} = 0.387$, which is a rather steep one. This favorable condition insures a structure of great stability, and is due to the low elevation of the natural ground-water at the site before building the dam and the favorable drainage for the water which passes through the soil down stream from the dam. After the dam is placed in service, the slope of the ground-water surface down stream from the dam will be $\frac{P_1}{h_1} = \frac{105}{2983.3} = 0.0352$, and, by Equation (8), the total discharge will be $q_1 = 0.00643 \times 0.0352 \times 110 = 0.0249$ cu. ft. per lin. ft. of dam per min. If the dam were 1 000 ft. long, the total seepage through and under it, therefore, would be 24.9 cu. ft. per min.

In applying Equation (11) to this problem, it has been assumed that the discharge past the dam, through the foundation soil, due to the seepage from the bottom of the reservoir, was a negligible quantity. It is plain that all the water passing the dam, whether it is seepage from the bottom of the reservoir or through the dam itself, must pass through the soil section, S_1, near the down-stream toe of the dam, in this case having an area of 110 ft. per lin. ft. of dam. The entire head in the 3 035 ft. is 125 ft. $= P$, and the average slope will be $\frac{P}{h} = 0.0417$. $q = 0.00643 \times 0.0417 \times 110 = 0.02945$ cu. ft. per min., which is only 0.00455 cu. ft. per min. different from the result already found by Equation (11). This difference is so small that it may be safely neglected. In the dam just discussed, the flow of the natural ground-water was so small that it might have been safely neglected without appreciable error, but it is believed that such flow should be considered in most cases, as it frequently has a marked effect on the position of the line of saturation in the cross-section of the dam.

In the method just given for determining the position of the line of saturation in Fig. 15, it was assumed that, at the fixed point, O, the ground-water has the same elevation after the dam is placed in service that it had before the dam was built. This is a safe assumption, and will not be productive of material error if the slope down stream from O becomes much steeper than it is up stream from O. This assumption may be tested by making such further approximations as may be necessary. Thus, it will be assumed that, in the investigation of the site for the dam shown in Fig. 15, the natural

ground-water at a point, O_1, 200 ft. down stream from O, was at Elevation 45. The average effective size of the material was found to be the same as that at the site. The average distance from the ground-water surface betwen O and O_1 to the ledge rock was found to be 50 ft. The point, O_1, is now considered as the fixed point, and it is desired to compute the rise in the elevation of the ground-water at O, and the resulting effect on the position of the line of saturation in the cross-section of the dam. The value of q, already computed, is equal to 0.0249 cu. ft. per min. In the equation, $q = \frac{K\,P}{h}\,S$, all quantities are known except P, and $P = \frac{q\,h}{S\,K}$.

Therefore,

$$P = \frac{0.0249 \times 200}{0.00643 \times 50} = 15.5 \text{ ft.}$$

As the elevation of O is 60 ft. above O_1 instead of 15.5 ft., one may be sure that there will be practically no rise in the elevation of the ground-water at O, due to the construction of the dam. By the foregoing equation, taking P as the difference in elevation between O and O_1, and solving for h, all other factors remaining the same, it is found that if h, instead of being 200 ft. had exceeded 770 ft., there would actually have been a rise in the elevation of the ground-water at O, due to placing the dam in service. In case the computed rise in ground-water elevation at O is small, it will be sufficient to consider the new ground-water line as parallel to the one previously computed, and thus obtain the point where it intersects the base of the dam. If the increase in elevation of the ground-water surface at O is considerable, it will be necessary to use Equation (11) again, with the new elevation for O. If the country is very flat, it may be necessary to choose still another point farther down stream, and find the true position of the point where the line of saturation cuts the base of the dam by several approximations. If O is on the bank of the river or of a gully leading down to the river, there will be a marked break in the slope of the surface of the ground-water at this point. Under such conditions, the effect on the position of the line of saturation in the cross-section of the dam, of such a slight rise in the elevation of the ground-water as would take place at O, would be negligible. What is likely to happen, in such cases, is the production of many springs in the vicinity of O after the dam is placed in service. In all these computations, it should be borne in mind that C is to be measured horizontally, along the path followed by the ground-water, and that O is not necessarily directly down stream from the section being investigated.

Position of the Line of Saturation in Fig. 16.—It is desired to build a 30-ft. earth dam at the site shown in Fig. 16. It has been ascertained that the ledge rock is at Elevation 90, or 25 ft. below the original surface. The ground-water elevation at the site is 93 ft. The elevation of the original ground surface is 115. At a point 500 ft. down stream from the dam there is a sudden break in the slope of the ground surface at the point, O, where the surface of the ledge rock is at Elevation 65. The natural ground-water

surface at *O*, before building the dam, is at Elevation 68. The average effective size of the particles composing the foundation soil at the site is 0.75 mm. (coarse sand) with a porosity of 36 per cent. The soil down stream from the dam has the same general effective size and porosity as the soil at the site. The ground-water temperature is 70° Fahr.

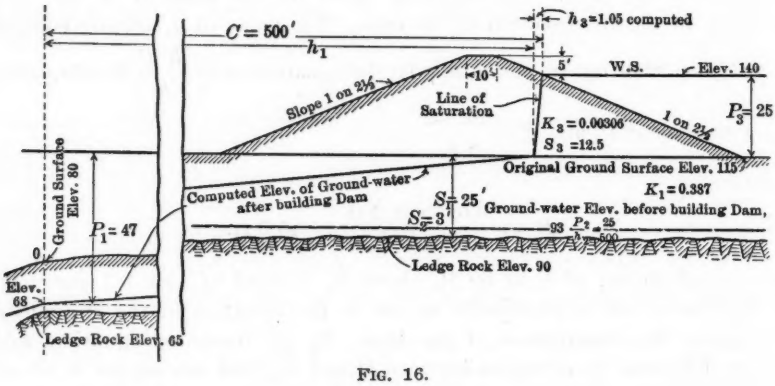

FIG. 16.

From Table 1 it is found that Slichter's transmission constant for an effective size of 0.75 mm. is 0.3365 for a porosity of 36% and a temperature of 60° Fahr. From Table 2 it is found that, for a temperature of ground-water of 70° Fahr., the constant obtained from Table 1 should be multiplied by 1.15. Hence, the transmission constant is $0.3365 \times 1.15 = 0.387$.

It is proposed to build the dam of material having an effective size of 0.09 mm., being a fairly fine sand, with a porosity of 30 per cent. Table 1 shows that Slichter's transmission constant for a material having an effective size of 0.09 mm. and a porosity of 30%, with ground-water temperature of 60° is 0.0026. The proper multiplier to use with this, in order to obtain the corresponding transmission constant at 70° Fahr. is 1.15, as before. Hence the transmission constant for the material of which the dam is to be composed is $0.00266 \times 1.15 = 0.00306$.

The other factors to be used in the formulas — from the foregoing and from Fig. 16 — are as follows:

$$P_3 = 25 \text{ ft.},$$
$$P_1 = 47 \text{ ft.},$$
$$C = 500 \text{ ft.},$$
$$P_2 = 25 \text{ ft.},$$
$$h_2 = 500 \text{ ft.},$$
$$S_1 = 25 \text{ ft.},$$
$$S_2 = 3 \text{ ft.},$$
$$S_3 = \frac{25}{2} = 12.5,$$
$$K_1 = 0.387,$$
$$K_2 = 0.00306.$$

Substituting in Equation (7), $q_2 = \frac{0.387 \times 25 \times 3}{500} = 0.058$ cu. ft. per min. per lin. ft. of cross-section.

Substituting in Equation (11),

$$h_3 = \frac{0.00306 \times 25 \times 12.5 \times 500}{0.387 \times 47 \times 25 + 0.058 + 0.00306 \times 25 \times 12.5} = 1.05 \text{ ft.}$$

$q_3 = 0.9$ cu. ft. per min. per lin. ft. of cross-section; and, $q_1 = q_2 + q_3 = 0.958$ cu. ft. per min. per lin. ft. of cross-section, being the quantity of seepage through and under the dam under full reservoir conditions.

In this case the material composing the dam itself is comparatively impervious, and the material composing the foundation soil is quite pervious. The difference between the water-carrying capacity of the two materials is very marked, the transmission constant for the foundation material being more than 100 times that for the material of which the dam was composed. This rather extreme case was selected for purposes of illustration.

As shown in Fig. 16, the effect is to produce a very steep line of saturation within the dam. The porous foundation material acts as a drain for the material above in such a way that there is practically no saturation in the dam except in that part directly under the water surface. Such a structure will be of great stability. The seepage through the foundation soil with full reservoir was found to be considerable. With a dam 1 000 ft. long the seepage would be 958 cu. ft. per min., or 16 cu. ft. per sec. It would be necessary to determine whether it would be economy to extend a core-wall or sheet-piling to ledge rock in order to reduce the seepage.

Position of the Line of Saturation in Fig. 17.—In Fig. 17 all conditions are the same as in Fig. 16, except that the foundation material is a fine sand having an effective size of 0.09 mm., the same as the material of which the dam is composed, with a transmission constant = 0.00306, as before. Hence, the constants to be used in Equation (11) are:

$$\begin{aligned} P_3 &= 25 \text{ ft.}, \\ P_1 &= 47 \text{ ft.}, \\ C &= 500 \text{ ft.}, \\ P_2 &= 25 \text{ ft.}, \\ h_2 &= 500 \text{ ft.}, \\ S_1 &= 25 \text{ ft.}, \\ S_3 &= 12.5 \text{ ft.}, \\ K_1 &= 0.00306, \\ K_3 &= 0.00306, \\ S_2 &= 3 \text{ ft.} \end{aligned}$$

$$q_2 = \frac{0.00306 \times 25 \times 3}{500} = 0.000455 \text{ cu. ft. per min.}$$

$$h_3 = \frac{0.00306 \times 25 \times 12.5 \times 500}{0.00306 \times 47 \times 25 + 0.000455 + 0.00306 \times 25 \times 12.5} = 107 \text{ ft.}$$

In the assumed section for the earth dam, Fig. 17, the horizontal distance from the point where the head-water surface intersects the up-stream

face of the dam to the down-stream toe is only 85 ft. As h_3, the horizontal distance from the point where the head-water surface intersects the up-stream face, to the point where the line of saturation intersects the base is 107 ft., it is clear that the line of saturation would intersect the down-stream face of the dam at some point above the toe. The section, therefore, should be increased by flattening the down-stream slope of the dam. If the down-stream slope is made 1 on 3.5, the horizontal distance will be 127.5 ft. from the intersection of the water surface with the up-stream face to the down-stream toe. The line of saturation will be 15.5 ft. inside the down-stream toe, which condition is satisfactory.

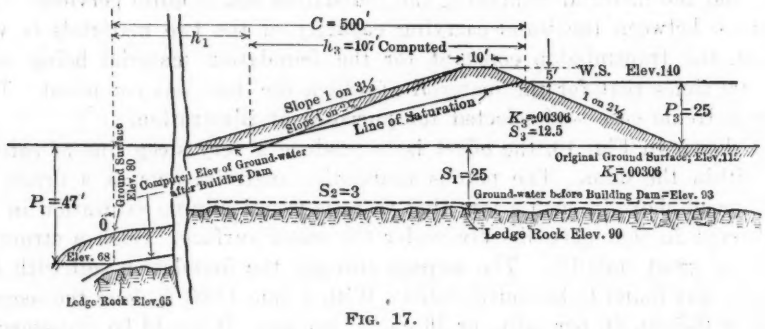

FIG. 17.

This increase in section is required because the material of the foundation is not capable of draining all the water passing through the dam. This condition might have been corrected by a proper system of drainage. If the additional material required were placed on the up-stream slope, the same result would not be achieved, as the distance from the point where the head-water intersects the up-stream face to the down-stream toe would be only 92.5 ft., which would be insufficient to contain the line of saturation.

For this reason, if the up-stream slope is sufficiently flat to be stable under the action of water there is a great advantage in making the down-stream slope the flatter, in order to make the percolation distance as great as possible and keep the line of saturation well buried, thus increasing the stability of the structure. This illustrates the principle that, after selecting an up-stream slope which is stable under water conditions, the greatest increase in the factor of safety of an earth dam is obtained by using additional material to flatten the down-stream slope.

Position of the Line of Saturation in Fig. 18.—In Fig. 18 all conditions are the same as those in Fig. 16, except that the foundation material at the site and the material of which the earth dam is to be composed have an effective size of 0.01 mm. (very fine silt), and the porosity of the soil is 30%, giving a transmission constant of 0.000033 at a temperature of 60° Fahr., which will also be assumed to be the temperature of the ground.

The constants to be used in Equation (11), for determining the position of the line of saturation, are as follows:

$$P_3 = 25 \text{ ft.},$$
$$P_1 = 47 \text{ ft.},$$
$$C = 500 \text{ ft.},$$
$$P_2 = 25 \text{ ft.},$$
$$h_2 = 500 \text{ ft.},$$
$$S_1 = 25 \text{ ft.},$$
$$S_3 = 12.5 \text{ ft.},$$
$$S_2 = 3 \text{ ft.},$$
$$K_1 = 0.000033,$$
$$K_3 = 0.000033,$$

$$q_2 = \frac{0.000033 \times 25 \times 3}{500} = 0.00000495,$$

$$h_3 = \frac{0.000033 \times 25 \times 12.5 \times 500}{0.000033 \times 47 \times 25 + 0.00000495 + 0.000033 \times 25 \times 12.5} = 105 \text{ ft.},$$

$$q_3 = \frac{0.000033 \times 25 \times 12.5}{105} = 0.000099 \text{ cu. ft. per min., which is the}$$

seepage through and under the dam per linear foot of cross-section. The slope of the line of saturation is $\frac{25}{105} = 23.8$ per cent. The material, therefore, is very impervious and the seepage is very small, but, because of the tightness of the material, both in the dam and its foundation, there is little chance for effective drainage, and the line of saturation has a relatively flat slope.

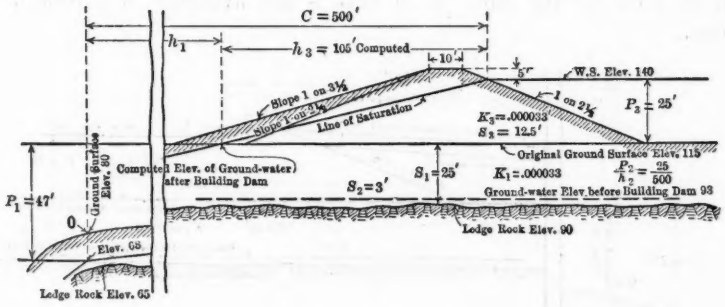

FIG. 18.

Position of the Line of Saturation in Fig. 19.—In Fig. 19 all the conditions are the same as in Fig. 16, except the difference in the materials composing the dam and the foundation. The effective size of the material in the dam is 0.95 mm., the porosity is 40%, and the ground temperature 70° Fahr. Table 1 shows that the proper transmission constant, at 60° Fahr., is 0.765. Table 2 shows that, for a ground-water temperature of 70°, the constant, 0.765, should be multiplied by 1.15. Then, $0.765 \times 1.15 = 0.88$. The foundation soil has an effective size of 0.09 mm. and a porosity of 30% with a ground-water temperature of 70° Fahr. As before, the transmission constant is 0.00306. Hence, in this case, the foundation soil is composed of very fine and the dam of very coarse sand. The values of the constants are:

$$P_3 = 25 \text{ ft.},$$
$$P_1 = 47 \text{ ft.},$$
$$C = 500 \text{ ft.},$$
$$P_2 = 25 \text{ ft.},$$
$$h_2 = 500 \text{ ft.},$$
$$S_1 = 25 \text{ ft.},$$
$$S_2 = 3 \text{ ft.},$$
$$S_3 = 12.5 \text{ ft.},$$
$$K_1 = 0.00306,$$
$$K_3 = 0.88,$$

$$q_2 = \frac{0.88 \times 25 \times 3}{500} = 0.00046 \text{ cu. ft. per min.},$$

$$h_3 = \frac{0.88 \times 25 \times 12.5 \times 500}{0.00306 \times 47 \times 25 + 0.00046 + 0.88 \times 25 \times 12.5} = 495 \text{ ft.}$$

which determines the position of the line of saturation.

$$q_3 = \frac{0.88 \times 25 \times 12.5}{495} = 0.555 \text{ cu. ft. per min.}$$

If the dam were 1 000 ft. long, the seepage would be 555 cu. ft. per min., or 9.25 cu. ft. per sec. The slope of the line of saturation is $\frac{25}{495} = 5\%$, or 1 on 20. In nearly any conceivable case, the very flat down-stream slope required would be so expensive as to be prohibitive. Some other material should be used for the dam, or, if this is not available, a core-wall should be added.

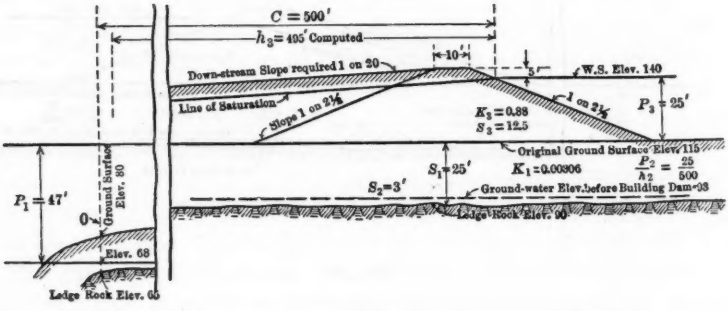

FIG. 19.

If it were not already clear from the foregoing that the material considered would be impracticable for the construction of a dam, it would be necessary to extend the analysis and take a point farther down-stream than *O*, where there is another sudden break in the surface of the ground, and of the ground-water, and then compute the rise in the surface of the ground-water at the point, *O*, and its effect on the position of the line of saturation, in accordance with the methods already outlined. The actual result would be that the slope of the line of saturation would be even flatter than shown in Fig. 19, which is based on the quite erroneous assumption that the ground-water elevation at *O* remains at the same elevation after the dam is in service.

Position of the Line of Saturation in Fig. 20.—As it is impracticable to build a dam of the cross-section shown to be required for the conditions outlined in Fig. 19, and as the material considered for use is assumed to be the only material available, the effect of using a core-wall and its proper depth, will now be investigated. As the foundation soil is quite impervious, it will be unnecessary to extend the core-wall to rock. It is assumed that the proposed core-wall will be merely sealed into the relatively impervious foundation soil. In Fig. 20 all conditions are the same as in Fig. 19, except that the core-wall has been added. First, assume that the core-wall extends only 5 ft. below the surface of the foundation soil. All the known factors will have the same values as in Fig. 19, except that S_3, instead of being the effective area of the triangle, will be the available discharge area below the core-wall. Assuming the core-wall to be tight, all the water must pass through the foundation soil, having a transmission constant, $K_1 = 0.00306$. The material composing the dam, therefore, plays a small part in preventing the passage of water through the dam, its function being merely to give stability. As ledge rock is 25 ft. below the surface, $S_3 = 20$ ft.

$$h_3 = \frac{0.00306 \times 25 \times 20 \times 500}{0.00306 \times 47 \times 25 + 0.00046 + 0.00306 \times 25 \times 20} = 148 \text{ ft.}$$

As the horizontal distance from the point where the head-water intersects the up-stream face to the down-stream toe is only 97.5 ft., if a slope of 1 on $2\frac{1}{2}$ is used, it appears that, even with a core-wall sealed 5 ft. into the foundation soil, a considerable part of the dam will be saturated. The water, after passing under the core-wall, will rise on the down-stream side until it reaches the line of saturation, as indicated in Fig. 20. With the condition assumed, a down-stream slope of about 1 on $4\frac{1}{2}$ would be necessary in order to contain the line of saturation.

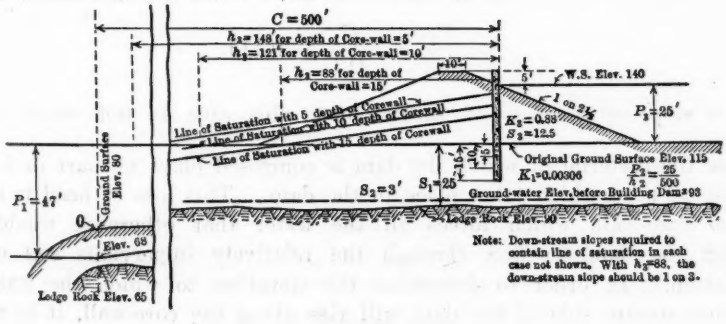

FIG. 20.

It is interesting, here, to note the effect of increasing the depth to which the core-wall is extended. All the known factors will have the same values as in the case just discussed, except S_3, which becomes a decreasing variable. If the core-wall is extended 10 ft. in the foundation, $S_3 = 15$, and, applying Equation (11), $h_3 = 121$ ft. Applying Equation (6), it is found that the seepage under the core-wall, $q_3 = 0.0095$ cu. ft. per min. per lin. ft. of dam.

With a core-wall placed 15 ft. in the foundation, $S_3 = 10$, and the value of h_3 would be 88 ft.

The line of saturation, therefore, will fall within the down-stream toe, about 10 ft., when the down-stream face has a slope of 1 on 2½. By Equation (6) it is found that the seepage under the core-wall, $q_3 = 0.00875$ cu. ft. per min. per lin. ft. of dam. If the dam were 1 000 ft. long, this leakage would be 0.146 cu. ft. per sec. In most cases, the loss of this quantity of water would not justify the additional expense of extending the core-wall to solid ledge rock. Where the additional work would not be expensive, it might be worth while to extend the core-wall to solid rock. However, in the case assumed, it would appear to be advisable to terminate the core-wall at a depth of 15 ft. in the foundation and use a down-stream slope of 1 on 3.

Where the core-wall was sunk 5 ft. into the foundation:

$$h_1 = C - h_3 = 500 - 148 = 352$$

and the slope of the ground-water below the dam would be:

$$\frac{P_1}{h_1} = \frac{47}{352} = 0.1336$$

Where the core-wall was sunk 10 ft. into the foundation soil:

$$h_1 = 500 - 121 = 379,$$

and the slope of the ground-water below the dam would be:

$$\frac{P_1}{h_1} = \frac{47}{379} = 0.124.$$

In the last case, where the core-wall was sunk 15 ft. into the foundation:

$$h_1 = 500 - 88 = 412,$$

and the slope of the surface of the ground-water below the dam would be:

$$\frac{P_1}{h_1} = \frac{47}{412} = 0.114.$$

The slope of the line of saturation in this case is not equal to $\frac{P_3}{h_3}$, because the material of which the dam is composed plays no part in causing the loss of head which takes place in the dam. This loss of head is caused by the core-wall, which forces all the water that otherwise would flow through the dam, to flow through the relatively impervious soil of the foundation. In order to determine the elevation to which the water on the down-stream side of the dam will rise along the core-wall, it is merely necessary to continue the slope—which has been determined for the surface of the ground-water—down stream from the dam, because, after the water has passed the core-wall, the loss of head per foot will be uniform, depending only on the ability of the foundation soil to transmit the water. In this discussion a theoretically tight core-wall has been assumed, which is never actually obtained in practice, but which may be closely approximated in high-class work.

Height of Water on Down-Stream Side of Core-Wall.—To determine the height above the base to which water will rise on the down-stream side of the core-wall, let

$$P = \frac{P_1}{h_1} h_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Thus, in Case 1, with the core-wall 5 ft. in the foundation, $P = 19.8$ ft. With the core-wall 10 ft. in the foundation, $P = 15$ ft., and with core-wall 15 ft. in the foundation, $P = 10.0$ ft.

The foregoing illustrates how the effect of a core-wall on the position of the line of saturation may be predicted. The same method applies to sheet-piling.

With cut-off walls of timber or steel sheet-piling, it should be remembered that they are seldom tight, and that some leak so badly that they have only little effect on the position of the line of saturation.

It was first noted by Mr. James B. Hays,* that the effect of sheet-piling on the loss of head of the water passing under the dam was similar to that produced by a partly closed valve in a pipe line. Similarly, as the sheet-piling or core-wall is placed nearer the ledge rock, the foregoing computations indicate that the effect is similar to that produced by closing a valve.

Position of Line of Saturation When Up-Stream and Down-Stream Parts of the Dam Are of Different Materials.—If the proper materials are available, it is generally recognized that the up-stream part of an earth dam should be of comparatively impervious material, and the down-stream part of as pervious material as is available. Such a dam will be much more stable than one built entirely of the material composing the up-stream part, and will tend to cause the line of saturation to fall much farther within the down-stream toe than would be the case with a homogeneous structure composed entirely of the relatively impervious material in the up-stream part of the dam.

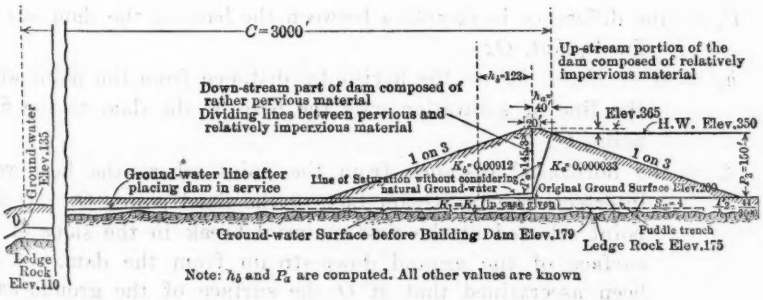
C=3000'
Up-stream portion of the dam composed of relatively impervious material
Down-stream part of dam composed of rather pervious material
Dividing lines between pervious and relatively impervious material
Elev.365
H.W. Elev.350
1 on 3
1 on 3
Ground-water line after placing dam in service
Line of Saturation without considering natural Ground-water
Original Ground Surface Elev.209
Ground-water Elev.135
Ground-water Surface before Building Dam Elev.179
Puddle trench
Ledge Rock Elev.175
Ledge Rock Elev.110
Note: h_3 and P_4 are computed. All other values are known

FIG. 21.

In Fig. 21 the up-stream part of the dam has been built of one material and the down-stream part of another. In the following discussion, the effect of the natural ground-water has been neglected. In order to apply the formula derived, without modification, the quantity of natural ground-water

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 20.

must be very small and its depth below the original surface at the site must be great. If there is much natural ground-water at the site, its effect on the position of the line of saturation may be computed in the manner described subsequently.

It is assumed that the material composing the down-stream part of the dam has a transmission constant, k_b, and that the soil composing the foundation and also the soil down stream from the dam has a transmission constant, k_1.

In Fig. 21, let,

k_a = the transmission constant for the material composing the up-stream part of the dam;

k_b = the transmission constant for the material composing the down-stream part of the dam;

k_1 = the transmission constant for the material composing the foundation soil;

P_3 = the difference in elevation between the head-water and the base of the dam;

P_a = the difference in elevation between the head-water and the point where the line of saturation cuts the plane dividing the two classes of material of which the dam is composed;

$P_b = P_3 - P_a$ = the difference in elevation between the above point and the base of the dam;

h_3 = horizontal distance from the head-water to the point where the line of saturation cuts the base of the dam $= h_a + h_b$;

h_a = the horizontal distance from the point where the head-water intersects the up-stream face to the vertical plane dividing the two classes of materials of which the dam is composed;

$h_b = h_3 - h_a$ = the horizontal distance from the vertical plane dividing the two classes of material;

P_1 = the difference in elevation between the base of the dam and the fixed point, O;

$h_1 = C - (h_a + h_b)$ = the horizontal distance from the point where the line of saturation cuts the base of the dam to the fixed point, O;

C = the horizontal distance from the point where the head-water intersects the up-stream face to the fixed point, O. (O is a point selected at the first material break in the slope of the surface of the ground down-stream from the dam. It has been ascertained that at O the surface of the ground-water changes and becomes much steeper. It is assumed that O is so far down stream that there is no rise in the elevation of the ground-water due to the construction of the dam. For this reason O is sometimes referred to as the fixed point.)

S_1 = the area of discharge per linear foot of cross-section through the foundation soil;

S_a = the effective area of discharge through the material composing the up-stream part of the dam = $\frac{2 P_3 - P_a}{2}$;

S_b = the effective area of discharge per linear foot of cross-section through the material composing the down-stream part of the dam. By the time the flowing water has passed through the up-stream part of the dam, it will be occupying the entire space between the line of saturation and bed-rock, accordingly, we have here taken

$$S_b = \frac{2 S_1 + P_3 - P_a}{2} \quad \ldots\ldots (14)$$

$$q_1 = k_1 \frac{P_1}{h_1} S_1 \quad \ldots\ldots (15)$$

This equals the total discharge through the foundation soil, in cubic feet per minute, after placing the dam in service.

$$q_b = \frac{k_b P_b S_b}{h_b} \quad \ldots\ldots (16)$$

This equals the total discharge through the material composing the down-stream part of the dam, in cubic feet per minute.

$$q_a = \frac{k_a P_a S_a}{h_a} \quad \ldots\ldots (17)$$

This equals the total discharge through the material composing the up-stream part of the dam, in cubic feet per minute. All discharges are, of course, per linear foot of dam.

As q_2, the discharge due to the natural ground-water at the site, is considered to be very small and its elevation low, $q_1 = q_a = q_b$. Therefore,

$$\frac{k_a P_a S_a}{h_a} = \frac{k_b P_b S_b}{h_b}$$

$$\frac{k_1 P_1 S_1}{h_1} = \frac{k_a P_a S_a}{h_a}$$

Substitute for S_b, its equivalent, $\frac{2 S_1 + P_3 - P_a}{2}$, and for h_1, its equivalent, $C - h_b - h_a$, and for P_b, its equivalent, $P_3 - P_a$. Equating and reducing, the following expressions are obtained for the values of the two unknowns, P_a and h_b:

$$P_a = -\sqrt{\frac{2 k_1 P_1 S_1 h_a + 2 S_1 P_3 k_b h_a + P_3^2 k_b h_a}{h_a k_a - k_b h_a - C k_a} + \left[\frac{C k_a P_3 - h_a k_a P_3 + S_1 k_b h_a + P_3 k_b h_a}{h_a k_a - k_b h_a - C k_a}\right]^2} - \frac{C k_a P_3 - h_a k_a P_3 + S_1 k_b h_a + P_3 k_b h_a}{h_a k_a - k_b h_a - C k_a} \quad \ldots\ldots (18)$$

$$h_b = \frac{k_b h_a (P_3 - P_a)(2 S_1 + P_3 - P_a)}{k_a P_a (2 P_3 - P_a)} \quad \ldots\ldots (19)$$

In solving for P_a, if an imaginary quantity is obtained, it indicates that the line of saturation does not cut the vertical plane dividing the two classes of material of which the dam is composed, but lies wholly within the up-stream part of the dam. If this occurs, Equation (11) should be used.

As previously stated, the foregoing equations do not take into account the effect of the natural ground-water at the site on raising the elevation of the line of saturation. If the quantity of ground-water is worth considering, its effect may be predicted in the following manner: After making the foregoing calculations, compute the discharge of the natural ground-water, q_2, using Equation (7). Add this discharge to that already found to be due to seepage through the dam. Apply Equation (8), with all factors known except $\frac{P_1}{h_1}$, the slope of the ground-water below the dam after the dam is placed in service. After computing this slope, the surface of the ground-water and the position of the line of saturation may be drawn or computed, starting from the fixed point, O.

Position of the Line of Saturation in Fig. 21.—In Fig. 21 it is assumed that the up-stream part of the dam is composed of a very fine silt having an effective size of 0.01 mm., that the porosity of the material is 30%, and that the ground-water temperature is 60° Fahr. From Table 1 the transmission constant is found to be $0.000033 = k_a$.

The down-stream part of the dam is assumed to be composed of an ordinary fine sand, having an effective size of 0.15 mm., a porosity of 32%, and a transmission constant of $0.00912 = k_b$ at a temperature of 60° Fahr. P_a and h_b are the unknowns, and the other factors involved are:

$$\begin{aligned} P_3 &= 150 \text{ ft.}, \\ h_a &= 40 \text{ ft.}, \\ P_b &= P_3 - P_a, \\ P_1 &= 65 \text{ ft.}, \\ h_1 &= C - (h_a + h_b). \\ C &= 3\,000 \text{ ft.}, \\ \frac{P_2}{h_2} &= \frac{44}{3\,000}, \\ S_2 &= 4 \text{ ft.}, \\ S_1 &= 25 \text{ ft.} \end{aligned}$$

Substituting in Equations (18) and (19), $P_a = 145.3$ ft. and $h_b = 123$ ft.

By Equation (8) the seepage is found to be 0.00523 cu. ft. per min. per lin. ft. of dam. The slope of the ground-water surface below the dam will be:

$$\frac{P_1}{h_1} = \frac{65}{2\,837} = 0.0229.$$

The slope of the line of saturation through the down-stream part of the dam will be:

$$\frac{P_b}{h_b} = \frac{4.7}{123.1} = 0.038.$$

The slope of the line of saturation through the up-stream part of the dam will be:

$$\frac{P_a}{h_a} = \frac{145.3}{40} = 3.75.$$

The effect of the natural ground-water on the position of the line of saturation has not been considered in the foregoing computations. Using Equation (7), the natural ground-water flow, $q_2 = 0.000535$ cu. ft. per min. By adding this quantity to q_1, already found, and solving Equation (8) for the new value of h, we have:

$$h_1 = \frac{0.00912 \times 65 \times 25}{0.00567} = 2630.$$

Hence, considering the effect of the natural ground-water, the line of saturation will fall well within the down-stream toe, as the distance from the point, O, to the down-stream toe is 2 490 ft.

Drainage of Earth Dams.—In many earth dams the down-stream part is drained artificially in order to lower the line of saturation and thus prevent the saturation of the down-stream toe. If the foundation is relatively impervious, as compared with the material of the dam, an artificial drainage system is especially applicable. If an earth dam is composed of fine sand and the foundation is of clay or rock, any water which finds its way into the comparatively porous dam cannot drain away through the soil of the foundation, and, consequently, the down-stream toe will be saturated, causing a dangerously unstable condition of the down-stream part of the bank.

In most cases of this kind a core-wall should be used, but even then artificial drainage may be necessary. In most instances where the foundation soil is of insufficient depth to provide natural drainage, or is too impervious to provide the necessary natural drainage, there should be a system of artificial drainage. Such drainage is often provided by digging trenches, several feet deep, perpendicular to the axis of the dam, which may be filled with broken stone, large one-man stones being placed on the bottom and the sizes decreasing toward the top, where fine crushed stone or gravel is used. Terra cotta sewer pipe may be laid in the trench, with open joints, and surrounded with crushed stone or gravel.

Drainage trenches should usually be perpendicular to the axis of the dam, but in some cases they should follow the contour of the ground. Several laterals, feeding the main drain, may be used to advantage. Such laterals generally consist of lines of 6 or 8-in. terra cotta sewer pipe laid, with open joints, directly on the ground. The joints should be amply protected by gravel or crushed stone.

The drains should not be laid too far apart, and, if drainage is to be effective, the distance between the main drains should not be more than one-fourth the thickness of the dam. The size and spacing of the drains and the quantity of water that they will collect may be predicted by the methods already outlined.

The drains should have an unobstructed outlet below the down-stream toe. As a part of such a system of drainage, heavy rock fills are often placed at the

down-stream toe of an earth dam. If rock is available, it is good practice to place a rock fill at the toe, as the stability of the structure is thus greatly increased, and the danger from sloughing is lessened. The maximum width of the base of such a rock fill should not, in general, exceed one-third the total thickness of the base of the dam.

General Remarks on the Position of the Line of Saturation.—Criterion 2 which requires that the line of saturation shall intersect the base of the dam well within the down-stream toe, is, perhaps, the most important consideration in a design, as it is the one most difficult to obtain with assurance both of safety and economy.

There are several points which must be emphasized, in applying the methods outlined previously for predicting the position of the line of saturation.

1.—The preliminary field data and sub-surface investigations must be thorough, as described previously.

2.—The position of the line of saturation depends, not only on the kind of material in the dam, but also on the kind of material below the base and down stream from the dam, and also on the depth to rock, clay, or other impervious material.

3.—Although the methods herein described enable the engineer to predict the approximate line of saturation in the cross-section under any given or assumed conditions, it is not wise to consider the results of the calculations as precise. There are so many uncertainties in the design of earth dams that it is well to err on the safe side.

4.—The laws governing the flow of water through soils, particularly as applied to earth dams, and the formulas and methods outlined should be thoroughly studied.

5.—The fixed point, O, should be selected at the first marked change in slope of the ground surface and the ground-water surface down stream from the dam site, as determined by borings or test-pits. The point, O, is seldom a fixed point, and good judgment is essential in its selection.

In most instances the assumption that the ground-water level at O does not rise after the dam is placed in service, would be erroneous. However, it has been shown that, after using the formula and determining the total quantity of water flowing through the soil, another fixed point, O', still farther down stream, at the next break in grade of the natural ground-water, may be selected. Then, with the total discharge known, the necessary rise in ground-water level at O can be computed and the other quantities then recomputed. If the break in grade at O is considerable, there will be less error in considering the point, O, as fixed than if the change in slope at O was slight.

6.—The use of S_3 equal to the mean ordinate from the base of the dam to the line of saturation, for the discharge area through the dam per linear foot of dam is somewhat arbitrary. When the dam is to be built of the same material as that in the foundation, or where it is coarser than that in the foundation, this assumption will not cause great error. If foundation soil,

however, is much coarser than the material in the dam, a serious error might be introduced by this assumption. In such cases, S_3 should be greater than the mean ordinate between the base of the dam and the line of saturation. In the extreme case where the material in the dam is entirely impervious and that of the foundation is very pervious, S_3 should be placed equal to S_1. In a dam with a core-wall, S_3 should be taken equal to the distance from the bottom of the core-wall to the first impervious layer below. If the core-wall is founded on an impervious layer of clay or rock, theoretically, there would be no water in the down-stream part of the dam, but actually some water would enter around the ends of the core-wall, through imperfections and cracks, and under it, through the ledge rock or clay. Even for these conditions the formulas may aid in predicting the probable line of saturation.

7.—It will be difficult to assign the proper effective size of the material to any particular cross-section, although numerous field samples may have been taken and analyses made. There may be a great variation both in the soil of the foundation and in the dam, and the change from a very impervious to a very pervious material is sometimes abrupt. An effective size should be selected somewhat higher than the average, so that, if an error is made, it will be on the safe side.

Criterion 3

Criterion 3 states that "the up-stream and down-stream slopes must be such that, with the materials used in the construction, they will be stable under all conditions."

Slope of the Up-Stream Face.—The slope of the up-stream face of the dam should generally be determined by the under-water angle of repose of the material. This angle, usually, will be flatter than the angle of repose for the same material in air. Other things being equal, a material having a high unit weight will stand on a steeper slope under water than one with a low unit weight. The slope of the up-stream face should be flatter than that indicated by the experiments as safe.

In general, the slope of the up-stream face, or, that part of it which is to be under water, should not be steeper than one-half that at which the same material would stand out of water.

The up-stream slope of some earth dams has been made as steep as 1 on 1½, but the writer believes that such steep slopes should be used only with the heaviest and most stable materials, and, even then, they should not be used in earth embankments more than 15 ft. high. A slope that is to be exposed to the action of water should generally not be steeper than 1 on 2.

Slope of the Down-Stream Face.—The slope of the down-stream face will generally be determined by Criterion 2, which requires the line of saturation to intersect the base at a point well within the down-stream toe. The slope of the down-stream face, however, should be flatter than the angle of repose of the material of that part of the dam even if the position of the line of saturation would permit the use of a steeper slope.

Berms.—In earth dams more than 30 ft. high, berms, from 6 to 20 ft. wide, should be used on the down-stream face. In high dams berms should

be used for about each 30-ft. difference in elevation. The main function of berms is to minimize the erosion from rain storms, the effects of which may be very severe. The outer edge of berms should be higher than the inner edge, to prevent rain-water from flowing over the edge and down the slope. A paved gutter should be placed at the inner edge of the berm to conduct the storm-water to the side of the valley, where other gutters conduct it to the toe of the dam. In many of the largest and highest earth dams, the storm-water from the berms is collected by catch-basins and conducted through storm sewers to the main drainage system at the down-stream toe of the dam.

On some dams, berms are built on the up-stream face, and a berm should always be used as a shoulder against which to build the bottom of the rip-rapping or concrete paving used as a protection against wave action.

Compacting the Embankment.—If the material in an earth dam is not compacted properly, shrinkage after the structure is placed in service may cause leaks or slides to develop. Dumping the material from trestles, without other means of compacting, is a most dangerous procedure and should not be tolerated. In India, the compacting has been done by the passage of the thousands of workmen employed. In some American dams, animals have been used to compact the material, but in most dams the material is compacted by rolling in thin layers or by the use of water.

When the soil is compacted with rollers, the material should be evenly spread in layers not more than 12 in. thick. With many materials the layers should not be more than 6 in. thick, and even then the top of a layer will be much more compact than the bottom. Under many conditions, compacting with water is the most economical and desirable method.

Excluding Vegetable Matter from Embankments.—Excessive quantities of vegetable matter should be excluded from the embankment, but small quantities of well-disseminated vegetable matter do no harm and may aid greatly in increasing the tightness of the embankment. As a rule, 6% is the limit of vegetable content that should be allowed in an earth dam embankment, and large roots, stumps, and trees should be excluded.

The effect of using soil containing a fairly high vegetable content was investigated during the construction of the Wachusett dikes. As a result of that study, soil containing 6 to 8% of vegetable matter was used in the up-stream parts of the dam and greatly increased the imperviousness of the structure. It was also decided that the time required for the vegetable matter to decay would be so great that it need not enter into the calculations.*

Settlement of Embankment.—With proper materials and proper compacting settlement will not be great, but some allowance for it should always be made. This allowance should be from 3 to 5% if the material has been properly compacted.

Should the Up-Stream or the Down-Stream Slope be the Steeper?—There has been some difference of opinion as to whether the up-stream or the down-stream slope should be the flatter. Some engineers contend that the up-stream slope should be the flatter and others have claimed the opposite.

* *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), p. 267.

A thorough study leads to the conclusion that neither viewpoint is correct, as sometimes the up-stream slope should be flatter than the down-stream, and *vice versa.*

Thus, if an earth dam is to be built of a fairly coarse material on a rather pervious foundation; the material in the dam will be stable under water on a 1 on 2 slope, but the line of saturation will be such that a slope of 1 on 4 is required for the down-stream slope.

Suppose, however, that the dam is to be built of a very fine, light, soil, which is quite impervious, but which has a flat angle of repose under water, and that the foundation soil has excellent drainage conditions, so that the slope of the line of saturation will be steep. It will be found that a down-stream slope of 1 on 2.5 will be sufficient to contain properly the line of saturation, and that the material will be stable in air at this slope; but the up-stream face will require a slope of 1 on 4 for stability under water.

These two examples show that each requires a separate and complete study.

Protection of Top and Down-stream Face of Dam.—All the exposed parts of the dam should be covered with 8 or 10 in. of rich top soil which should be treated with about 600 lb. of good fertilizer per acre. The surface should then be freshly raked and seeded to grass. A good seed mixture for this purpose is Red top, 12 lb., white clover, 6 lb., and Canadian blue grass, 10 lb. of the recleaned seed being used per acre. In the South, Bermudez grass and "wire grass" have been used with great success.

Matrimony Vine.—Matrimony vine (*Lycium Vulgare*) has been successfully used for bank protection along the Susquehanna River at Harrisburg, Pa., and also in protecting the slopes of the Wildwood Reservoir embankment, near Harrisburg. The shoots of this vine are planted, and the vine grows along the ground sending out roots, which enter the ground for some depth and then start other shoots. The vine, which was brought from the Mediterranean, is very hardy. In a few seasons, the bank presents the appearance of a continuous tangle of vines, and the top soil becomes a tangle of roots. Banks thus protected have withstood successfully occasional wave and stream action, but, of course, could not be depended on to resist continuous wave action.

Protection of Up-Stream Slope.—It is necessary on many dams, to protect the up-stream face from wave action. Stephenson gives the following equation for determining the height of waves in reservoirs:

$$H = 1.5 \sqrt{D} + 2.5 - \sqrt{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which H = the height of the waves, in feet, and D = the exposure, or "fetch", in miles. This formula would ordinarily require that the protection from wave action should extend from 2 to 5 ft. above normal water level. The depth of the protection depends on the depth to which the reservoir is likely to be drawn down.

Log booms placed in front of the slope have proved to be effective in breaking the waves before they strike the slopes. For many years the Spring Valley Water Company, of San Francisco, Calif., has protected the earth

slopes of its reservoirs by anchoring booms about 3 ft. from the up-stream slope.

In reservoirs that are emptied frequently the bank protection should begin at the up-stream toe. If it can be determined that the reservoir will be drawn down below a certain elevation only once in a term of years, it is sound engineering not to carry the bank protection much below that elevation, at which point a berm should be provided. If the water happens to be drawn down below that point, wave action will not cause sliding or injury to the paving or rip-rap.

Rip-rap.—There is no material superior to rip-rap for the protection of the up-stream face of an earth dam from wave action. Rip-rap is of two classes, random rip-rap and hand-placed. The former consists of stones dumped in place from cars or wagons or tossed in place by hand. If random rip-rap is dumped in place from cars, the individual stones may be any size up to the capacity of the steam shovel. Such rip-rap is sometimes termed cyclopean rip-rap.

Hand-placed rip-rap consists of one-man stones laid on edge on a bed that has been prepared and graded. An effort is made to break joints as much as possible, and the voids are filled with smaller stones. Hand-placed rip-rap is generally from 12 to 18 in. thick, the minimum size of individual stones frequently being specified as 12 by 12 by 3 in.

The best hand-placed rip-rap approaches good dry rubble in quality and appearance. The bottom of the band of rip-rap on the up-stream face of an earth dam should rest on a shoulder or berm in the embankment; otherwise, the weight of the rip-rap might be sufficient to cause it to slide down the saturated up-stream face. The bottom course of the rip-rap should be formed with headers composed of stones twice as deep as the other stones, and set into the bank in a trench. This aids in holding the upper courses in place.

Concrete-Lined Distribution Reservoirs.—Municipal distribution reservoirs are often lined with concrete, which is generally laid in the form of blocks, on graded slopes. Such linings are generally not relied on for water-tightness, but are intended primarily to furnish a surface that may be washed and cleaned when the reservoir is drawn down.

Monolithic Concrete Lining.—A concrete lining is sometimes used on the up-stream face of earth dams. It is generally poor practice to place much reliance on a concrete pavement for keeping water from entering the embankment. The function of such a lining, in the writer's opinion, is wave protection. Sometimes the concrete lining is built as a reinforced monolith, the percentage of steel being about 0.3% of the effective area of the cross-section of concrete. The writer believes that such monolithic linings are, in most cases, of questionable value. When the bank settles, the reinforced concrete lining will be left unsupported at some points, at which the slab will crack, and wave action may eventually break it up, allowing the waves to enter the gaps and work disastrous effect.

Concrete Lining of Square Blocks.—It is best to make the concrete lining of square blocks, generally not larger than 6 by 6 ft. It is not usually

necessary to reinforce these blocks. The thickness of the block in inches should be the same as the dimension of the block in feet, that is, a block 6 ft. square should be 6 in. thick. The square should be poured alternately, the blocks being separated by layers of 3-ply tar paper, so that they will adjust themselves to the surface of the embankment in case of settlement. Sometimes, pre-cast concrete slabs of much smaller size are used. Small blocks of wood, ½ in. thick, may be used to separate the slabs, thus insuring their settlement with the embankment and preventing bridging. Parallel to the center line of the embankment a concrete curb should be built against which to place the concrete lining. This curb should be at the inner edge of a berm or shoulder in the embankment and should extend not less than 18 in. below the bottom of the concrete lining.

If the concrete paving is monolithic, or if the concrete blocks are set without spaces between them, it is essential to provide numerous weep-holes through the concrete, in order to allow the water in the embankment to drain away when the reservoir is drawn down quickly. Otherwise, hydrostatic pressure behind the concrete lining might break it or cause it to slide down the slope, and possibly, also, cause the sloughing of some of the saturated embankment material.

Use of Concrete or Rip-rap a Question of Cost.—The use of concrete or rip-rap for protection against wave action should depend on the relative cost of the two materials at the site. A concrete lining, however, should never be credited with performing any of the functions of a core-wall, as such an assumption is a dangerous fallacy.

Relative Cost of Protecting Up-Stream Slope.—The protection of the up-stream face of dams from wave action may cost from one-half to one-fourth as much as the embankment. The protection of the up-stream slope of earth dams from wave action is very expensive, and is often a considerable proportion of the total cost of the structure. This protection often proves to be a much larger part of the total cost than is realized in advance of construction.

Flatter Up-Stream Slope Used Instead of Rip-rap or Concrete Lining for Wave Protection.—The slope which will withstand wave action successfully depends on the nature of the material composing it, and the height of the waves.

Heavy materials, containing from 5 to 10% of gravel and stones on a slope of 1 on 3, will, with any ordinary reach of reservoir, withstand wave action quite successfully. Unless the reach of the wind on the surface of the reservoir is small, fine sand and clay require such flat slopes, that for reasonable safety against wave action, it is cheaper to use rip-rap or other protection.

The writer recalls that because the up-stream face of a certain embankment was placed on a slope of 1 on 4, it was thought unnecessary to protect the slope with either concrete or rip-rap. The material was fine sand and the distance across open water in front of the face was about 2 miles. When the dam was placed in service, wave action promptly demonstrated that protection was essential. In a few days this exposed part was worn down to a slope averaging 1 on 10.

If the material of the up-stream face contains a little stone and gravel, some engineers adopt a fairly flat slope and use no rip-rap or other bank protection at the time the dam is built, with the understanding that the cost of such protection is merely being deferred. If the embankment withstands the wave action successfully, the expenditure for rip-rap may be saved or postponed.

Considerable danger is involved in this course, because after the engineer has turned the works over to the owners, serious erosion of the embankment may take place. The operating officials may not realize the importance of protection, and action is postponed until an alarming condition exists and the partial or total failure of the embankment results. The engineer who designed and built such a structure will be sure to receive much of the blame for the failure. Hence, it is generally better to be sure that the slope of the up-stream face is sufficiently flat for stability against wave action, or protect it when the dam is built.

Core-Wall to Protect Up-Stream Face Against Wave Action.—If the core-wall is placed so that it intersects the up-stream face of the dam about at the elevation of normal head-water, as in Fig. 22, it may frequently be used to protect the embankment against wave action. In Fig. 22 the distance, *a*, from the elevation of normal tail-water to the top of the core-wall, should be somewhat more than the highest waves to be expected. The pond, it is assumed, is never to be drawn down below normal head-water elevation more than the predetermined distance, *b*. Then, when the reservoir is drawn down to minimum elevation, wave action may wear the slope down to an elevation equal to head-water elevation minus $(a + b)$. If the wall is sufficiently strong to support the bank for the distance $(a + b)$, rip-rapping of the up-stream slope may be omitted. After the up-stream face has been worn down to the dotted line shown, the core-wall will be effective in protecting the remainder of the embankment.

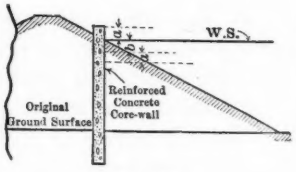

FIG. 22.

Core-walls such as those shown in Figs. 22 and 12 have been used in many power dams throughout the Middle West. At the Wissota hydro-electric project, near Chippewa Falls, Wis., this procedure saved about $60 000 worth of rip-rap.

Other advantages of placing the core-wall in this position in the cross-section are the increase in stability and the fact that the line of saturation is thus caused to fall farther inside the down-stream toe.

Slopes of Up-Stream and Down-Stream Faces.—Up-stream and down-stream slopes, which might otherwise be satisfactory, became unstable on an earth dam having an unstable foundation. It is sometimes necessary to flatten both the up-stream and down-stream slopes in order to obtain increased bearing area. A foundation soil that is stable and capable of sustaining a great load in its natural condition, may become very unstable when it becomes saturated after the dam is placed in service, and cause the sliding and subsidence of the slopes.

The following incident illustrates the manner in which the saturation of the foundation soil will sometimes produce a very unstable condition. The writer was connected with the construction of a hydro-electric project where it became necessary to re-locate a part of main line railroad. Where the railroad crossed an arm of the reservoir, a bridge was built, having 60-ft. reinforced concrete cantilever abutments which were also to serve a highway bridge. The foundation material was a fine sand of good bearing power, but the abutments were supported on 40-ft. piles which were jetted through the sand into a layer of gravel. The approach fill was built of coarse sand with side slopes of 1 on 4. The abutments and the fill were completed without mishap, but when the elevation of the water in the reservoir was within about 20 ft. of normal head-water, the fill back of the abutment suddenly subsided, more than 20 000 cu. yd. of material disappearing in an apparently unaccountable manner. The accident, luckily, happened on the highway side of the abutment, and the subsidence did not extend to the railroad. The abutment was not disturbed and careful measurements showed that the floor of the valley in front of it and outside the wing-wall had risen for several hundred feet, the maximum rise being 4 ft. The investigation showed that the fill had flowed out under the abutment and between the piles without disturbing the abutment. In order to refill the abutment, it was first necessary to place a counterweight fill of about 25 000 cu. yd. outside the abutment and wing-wall.

If an earth dam has an unstable foundation soil somewhat like the one just mentioned, a slide is much more likely to occur when the reservoir is partly filled, because at that time the foundation is saturated and practically the entire weight of the embankment is bearing on it.

When the reservoir is full, the effective weight of the material in the saturated part of the dam is decreased. As the down-stream part of the foundation is drained more or less, a slide, under the conditions just outlined, is more likely to occur on the up-stream face when the reservoir is only partly filled. This probably accounts for certain slides on the inside of reservoirs when they were drawn down.

Bearing Power of Foundation Soil.—Unless the bearing power of the foundation soil is unquestionable, it should be thoroughly investigated prior to construction. The Progress Report* of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations, etc., provides standard methods of making such tests.

Removal of Unstable Material.—If much unstable material is found in the foundation of an earth dam, it should be removed if possible. Owing to the depth to which the objectionable material extends, this procedure would sometimes be uneconomical. In such a case, a system of drainage should be provided in the down-stream half of the foundation, as described on page 891 under "Drainage". The slopes proposed for the structure should then be flattened, so that the actual foundation pressure would be within the allowable limits.

* *Proceedings*, Am. Soc. C. E., March, 1922, Papers and Discussions, p. 523.

Statistics of Earth Dams.—Table 7 gives data on a number of successful earth dams that have been gathered from various sources,* and from personal records.

Criterion 4

Criterion 4, for the design of earth dams, states that, "there must be no opportunity for the free passage of water from the up-stream to the down-stream face", which means the passage of water, through some well-defined seam or passageway, in quantities greater than would be indicated by the term seepage. If water under pressure has access to such a passageway, particles of the embankment will be dislodged, and the opening thus enlarged may cause the destruction of the dam if prompt action is not taken.

Cause of Free Passage of Water Through Dams.—Seams or openings through an earth dam may be caused in a number of ways, chief among which are the following:

1.—By water following the exterior surfaces of pipes or conduits through the embankment;
2.—By burrowing animals, such as muskrats;
3.—By the placing of very pervious material, such as large stones, in an otherwise impervious embankment, in such a manner as to make a blind drain from the up-stream to the down-stream face;
4.—By failure to bond and compact the succeeding layers of the embankment properly;
5.—By the use of a layer of very pervious material over a layer of impervious material;
6.—By failure to bond the lower layers of an earth dam properly to the foundation;
7.—By water following the smooth surfaces of concrete abutments or other concrete structures.

Danger from Pipes or Conduits Passing Through Earth Dams.—Pipes or conduits passing through earth dams have been a frequent source of trouble, and have caused the failure of many dams. Water from the reservoir has a tendency to follow along the smooth surface of such pipes or conduits. Concrete cut-offs, 12 in. thick or more, entirely surrounding the pipe and projecting several feet from it should always be used. A number of such cut-offs (never less than three) should be cast on any pipe line through an embankment. Even if the pipe through the embankment is well provided with concrete cut-off walls, the practice is not to be encouraged. The embankment will settle, and if the settlement is unequal, the pipe will be disturbed and distorted, the joints will leak or even break apart, probably causing a blow-out and the destruction of the embankment. The questionable procedure of supporting the pipe on piers from the foundation is sometimes adopted. If the pipe between supports is not sufficiently strong to support the embankment above, it will probably break when the material settles from under the

* Including, *Engineering News-Record, Engineering News, Engineering Record,* "Earth Dams", by Burr Bassell, *Transactions,* Am. Soc. C. E., "American Civil Engineer's Handbook", "Water-works Handbook".

pipe. If the pipe between supports is sufficiently strong to sustain the load, the earth shrinking away from the under side of the pipe leaves an opening for the free passage of water.

Pipes Through Earth Embankments.—The writer strongly believes that pipes and conduits should never pass through an earth embankment unless they rest on the original foundation material or are built on a solid masonry wall (provided with cut-offs) founded on the original foundation material and extending throughout the embankment. The best practice is to place pipes and conduits in trenches excavated in the original foundation material or in a trench excavated through the hill at the side of the dam.

Construction of Cut-offs in Pipe Trench.—Pipes placed in trenches excavated in the foundation should have a number of concrete collars. Where the concrete collars are to be constructed, a cross-trench should be dug in the bottom and sides of the main trench at least 24 in. Forms should be built so as only to block the main trench, thus allowing the concrete to fill all the space of the cross-trench. The concrete collar should be built at least 2 ft. above the top of the main trench. The concrete used for collars should be of about 1:2:4 mix, and just wet enough to puddle readily, as more water will cause shrinkage. There should be not less than three cut-offs throughout the width of the embankment. After the cut-off forms have been removed, the pipe trench should be carefully refilled, the earth being deposited in layers 3 or 4 in. thick and rammed thoroughly before the next layer is placed. Before depositing the next layer, the material in place should be lightly sprinkled with water, if it is not sufficiently moist, so as to insure a bond between layers. Trenches of this kind should not be puddled, as puddling may cause the fill to shrink away from the walls of the trench.

It would seem that the construction of a concrete cut-off is so simple as to require little attention. The writer, however, has seen many ineffective cut-off walls constructed. In some cases, the cut-off did not extend to the walls of the trench, whereas with others, the concrete merely touched, instead of projecting into, the sides of the trench.

The main advantage of a cut-off wall of the type advocated herein is the large surface contact of the cut-off with the undisturbed material of the foundation. In order to follow around the cut-off wall, water must pass through the original undisturbed foundation material. Fig. 23 shows the arrangement of such a cut-off wall.

Burrowing animals, particularly the muskrat, have been blamed for the failure of many earth dams. Woodchucks, snakes, beavers, eels, crawfish, mice, and rats, are also said to have injured earth dams. The muskrat, however, is apparently more dangerous than any of the others. Many of the recorded failures in America are due to his burrowing habits. In the days of the old canals, he was a terror to those charged with the maintenance of the embankments. Whenever a mill dam failed and the reason was not entirely clear, the muskrat received the blame. It is probable that many earth dam failures attributed to burrowing animals were really due to some

other cause. Most of such failures have occurred in small and unimportant structures.

Protection Against Burrowing Animals.—If a core-wall extending above high-water mark is included in the design, there is nothing to fear from burrowing animals. If the up-stream face is protected with rip-rap or with concrete slabs, they will have no opportunity to injure the embankment. If the material of the embankment is compacted thoroughly, the muskrat will go elsewhere in search of a home. The freeboard and top width of high earth dams are generally so great that the muskrat does not burrow through to the down-stream face.

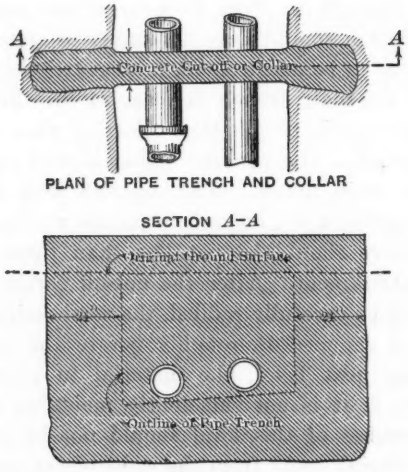

FIG. 23.

Accidental Blind Drain Through Embankment.—The careless placing of materials occasionally causes what is, in effect, a blind drain, from the up-stream to the down-stream face. Such a condition is avoided by preventing coarse material from forming a continuous line from the up-stream to the down-stream face. If the central pool of a hydraulic-fill dam is narrow, there is a tendency for tongues of coarse material to be projected across the pond. This is often combated by breaking up such tongues with plows.

Passage of Water Through Uncompacted Material.—If succeeding layers of the embankment are not properly compacted and bonded, there is a great chance for water to pass through the loose material. After work on an embankment has been suspended for a time, the surface may become hard and smooth. If additional layers are placed, without special precautions to secure a bond between the new work and the old, there will be danger of a leak along the plane of contact with the pervious uncompacted layer. The surface of the hardened layer should be scarified and moistened before the new layer is placed.

Passageway for Water Through Pervious Layer.—A comparatively free passageway for water is sometimes provided through the dam by a pervious layer lying between two impervious layers. Under such conditions the per-

vious layer forms a pipe, as it were, and water under a head flows through the layer to the down-stream face with sufficient velocity to move some of the particles of the material. Such layers in the up-stream part of the dam should be prevented by rejecting such material from the borrow-pits for use on the up-stream side of the dam and diverting it to the down-stream side.

When an embankment is built by using trains of dump cars running on track laid directly on the embankment, it is no simple matter to place the tight material on the up-stream part of the embankment, and the pervious material on the down-stream part. If the character of the material varies greatly, it is a source of great annoyance to the engineer and expense to the contractor.

Bonding Earth Dams to Their Foundations.—It is of the utmost importance to bond the lower layers of earth dams properly to the foundation material, in order to prevent the passage of water along any possible dividing plane between the foundation and the dam. All perishable material, such as stumps, brush, sod, roots, and vegetable matter, should be removed from the foundation. All top soil containing more than 6% of vegetable matter should also be removed. Otherwise, when the vegetable matter disintegrates, a plane of weakness will be left through the embankment along which water may find a more or less free passage.

After the ground surface at the site has been stripped, the earth should be plowed and scarified so as to form a good bond with the material of the dam. If no core-wall is planned, a trench should be excavated along the center line of the dam, or up-stream from the center line, and refilled with suitable material properly compacted.

Earth dams are sometimes built without the top soil or even the sod being removed. Such practice is reprehensible for reasons previously stated. Some engineers are so particular about the removal of all vegetable substances that they require the removal of all roots, no matter how small or how widely separated. As a matter of fact, a few small segregated roots in an embankment or foundation could not do the slightest harm.

Core-Wall to Prevent Passage of Water Through Dam.—The use of a substantial core-wall on a proper foundation will tend to prevent the free passage of water through the embankment. It is necessary, however, to observe the various precautions outlined herein, and to consider the use of a core-wall merely as a means of making assurance doubly sure.

Cut-Off Buttresses on Spillway and Power-House Walls.—Spillway abutments, power-house walls, or other concrete walls extending through the dam in an up and down-stream direction should be provided with cut-off buttresses projecting well into the embankment. There should be at least two buttresses on each wall passing through the embankment. It is unnecessary to make these buttresses more than 12 in. thick, if they serve only as cut-offs, but they should be lightly reinforced in order to prevent their separation from the main wall. The cut-off walls and the rear face of the main wall should have a slight batter (about 1 on 20), so that shrinkage of the embankment will tend to bring the earth in closer contact with the concrete.

Criterion 5

Criterion 5 for the design of earth dams requires that water which passes through or under the dam, must have a velocity so small that it is incapable of moving any of the material of the dam when it rises to the surface below the toe.

Piping and Blow-Outs.—It is conceivable that an earth dam might be built of materials that would prohibit water from passing through the dam. Such a dam, however, might be placed on a foundation so permeable that water would pass through it and issue on the down-stream side with sufficient velocity to move the particles of the foundation. Such action, which is generally called piping, is likely to cause a blow-out. Under certain conditions, piping may take place through the embankment itself, due to insufficient thickness.

Velocity of Water Required to Move Particles of Various Sizes.—The theoretical scouring velocity of the water passing through an earth dam or through its foundation may be found by computing the pressure due to jet action necessary to overcome the effective weight (weight in water) of the various sized particles. The equation for the pressure produced by a jet is:

$$P = \frac{2\,F\,C^2}{2\,g}\,Y^* \qquad (21)$$

in which,

P = the pressure produced;
C = the velocity of the water;
F = the area, in square feet, exposed to the jet;
g = the acceleration of gravity; and,
Y = the weight of a cubic foot of water.

For a condition of equilibrium which must exist, if the topmost particle, for instance, at the surface of the ground just below the dam, is to be just on the point of being moved, P must equal W_1, the effective weight of the particle. Hence, substituting W_1 for P

$$W_1 = P = 2\,F\,\frac{C^2}{64.4}\,62.4 \qquad (22)$$

from which,

$$C = \sqrt{\frac{W_1}{2\,F}} \text{ (very nearly)} \qquad (23)$$

in which,

C = the velocity of the water as it leaves the ground;
W_1 = the effective weight of a particle of the material; and,
F = the area of the particle exposed to jet action.

As a basis for the computation of Table 3, it has been assumed that W, the weight of the material, is 159.5 lb. per cu. ft., and that W_1, the effective weight, is 97 lb. per cu. ft.

Application of Table 3, Permissible Velocities.—In using Table 3, a high factor of safety should be provided; first, because the theory is not precisely

* Church's "Mechanics", p. 803.

applicable to the action known as piping; and, second, because the actual effective sizes of the particles in any plane will vary greatly from the size selected. Even coarse gravel contains some fine particles, and if the velocity of the water flowing through it is sufficient to move these fine particles, the velocity will increase until larger and larger particles are moved. Also, water passing through the gravel layer, at high velocity, will dislodge and move finer material immediately adjacent. For these reasons, it would not be safe to use a permissible velocity based on that required to move particles of the material. For example, with a material having an effective size of 1.0 mm. (fine gravel), the theoretical velocity required to move particles is (Table 3) 19.4 ft. per min. With a factor of safety of 4, a velocity of 4.8 ft. per min. is obtained, which is sufficient to dislodge and move all particles up to 0.8 mm. in diameter. For this reason a permissible velocity should be chosen so small, in most cases, that it will not move even the fine silt. The theoretical velocity required to move silt having particles with a diameter of 0.01 mm. is found from Table 3 to be 1.94 ft. per min. With a factor of safety of 4, a permissible velocity of 0.5 ft. per min. is obtained, which is the permissible velocity recommended for general use. The permissible velocity through clay would be much less, but the thickness of clay required to reduce the motion of the water passing through it, to less than a scouring velocity, is so small that it is not pertinent to this discussion. It will be shown subsequently that, under a 120-ft. head of water, a thickness of 0.237 ft. of silt (having an effective size of 0.03 mm.) is sufficient to reduce the velocity of water passing through it to 0.34 ft. per min., which is too small to move even the finest particles.

TABLE 3.—THEORETICAL VELOCITY REQUIRED TO MOVE PARTICLES OF VARIOUS SIZES BY JET ACTION.

Diameter of particle, in millimeters.	Velocity, in feet per minute, required to move by jet action.	Diameter of particle, in millimeters.	Velocity, in feet per minute, required to move by jet action.
5.0	43.4	0.1	6.0
3.0	33.7	0.08	5.5
1.0	19.4	0.05	4.3
0.8	17.4	0.03	3.4
0.5	13.8	0.01	1.94
0.3	9.6		

Necessary Thickness of Dam to Prevent Piping.—For any given height of dam and given material, the thickness along any horizontal plane must be sufficient to reduce the velocity of the water passing through the material, so that the finest particles composing the embankment will not be moved. Slichter's formula is $Q = K\frac{p\ s}{h}$, in which,

Q = the discharge, in cubic feet per minute;

s = the cross-section of area, in square feet, of the material through which the discharge is taking place;

K = the transmission constant of the material (see Table 2) composing the dam or foundation;
p = the difference in elevation between the two given points; and,
h = the horizontal distance between these two points.

For unity cross-sectional area,

$$Q = K\frac{p}{h} \quad \ldots\ldots(24)$$

In order to obtain an expression for velocity, it is merely necessary to divide K by P, the porosity of the material. This velocity must not exceed the permissible velocity, which for the general case may be taken as 0.5 ft. per min. Therefore,

$$v = \frac{K\,p}{P\,h} \quad \ldots\ldots(25)$$

in which all quantities are known except h.

Therefore,

$$h = \frac{K\,p}{P\,v} \quad \ldots\ldots(26)$$

in which, h is the necessary thickness of embankment or base at the given head, p, to prevent piping. In using this formula, it should be borne in mind that, with very fine materials, h might be less than 1 ft.; in which case the width of base and the side slopes will be determined by other criteria. With some materials, h will require a great width of base, and this criterion may determine the dimensions of the structure.

Required Thickness of Base for Safety Against Piping.—Assume that the height of an earth dam is to be 100 ft., that the material of the dam is very impervious, and that the foundation soil is quite pervious, having an effective size of 0.80 mm. and a porosity of 34 per cent. From Table 1, the transmission constant for this material is found to be 0.3175 for a temperature of 60° Fahr. That is, if the difference in head is 1 ft. and the path of the water through the material is 1 ft., the discharge will be 0.3175 cu. ft. per min. per sq. ft. of cross-section. The discharge per square foot of water area will be $\frac{0.3175}{0.34} = 0.934$, which is also equivalent to the velocity of the water, in feet per minute. Assuming a permissible velocity of 0.5 ft. per min., and substituting in Equation (26), we have,

$$h = \frac{0.3175 \times 100}{0.34 \times 0.5} = 186.7,$$

which is the necessary thickness of the embankment at the base to insure safety against piping. As a dam 100 ft. high with slopes as steep as 1 on 2 would be about 420 ft. thick at the base, it is clear that the proposed structure will be safe against piping.

Taking another case: Assume that the height of the dam is to be 120 ft., that the effective size of the material of the dam and the foundation is 0.03 mm. (silt), with a porosity of 30 per cent.

From Table 1, $N = 0.000296$ at 60° Fahr. Let the permissible velocity, $V = 0.5$ ft. per min. Substituting in Equation (26), we have,

$$h = \frac{0.000296 \times 120}{0.30 \times 0.5} = 0.237$$

A very thin layer of this fine material will reduce the velocity of the water through it so that it will be insufficient to move any of the material. Other considerations control the necessary thickness of an embankment, where such fine materials are used. It may also be stated that the formula is useful in determining the thickness of fine material to be placed on the up-stream face of a leaky embankment in order to reduce the leakage to a predetermined quantity.

As another illustration of the use of the formula, assume that, at the site of a proposed earth dam, the foundation is of gravel, about 20 ft. in thickness, below which is a layer of more impervious material. Loss of water, it will be assumed, is not important if it does not affect the stability of the dam. It is desired to determine whether it will be necessary to remove this gravel, in whole or in part, from the foundation, or whether a safe and economical structure can be built on this material by increasing the width of the base. The effective size of the gravel in the foundation is 4.0 mm., and the porosity is 38 per cent. The maximum head of water back of the dam is to be 120 ft. Table 1 shows the transmission constant to be 11.45, at 60° Fahr. V, the permissible velocity, is taken as 0.5 ft. per min.

$$h = \frac{11.45 \times 120}{0.38 \times 0.5} = 7\ 230 \text{ ft.}$$

For safety, then, the thickness of the dam would have to be at least 7 230 ft., and the leakage, as computed by the Slichter formula, would be excessive. This thickness of embankment is impracticable, and some other means of overcoming the difficulty must be adopted such as, for instance, the excavation of a puddle trench, or the placing of a core-wall or driving sheet-piling to an impervious stratum.

In another case, it will be assumed that the effective size of the material is 1.0 mm. and the porosity is 36 per cent. The height of the dam is 120 ft., equal to the maximum head of water. The transmission constant, from Table 2, is 0.5980 at 60° Fahr. V, the permissible velocity, is taken as 0.5 ft. per min.

$$h = \frac{0.5980 \times 120}{0.36 \times 0.5} = 399 \text{ ft.}$$

Hence the thickness at the base of the dam must not be less than 399 ft. for safety against piping.

The illustrations just given serve to show the manner in which Equation (26) may be used to obtain the necessary thickness at the base of an earth dam (or at any other point), to secure safety against piping.

Necessary Depth of Sheet-Piling or Cut-Off Wall.—It is immaterial how the distance, h, is obtained, that is, it may be more economical to get part of h by driving sheet-piling, instead of building a thicker embankment or by placing a heavy blanket of impervious material on the bottom of the reservoir

up stream from the dam. In a dam with a core-wall or sheet-piling, h should be measured in a straight line from the up-stream toe to the bottom of the core-wall or sheet-piling, and thence in a straight line to the down-stream toe. Equation (26) is believed to furnish a more logical method of determining the proper length of sheet-piling or depth of core-walls for safety against piping, than other methods in general use. The depth to which sheet-piling should be driven has been generally determined by empirical formulas and individual judgment.

Koenig's Formula for Depth of Sheet-Piling.—The following empirical formula for the proper depth of single-row, sheet-piling under dams, is given by Mr. Arnold C. Koenig:*

$$d = 2h \text{ for low heads, up to 8 ft} \quad \ldots\ldots(27)$$

$$d = h + 5 \text{ for heads up to 25 ft} \quad \ldots\ldots(28)$$

$$d = h \text{ for heads of more than 30 ft} \quad \ldots\ldots(29)$$

In these equations, d is the necessary depth of penetration of the sheet-piling and h is the maximum head of water back of the dam. The equations are intended to apply particularly to overflow dams where the width of base, including the apron, is about 2.5 h. The equations are not logical, as the vastly varying capacity of different soils to transmit water is not considered. The depth of sheet-piling required for an impervious material would be the same as that for porous sand. These equations are of interest, principally in showing the rough empirical method that has generally been used. Sheet-piling under a masonry structure on a soft foundation, in addition to making the structure safe against piping, must sometimes perform the function of making it safe against blow-outs. Thus, a foundation soil might be practically impervious, so that there was no possibility of piping, but the structure might be so narrow that there would be a blow-out. This factor does not require consideration in the design of earth dams, because of the relatively great width of the base.

Proper Depth of Sheet-Piling Under an Earth Dam.—As stated previously, the foundation material of an earth dam may be so pervious as to require an excessive thickness of base for safety. Thus, in the case discussed previously, of an earth dam 120 ft. high, founded on a gravel foundation having an effective size of 4.0 mm. and a porosity or 38%, it was found that, for safety against piping, the thickness of the base would have to be 7 230 ft.

Now assume that at a depth of 25 ft. there was a thick layer of fine sand having an effective size of 0.10 mm. and a porosity of 30% at 60° Fahr., and consider what may be accomplished by extending a core-wall 6 ft. into this fine sand, giving a total depth of 31 ft. for the core-wall below the base of the earth dam. As the material of the dam is assumed to be quite impervious, it is only necessary to extend the core-wall up into the dam enough to secure a seal, say, 6 ft. Whether this distance is sufficient can readily be determined by the methods used in determining the proper distance to extend the core-wall into the layer of fine sand. If the dam is to be built of pervious material, it would be necessary to extend the core-wall above high-water mark.

* *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), p. 182.

As the core-wall is to extend 6 ft. into the layer of fine sand, the shortest path of the water will be down along the up-stream side of the core-wall and thence up along its down-stream side, thus making 12 ft. the minimum length of travel. Most of the water, however, will travel a much greater distance than 12 ft. through the fine sand. From Table 1 the transmission constant for this material at 60° Fahr. is found to be 0.003282. Taking Equation (26) and substituting V_i (the issuing velocity) for V (the permissible velocity),

$$V_i = \frac{K\,p}{P\,h} \qquad (30)$$

Substituting,

$$V_i = \frac{0.003282 \times 120}{0.30 \times 12} = 0.1094 \text{ ft. per min.}$$

By reference to Table 3, it will be noted that this velocity will not move even the finest silt. It apears therefore, that, to make the structure safe against piping, it would be necessary to construct a core-wall at least 6 ft. into the fine sand. Steel sheet-piling might be substituted for the core-wall, but there would be no assurance of accomplishing the same result, as one can never be sure that the sheet-piling is tight. The difficulty might be overcome by excavating a cut-off trench into the fine sand and refilling it with impervious soil, properly compacted. Fig. 24 shows the probable saving in the dimensions of this dam by the substitution of the core-wall. It should be remembered that, because an earth dam satisfies one criterion, it is not proof that it is safe, because it must, in addition, satisfy each of the other criteria.

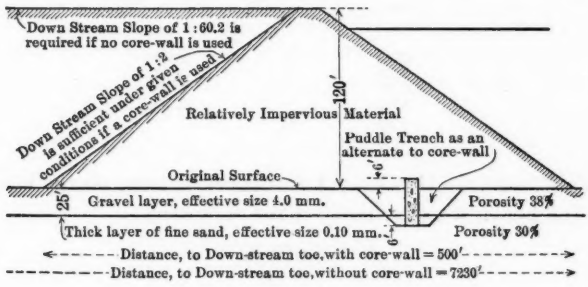
Down Stream Slope of 1 : 60.2 is required if no core-wall is used
Down Stream Slope of 1 : 2 is sufficient under given conditions if a core-wall is used
120'
Relatively Impervious Material
Puddle Trench as an alternate to core-wall
Original Surface
25
Gravel layer, effective size 4.0 mm.
Porosity 38%
Thick layer of fine sand, effective size 0.10 mm.
Porosity 30%
Distance, to Down-stream toe, with core-wall = 500'
Distance, to Down-stream toe, without core-wall = 7230'

FIG. 24.

CRITERION 6

Criterion 6 for the design of earth dams states that the freeboard must be such that there will never be any danger of overtopping. Freeboard may be defined as the difference in elevation between the top of the spillway and the top of the dam. Freeboard seems to be determined quite largely by custom. Small and unimportant earth dams have generally been given a freeboard of 3 to 5 ft., ordinary structures from 6 to 10 ft., and the larger and more important earth dams 10 to 30 ft.

Top Width of Earth Dams.—The top width of earth dams is a factor in the protection of the structure against wave action, as a wide top is much safer against wave action than a narrow one. A usual rule is to make the top

width equal to one-fifth of the height of the dam plus 5 ft. Another rule is to make the top width of the dam equal to 25% of its height. The top width of the earth dam, however, should never be less than 10 ft.

Failure of Earth Dams by Overtopping.—Partial or complete failure is certain if an earth dam is overtopped. Many earth dams have failed from this cause. The Jamestown flood is perhaps the most startling illustration of such a failure. The spillway capacity must be sufficient to care for any flood which can be predicted in a long term of years.

Factors Making Up the Proper Allowance for Freeboard.—The proper freeboard for any particular dam depends on three factors, to which should be added an allowance for safety. These three factors are: (1) The maximum depth to which frost will penetrate; (2) the height to which waves will rise on the up-stream slope; (3) the height above the spillway or flashboard crest to which the greatest floods will raise the water.

(1).—Depth of Frost.—For any locality, data on the depth of the frost line may be readily obtained, particularly from municipal water departments, and the proper allowance should be made.

(2).—Height of Waves on Up-Stream Face of Dam.—The Stevenson formula is most generally used for computing the probable height of waves. Table 4 gives the height of waves, in feet, for various lengths of exposure, in miles, and is taken from Fanning.*

TABLE 4.

Exposure, in miles.	Height of waves, in feet.	Exposure, in miles.	Height of waves, in feet.
0.25	2.54	2.00	3.33
0.50	2.76	3.00	3.78
0.75	2.87	5.00	4.44
1.00	3.00	10.00	5.47
1.50	3.03		

Hawksley's Formula.—Hawksley,† from observations on the height of waves produced by the heaviest gales, derived the following:

$$h = 0.025 \sqrt{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which

h = the height of the waves, in feet; and
l = the fetch, in feet.

The formula is also given in the following form.

$$h = 1.95 \sqrt{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which

h = the height of the waves, in feet; and
f = the fetch, in miles.

Henny's Wave Formula.—Mr. D. C. Henny‡ gives data on the observed height of waves in the Belle Fourche Reservoir, in South Dakota. The

* "Practical Treatise on Water Supply Engineering."
† *Minutes of Proceedings*, Inst. C. E., Vol. XX, p. 361.
‡ *Engineering News*, Vol. 75, p. 1235.

heights of waves were observed at different times during 1915, with the results shown in Table 5.

TABLE 5.

Wind velocity, in miles per hour.	Height of waves, in feet.	Wind velocity, in miles per hour.	Height of waves, in feet.
35	2.0	50	3.5
40	2.5	56	4.0
44	2.5	75	5.0
48	3.5		

The water in the reservoir during the period of observation was within 5 to 8 ft. of full reservoir elevation, and the fetch varied little from 5 miles.

It was found that the formula:

$$H = 0.075\ (V - 8.5) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

gave results agreeing closely with the observed data. In the Henny formula, Equation (33), H is the total height of the waves, in feet, from trough to crest, and V is the velocity of the wind, in miles per hour. The Stevenson formula for a fetch of 5 miles gives 4.35 ft. as the maximum wave height, which corresponds with the results obtained by using the Henny formula for the Belle Fourche Reservoir with a velocity of 60 miles per hour. Stevenson also gives the formula:

$$h = 1.5\ \sqrt{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which h is the height of the waves, in feet, and f is the fetch, in miles, for great distances of fetch and deep-water conditions.

Table 6, gives a comparison between the observed and computed height of waves.*

TABLE 6.

Location.	Fetch, in nautical miles.	Observed height, in feet.	Calculated Height, by Formula.		
			a.	*b.*	*c.*
Duluth Basin	0.375	1.5	....	2.6	1.2
" "	0.428	2.3	....	2.7	1.3
" "	0.641	2.0	....	2.8	1.6
" "	0.748	3.8	....	2.9	1.7
St. Louis Bay	0.916	2.0	....	2.9	1.9
Portage Lake	1.086	3.0	....	3.1	2.0
Firth of Forth	1.3	1.8	....	3.0	2.2
St. Louis Bay	1.923	4.5	....	3.4	2.7
Stannard Rock	41.45	11.0	9.7	....	....
San Pedro Bay, Calif	15.64	7.0	5.0	....	....
Marquette, Mich	116.63	15.0	16.2	....	....
Portage Breakwater	82.5	16.5	13.6	....	....
Duluth Canal	258.62	23.0	24.1	....	....
Clyde	9.0	4.0	4.5	....	....
Lake Geneva	30.0	8.2	8.2	....	....
Sunderland	165.0	15.0	19.3	....	....
Petershead	400.0	22.6	30.0	....	....

* From Mead's "Hydrology", p. 106.

In Table 6: a is Stevenson's formula,

$$h = 1.5 \sqrt{f}$$

b is Stevenson's formula,

$$h = 1.5 \sqrt{f} + (2.5 - \sqrt{f})$$

c is Hawksley's formula,

$$h = 0.025 \sqrt{l}$$

Cunningham* gives the following recorded height of waves in heavy storms:

Lake Geneva	10 ft.
German Ocean	12 to 15 ft.
Mediterranean Sea	15 to 20 ft.
Bay of Biscay	25 to 30 ft.
Atlantic Ocean	30 to 40 ft.
Pacific Ocean	50 to 60 ft.

The formulas just presented and the data in Table 6 give the height of waves from trough to crest. Even more important than this to engineers is the height to which waves will rise above mean still water. Gaillard† gives the formula:

$$a = \frac{h}{2} 8 c' \frac{h^2}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which,

a = the height of the wave above mean still water;
h = the total wave height;
l = the wave length; and
c' = a coefficient, found to be about 2 for a mean depth of 26 ft. in the Duluth Canal.

The length of waves is generally about twenty times their height.

Waves will mount higher on an embankment than their own height, depending on the slope of the embankment. They will mount higher on a flat slope than on a steep one. There are no exact data available on this matter, but if a factor of safety of 2 is allowed, it will generally be sufficient. Hence, the freeboard allowance for wave action should be $2a$, or, roughly, an allowance equal to h should be sufficient.

(3) Freeboard Must Include Allowance for Greatest Height of Floods.—In deciding on the freeboard for any particular dam, an allowance, in addition to those made for frost and height of waves, must be made to cover the height above the spillway to which the greatest flood may be expected to rise, plus a factor of safety. It is not within the province of this paper to discuss the design of spillways.

The length of spillway adopted will depend largely on economic considerations. Thus, if the cost per linear foot of spillway is very high in proportion to the cost per linear foot of earth dam section, it may be economical to use a shorter spillway and increase the height of the dam. This condition might

* "Harbor Engineering".

† "Wave Action in Relation to Engineering Structures", *Professional Papers No. 31*, U. S. Army, 1904.

TABLE 7.—STATISTICS OF SUCCESSFUL EARTH DAMS.

Name of dam.	Location.	Maximum height, in feet.	Freeboard, in feet.	Up-stream slope.	Down-stream slope.	Top width, in feet.	Remarks.
*Calaveras	Calif.	240	10	1 on 3	1 on 2½	25	Semi-hydr.-fill.
Little Bear Valley	Calif.	200	15	1 on 2½	1 on 2	20	Concr. core-wall.
*Necaxa, No. 2	Mexico	190	16	1 on 3	1 on 2	54	Puddle core, hydr.-fill.
Terrace	Colo.	180	15	1 on 3	1 on 2	25	Concr. core-wall and puddled core. Hydr.-fill.
Swift (a)	Mont.	165	.....	1 on 1.25	.....	.....	.....
*Linville	N. C.	160	24	1 on 3	1 on 2½	20	Semi-hydr.-fill.
Paddy Creek	N. C.	160	30	1 on 2.715	1 on 2.268	35	Puddled semi-hydr.-fill.
Terrace	Colo.	157	5	1 on 4	1 on 2	20	.....
Goose Creek	Idaho	145	7.5	1 on 3	1 on 2	16	Reinf. concr. core-wall.
Idaho Irr. Co.	Idaho	135	.....	1 on 3	1 on 2.5	40	.....
Patillas	Porto Rico.	135	.....	1 on 3 and 1 on 2	1 on 2	20	Semi-hydr.-fill.
Ochaco	Ore.	126	15	1 on 2.5	1 on 2	18	Hydr. sluicing.
San Leandro	Calif.	125	5	1 on 3	1 on 2.5	28	Puddled hydr. sluicing.
Catawba	N. C.	120	24	1 on 3	1 on 2.5	20	Semi-hydr.-fill.
Englewood	Ohio	124	16.5	1 on 2 to 1 on 4	1 on 2 to 1 on 4	25	Hydr.-fill.
Tabegud	Calif.	123	8	1 on 2.5, 1 on 3	1 on 2.5	20	6 and 8-in. layers, rolled.
Owl Creek	S. Dak.	122	15	1 on 2 and 1 on 5	1 on 2	19	6-in. layers, rolled.
Lahontan	Nev.	120	12	1 on 3	1 on 2	20	Thin layers, rolled.
Apishapa	Colo.	120	7.8	1 on 2 and 1 on 3	1 on 2	19	12-in. layers, rolled.
Druid Lake	Md.	119	5	1 on 4	1 on 2	60	Puddled core.
Dodder	Ireland	115	.....	1 on 3	1 on 3	22	Puddled core.
Belle Fourche	S. Dak.	115	15	1 on 2 and 1 on 3	1 on 1.75 and 1 on 2	20	Timber core, 6-in. layers.
Temescal	Calif.	115	5	1 on 3	1 on 5	18	.....
*Standley Lake	Calif.	113	5	1 on 2 and 1 on 3	1 on 2	20	Puddled core, no rolling.
Ashokan Dikes	N. Y.	110	20	1 on 2 and 1 on 2.5	1 on 2 and 1 on 2.75	34	Concr. core; Rolled layers, 4 and 6-in.
Titicus	N. Y.	110	9	1 on 2.4	1 on 2.5	30	Masonry core. Rolled layers.
Beaver Park	Colo.	108	.....	1 on 2	1 on 1.5	16	Reinf. concr. paving, up-stream face.
Mudduk	India	108	.....	1 on 3	1 on 2.5	....	.....
Gatun	Panama	115	30	1 on 4 and 1 on 7.67	1 on 4, 1 on 8, and 1 on 16	....	Semi-hydr.-fill.
Germantown	Ohio	107	15	1 on 2 to 1 on 4	1 on 2 to 1 on 4	25	Hydr.-fill.
Quemahoning	Pa.	106	13	1 on 3 and 1 on 4	1 on 3 and 1 on 4	20	Hydr. sluicing.
Cavite	Porto Rico.	105	15	1 on 2.75	1 on 2	20	Rolled layers.
Somerset	Vt.	105	10	1 on 3	1 on 2.5	....	Semi-hydr.-fill.
Cummum	India	102	12	1 on 3	1 on 1	....	.....
Dale Dike	England	102	.....	1 on 2.5	1 on 2.5	12	.....
Morris	Conn.	100	8.75	1 on 2.5 and 1 on 3	1 on 2	20	Concr. core. 6-in layers.
Crane Valley	Calif.	100	.....	1 on 2	.....	20	Hydr. sluicing.
Waialua	Hawaii	98	10	1 on 4	1 on 1.5	....	Timber core. Down-stream part, rock fill.
Cold Spring	Ore.	98	10	1 on 3	1 on 2	20	Rolled layers.
Honey Lake	Calif.	96	6	1 on 3	1 on 2	20	Puddled core.
Pilarcitos	Calif.	95	5	1 on 2	1 on 2	24	Puddled core. Rolled layers.
Waghad	India	95	14	1 on 3	1 on 2	6	.....
Bradfield	England	95	.....	1 on 2.5	1 on 2.5	12	.....
Temescal	Calif.	95	.....	1 on 4.5	.....	12	Puddled core.
San Andreas	Calif.	93	5	1 on 3.5	1 on 3	24	Puddled core. Rolled layers.
Edgelaw	Scotland	93	.....	1 on 3	1 on 2.5	....	.....
Sevier Bridge	Utah	92	13	1 on 3	1 on 4	32	Core-wall, steel and concr.
Haiwee	Calif.	91	10	1 on 2.5	1 on 2.5	20	.....
Lewiston Sweetwater	Idaho	85	.....	1 on 3	1 on 2	16	Semi-hydr. fill.

*These dams had slides during construction.

TABLE 7.—(*Continued.*)

Name of dam.	Location.	Maximum height, in feet.	Freeboard, in feet.	Up-stream slope.	Down-stream slope.	Top width, in feet.	Remarks.
Yarrow	England	87	6	1 on 3	1 on 2	30	Puddled core.
Forrest Park	Md	87	5	1 on 3	1 on 2.5	15	Puddled core.
Turdoff	Scotland	85	.....	1 on 3	1 on 2.5	10	.....
Main Channel, Snake River	Idaho	86	.....	1 on 4	1 on 1.5	20	Rock fill.
Vehar	India	84	6	1 on 3	1 on 2.5	24	Rock fill.
North Dike, Wachusett	Mass.	82	15	1 on 2	1 on 33⅓ and 1 on 16⅔	189	Timber core. Rolled layers.
Sherburne Lake	Mont	83	10	1 on 3	1 on 2	22	6-in. layers, rolled.
Cold Spring	Ore.	82	7	1 on 3	1 on 2	20	Hydr. sluicing.
Middle Dam, Snake River	Idaho	81	.....	1 on 4	1 on 1.5	20	Timber core. Down-stream part rock fill.
Roddlesworth	England	80	12	1 on 3	1 on 2.5	16	.....
Minidoka	Idaho	80	10	.....	.....	25	Concr. core-wall.
Gladhouse	England	79	10.5	1 on 3	1 on 2.5	12	.....
Taylorsville	Ohio	78	19	1 on 2 and 1 on 3	1 on 2 and 1 on 3	25	Hydr.-fill.
Rake	England	78	.....	1 on 3	1 on 2	....	.....
Lockington	Ohio	78	16	1 on 2 to 1 on 3	1 on 2 to 1 on 3	25	Hydr.-fill.
Silsden	England	78	.....	1 on 3	1 on 2	....	.....
Talla	Scotland	78	.....	1 on 4	1 on 3	20	Puddled core.
Dobbins Creek	Calif.	77	6	1 on 3	1 on 2	6	Hydr.-fill.
Glencourse	Scotland	77	.....	1 on 3	.....	....	.....
Throttle	N. Mex.	77	8	1 on 2.5	1 on 1.5	25	Steel core.
Wayoh	England	76	.....	1 on 3	1 on 2.5	22	.....
Ekruk	India	75	.....	1 on 3	1 on 2	20	.....
Upper Crystal Springs	Calif.	75	.....	1 on 2	1 on 2	....	Puddled core. Rolled layers.
Sanguijuella	N. Mex.	75	.....	1 on 3	1 on 2	20	Semi-hydr.-fill.
Ceros (7 Dams)	Spain	75 to 30	...	1 on 3	1 on 2	....	Rolled layers.
Wissota	Wis.	75	10	1 on 2 and 1 on 3.5	1 on 2	20	Reinf. concr. core.
Leeming	Ireland	73	.....	1 on 3	1 on 2	10	.....
Huffman	Ohio	73	15	1 on 2 to 1 on 3	1 on 2 to 1 on 3	25	Hydr.-fill.
South Fork	Pa.	72	.....	1 on 2	1 on 1.5	20	.....
Dixville	N. H.	70	5	1 on 2	1 on 1.5	25	Reinf. concr. core-wall.
Lake Francis	Colo.	70	.....	1 on 3	1 on 2	20	Hydr. sluicing.
Zuni	N. Mex.	70	10	1 on 3	1 on 1.25	20	Hydr. sluicing.
Upper Deer Flat	.....	68	7	1 on 3	1 on 2	20	Thin layers, rolled.
Phelps Brook	Conn.	68	12	1 on 2, 1 on 3	1 on 2.5	15	Concr. core-wall.
South Dam, Snake River	Idaho	66	.....	1 on 4	1 on 1.5	20	Timber core. Down-stream part, rock fill.
Lough Vartry	Ireland	66	.....	1 on 3	1 on 2.5	28	.....
La Messa	Calif.	66	.....	1 on 1.5	1 on 1.5	20	.....
Mukti	India	65	.....	1 on 3	1 on 2	10	.....
Bog Brook	N. Y.	65	8	1 on 2	1 on 2.5	25	Masonry core.
Piedmont	Calif.	65	.....	1 on 2	1 on 2	....	.....
Sudbury	Mass.	64	7	1 on 2	1 on 2 and 1 on 2.5	14	Concr. core.
Snake River	Calif.	64	.....	1 on 2	1 on 1.5	12	.....
Glenwild	N. Y.	64	7	1 on 2	1 on 2.5	13	Masonry core.
Stubden	Ireland	63	.....	1 on 3	1 on 2	12	Puddled core.
Junction	Mich.	61	12	1 on 4	1 on 3	12	Hydr.-fill concr. core.
Tytam Bay	Hong Kong	60	.....	1 on 3	1 on 2	30	Core-wall.
Loganlea	Scotland	59	.....	1 on 3	1 on 2.5	10	.....
Conconully	Idaho	59	.....	1 on 3	1 on 2	20	Hydr.-fill.
Ashti	India	58	.....	1 on 3	1 on 2	6	.....
Chollas Heights	Calif.	56	5	1 on 3	1 on 2	18	Steel core. Rolled layers.
Cedar Grove	N. J.	55	6	1 on 3	1 on 2	18	Concr. core. Rolled layers.
Hinckley	N. Y.	50	8	1 on 2.5, 1 on 3.5	1 on 2	20	Concr. core. Rolled layers.
Merced	Calif.	50	.....	1 on 3	1 on 2	20	Thin layers.
Rotten Park	England	50	.....	1 on 3	1 on 2	20	Puddled core.

TABLE 7.—(*Continued.*)

Name of dam.	Location.	Maximum height, in feet.	Freeboard, in feet.	Up-stream slope.	Down-stream slope.	Top width, in feet.	Remarks.
Vale House	England	47	.....	1 on 3	1 on 3	18	Puddled core.
Jackson Lake	Wyo.	45	8	1 on 3	1 on 1⅞	20	Hydr.-fill.
Mormon Canyon	Calif.	45	.....	1 on 1.5	1 on 1.5	6	.....
Uley Brook	England	42	.....	1 on 3	1 on 2	25	Puddled core.
Lower Deer Flat	.....	43	7.5	1 on 3	1 on 2	20	Rolled layers.
Peary	Ireland	40	.....	1 on 3	1 on 2	12	Puddled core.
Cuyamaca	Calif.	40	6.5	1 on 2	1 on 2.5	15	Puddled core.
Monument Creek	Colo.	40	.....	1 on 3	1 on 2	20	.....
Swift	Mont.	40	.....	1 on 3	1 on 1.5	20	Concr. cut-off. Rock fill against earth.
Milton	Ohio	40	9.5	1 on 2	1 on 2	20	Rolled layers.
Sugar Loaf	Colo.	38	8	1 on 3	1 on 2	25	Rolled layers.
Cache le Poudre	Colo.	38	.....	1 on 3	.....	....	.....
South Dike of Sweetwater	Calif.	37	.....	1 on 3 to 1 on 1½	1 on 2	....	Rolled layers.
Little Horse Creek	S. C.	35	.....	1 on 3	1 on 1.5	....	Masonry core.
Llanefydd	Wales	32	.....	1 on 3	1 on 2	10	Puddle trench, 154 ft. deep.
Charleroi	Pa.	25	.....	1 on 1.5	1 on 1.5	12	Rolled layers.
Bog Brook	N. Y.	25	.....	1 on 2	1 on 2	12	Masonry core.

also require the use of flood-gates which give an enormous discharge per linear foot of crest. Each dam must be studied to determine the proper economical relation between length and capacity of spillway per linear foot of crest, and the amount of rise in head-water elevation which must consequently be provided for by an increased height of embankment.

The considerations just discussed: (1) depth of frost line; (2) height of waves; and (3) height to which greatest floods will raise water in the pond, taken together will determine the proper freeboard to be provided. Assuming that it has been determined, for a particular dam, that the depth of the frost line does not exceed 2 ft., that the greatest height of waves to be expected is 5 ft., and that the height to which the greatest floods will raise the head-water above the spillway crest is 6 ft.; the proper freeboard to provide will be the sum of these three factors, or 13 ft., as each of them is supposed to include a proper factor of safety.

Hydraulic-Fill Dams and Semi-Hydraulic-Fill Dams.—The same criteria of design must be applied to such dams as are applied to earth dams built by other methods. During and immediately after construction, while consolidation and drainage are taking place, hydraulic-fill and semi-hydraulic-fill dams are sometimes subject to internal stresses due to the outward pressure of a semi-liquid or plastic core. In consideration of these construction stresses, a wider section or heavier toe fills of rock are used than would otherwise be required. Mr. Allen Hazen has suggested that such stresses can be avoided by limiting the size of core particles. The special problems of hydraulic-fill dam construction and design are treated in two papers recently presented before the Society.*

* "Hydraulic-Fill Dams", by Allen Hazen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1713; and "Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District", by Charles H. Paul, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1181.

Earth Dams Meeting the Six Criteria Will Prove Safe.—The design of earth dams has been discussed under the headings of six different criteria. To be safe, any earth dam must meet successfully each and all of these criteria (except in a few special cases where particular methods for safety are adopted). An earth dam must also fulfill the preliminary requirement for safety against sliding, which, however, will usually be satisfied if the other requirements are met. It may be stated confidently that any earth dam which meets these requirements, after the construction stage has been passed, will be safe, and that any earth dam that fails to meet each and all of them is likely to be unsafe. It should be remembered, also, that the best of designs may be ruined and failure insured by inefficient construction undertaken without proper engineering supervision.

THE WATER POWER PROBLEM*
A SYMPOSIUM

WATER POWER POTENTIALITIES OF THE PACIFIC COAST

By Frederick Hall Fowler,† Assoc. M. Am. Soc. C. E.

Synopsis

This paper approaches the water-power potentialities of the Pacific Coast from the standpoint of market demand for their future development. It gives the outputs from water-power and fuel-burning plants in the seven States within the Pacific drainage for 1920 and 1921 and, from more extensive records, it computes the rate of growth in the seven principal market centers within this area during the last decade and forecasts the loads in the same markets for 1930 and 1940. These market demands are shown to scale on a map (Fig. 28) of the Pacific drainage, on which also are shown, to the same scale, the flat load power capacities and approximate locations of many of the more important undeveloped water-power projects or sites. The general conditions on the principal water-power streams are outlined in the text.

The paper is not intended as a complete inventory of all undeveloped water powers of the Pacific drainage, but the study has been carried far enough to show the general relation between present and future demand and supply. No attempt has been made to cover the market in the relatively small parts of Montana, Wyoming, Colorado, and New Mexico within the Pacific drainage.

Market Conditions and Growth of Load

Interest in potential water power is measured largely by present and prospective power needs and the relative expense of producing energy by various well established methods. If in any region there is an urgent present and prospective power demand, some source of energy, usually that which at the time is cheapest, will be developed. It may turn out later that the original conditions are changing and to the extent of that change, the trend of development will change. It is reasonable, therefore, to consider the potential water powers in the streams west of the Continental Divide in relation to the demand for their utilization in competition with power developed by fuel.

Four types of fuel have been used within this territory for the commercial generation of power: Oil, coal, natural gas, and wood, listed in the order of magnitude of their use.

Oil, supplied chiefly from California fields, is practically the only fuel utilized for the development of power in steam-electric stations over a large

* Presented at the meetings of October 4 and 5, 1922, and continued from March, 1923, *Proceedings*.

† Cons. Engr., San Francisco, Calif.

area in the Western States. The greatest use of fuel oil is in California where 5 600 000 bbl. were burned in 1920 and 3 300 000 bbl. in 1921. It is used to a more limited extent in Arizona, Washington, and Oregon. Immediately after the World War, when the price of fuel oil advanced rapidly, there was a tendency for Colorado and Wyoming coal to be shipped westward and to displace the use of oil on the railroads, but only a long continued rise in the price of oil would displace oil in steam stand-by stations on the hydro-electric systems.

A readily available supply of good steam coal is found in only limited areas west of the Rocky Mountains. It so happens, however, that in the areas where coal is found, water power is also abundant and, for the West, unusually cheap. It is, therefore, a strong competitor for the power market. Washington is the only one of the seven States in which coal has been used to any great extent for the generation of electric power.

Increased labor costs and difficulties of mining coal and the growing demand for oil for purposes other than fuel (coupled with the marked tendency of every oil field to drop rapidly in rate of production from the day that it is first brought in) will doubtless put these two sources of energy at an increased disadvantage as primary sources of power.

Natural gas is available in only a few comparatively restricted areas in California. The most important area of this type is in the southwestern side of the San Joaquin Valley where one of the operating electric companies is now generating steam power from gas, not only for use in its own system, but for transmission southward to the companies serving the power market in and around Los Angeles.

Wood has been used for fuel in electric generation only to consume the waste of some of the largest lumber mills in Oregon, Washington, California, and Idaho.

The relative importance of the various types of fuel may be seen by reference to Table 17 which is transcribed from the general compilations of the U. S. Geological Survey.

In the future, as in the present and past, steam stand-by stations using oil or coal for fuel will play an important part in the hydro-electric systems of the West, but from present indications, a constantly decreasing percentage of the total output from steam power and an increasing percentage from water power may be expected.

Most of these facts have long been common knowledge to those directly engaged in Western water-power development, but are presented here for the benefit of those who have not been in close contact with Western power problems.

Few persons realize that even now inter-connecting transmission systems extend, except for minor breaks, from Eastern Idaho and Central Utah westward and northward through the Columbia and Snake River Valleys to the Pacific Coast in the Puget Sound region and thence southward through Washington, Oregon, and California, to the Mexican line and Western Arizona. The lines in west Central Nevada, now isolated, are soon to be connected with Central California, and short connections only will be necessary to unite the

TABLE 17.—PRODUCTION OF ELECTRIC POWER AND CONSUMPTION OF FUELS BY PUBLIC UTILITY POWER PLANTS

TABLE 17.—PRODUCTION OF ELECTRIC POWER AND CONSUMPTION OF FUELS BY PUBLIC UTILITY POWER PLANTS IN THE SEVEN STATES ENTIRELY WITHIN THE PACIFIC DRAINAGE, 1920 AND 1921, REPORTED BY U. S. GEOLOGICAL SURVEY.

	Total Power Produced.		Produced by Water Power.			Produced by Fuel Power.			Coal Consumed.		Fuel Oil Consumed.		Gas Consumed.		Kilowatt-hours produced by wood fuel.
	Kilowatt-hours.	Percentage of United States total.	Kilowatt-hours.	Percentage of State total.	Percentage of United States total.	Kilowatt-hours.	Percentage of State total.	Percentage of United States total.	Short tons.	Percentage of United States total.	Barrels.	Percentage of United States total.	Thousands of cubic feet.	Percentage of United States total.	
1920															
Arizona	159 228 000	0.37	106 527 000	66.9	0.66	53 701 000	33.1	0.19	5 579	0.01	238 517	1.62	0	.....	.....
California	8 795 645 000	8.58	2 568 541 000	68.8	15.91	1 167 104 000	31.2	4.26	0	0.00	5 646 085	43.02	2 970 956	12.03	6 671 000
Idaho	590 864 000	1.37	598 978 000	99.7	3.64	1 886 000	0.3	0.01	435	0.00	40	0.00	0	.....	1 284 000
Nevada	33 549 000	0.08	32 690 000	97.4	0.20	859 000	2.6	0.00	2 847	0.01	2 937	0.02	0	.....	0 000
Oregon	475 543 000	1.09	370 867 000	78.0	2.29	104 676 000	22.0	0.38	2 779	0.01	24 052	0.18	0	.....	99 972 000
Utah	312 321 000	0.19	212 243 000	100.0	1.31	78 000	0.0	0.00	212	0.00	0	0.00	0	.....	0 000
Washington	1 196 725 000	2.75	1 154 185 000	96.5	7.14	42 590 000	3.5	0.16	43 891	0.12	141 523	1.08	0	.....	9 888 000
1920	6 403 875 000	14.73	5 083 981 000	78.6*	31.15	1 369 894 000	21.4*	5.00	55 743	0.15	6 053 104	46.12	2 970 956	12.03	117 815 000
1921															
Arizona	182 434 000	0.32	93 120 000	70.3	0.62	39 314 000	29.7	0.15	2 606	0.01	180 406	1.50	0	0.00	.....
California	3 962 938 000	9.72	3 227 261 000	81.0	21.55	755 677 030	19.0	2.90	0	0.00	3 348 269	27.76	3 167 454	13.35	7 269 000
Idaho	550 149 000	1.34	546 896 000	99.4	3.65	3 253 000†	0.6	0.01	0	0.00	0	0.00	0	0.00	3 253 000
Nevada	36 532 000	0.09	36 139 000	98.9	0.25	393 000	1.1	0.00	1 885	0.01	598	0.00	0	0.00	0 000
Oregon	468 534 000	1.14	374 465 000	79.9	2.50	94 469 000	20.1	0.36	1 165	0.00	36 613	0.30	0	0.00	84 156 000
Utah	119 551 000	0.29	119 532 000	100.0	0.80	19 000	0.0	0.00	56	0.00	0	0.00	0	0.00	0 000
Washington	1 176 662 000	2.87	1 139 909 000	96.9	7.61	36 753 000	3.1	0.14	31 695	0.10	103 889	0.86	0	0.00	12 357 000
1921	6 466 800 000	15.77	5 586 922 000	85.62*	36.98	929 878 000	14.38*	3.56	37 407	0.12	3 664 770	30.42	3 167 454	13.35	107 035 000

*Percentage of total for the group of seven States.
†Produced by plants using wood fuel.

systems of Southern California and Arizona, Utah and Colorado, Idaho and Eastern Montana, and Idaho and Oregon. On the completion of these ties the entire inhabited country of the Pacific drainage will be served by an interconnecting network. Some of the present inter-connections are weak, but they may be strengthened at relatively small cost as market conditions warrant. The large investment in existing systems will naturally have a marked effect on the order of development of new projects.

As a basis for comparing the load carried by the systems serving the Pacific drainage with the total load for the United States, the figures of the U. S. Geological Survey are of great value. For 1921, the survey reported 40 976-000 000 kw-hr. for the entire country of which 14 970 000 000 kw-hr., or 36.5%, was produced by water power, and the remainder by fuel. The total power output of the seven States (Arizona, California, Idaho, Nevada, Oregon, Utah, and Washington) entirely within the Pacific drainage was 6 467 000 000 kw-hr., or 15.77% of the total for the country, and the water power output in these seven States was 5 537 000 000 kw-hr., or 85.62% of the total production within the group of States, and 36.98% of the water-power output for the entire country. The detailed figures by States for the group are given in Table 17.

The outputs reported by the U. S. Geological Survey check closely the 1920 and 1921 outputs available from other sources, and these independent records extend back for a full decade; they are in varying detail, in different sections of the West, but in all sections are sufficiently complete and accurate to furnish a basis for computations of past growth in demand for power, and afford also the only logical basis for forecasting the loads of the future. On the rate of growth of these future loads will depend the demand for the potential water powers. The following paragraphs outline the past growth in output, and on Fig. 28 are shown in simple graphic form:

(*a*) The actual load at seven given market centers in 1921 and the estimated loads at the same points in 1930 and 1940; and
(*b*) Some of the important undeveloped powers that are now scheduled for construction by public utilities and municipalities or that have been investigated in more or less detail by Federal or State authorities.

Both loads and power capacities are shown on the same scale so that they may be readily compared, and arcs have been described around the various load centers to show the approximate transmission distances.

This diagram is not intended as a catalogue of potential power, and a short study will show that many important sites that have not been fully investigated, or for which there is small prospective demand, have been omitted. Enough power possibilities have been recorded, however, to show several outstanding facts, among which may be mentioned the wonderful potential powers of the Columbia River (so large in many instances as not to be economically feasible unless new industries are established to utilize their output); the importance of the California sites still remaining undeveloped; and the availability of the southernmost of these sites in competition with the principal sites on the Colorado, that have recently received much publicity.

In some instances, the so-called "markets" are definite centers and, in others, are broad territories. They may be classified, or described, as follows.

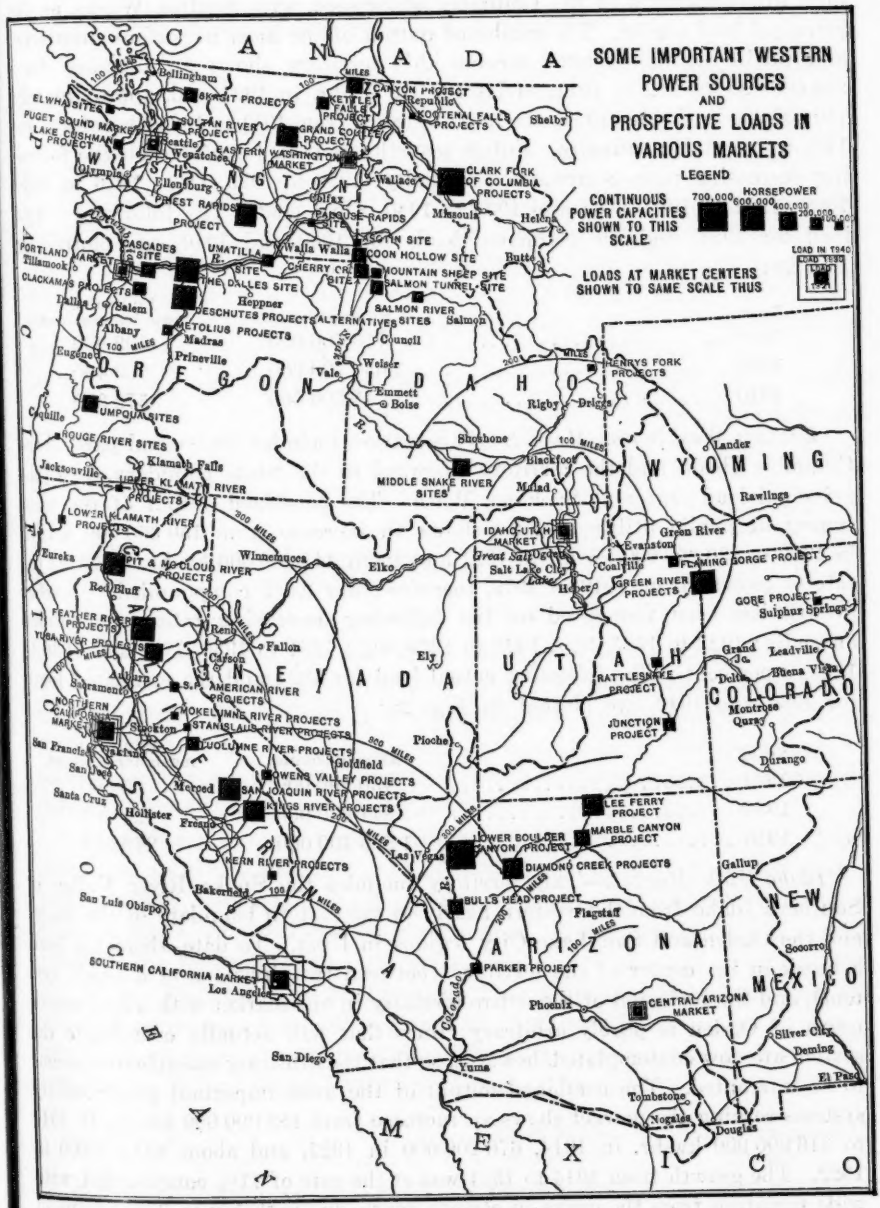
SOME IMPORTANT WESTERN POWER SOURCES AND PROSPECTIVE LOADS IN VARIOUS MARKETS
LEGEND
CONTINUOUS POWER CAPACITIES SHOWN TO THIS SCALE
HORSEPOWER
700,000 600,000 400,000 200,000 100,000
LOADS AT MARKET CENTERS SHOWN TO SAME SCALE THUS
LOAD IN 1940 HEAD 1930
CANADA
OREGON
IDAHO
WYOMING
NEVADA
UTAH
COLORADO
ARIZONA
NEW MEXICO
MEXICO
WASHINGTON
CALIFORNIA
Bellingham
Seattle
Wenatchee
Ellensburg
Olympia
Spokane
Wallace
Missoula
Helena
Butte
Shelby
Kootenai Falls Projects
Kettle Falls Project
Z Canyon Project
Republic
Grand Coulee Project
Pasayten Rapids Project
Priest Rapids Project
Elwha Sites
Puget Sound Market
Lake Cushman Project
Skagit Projects
Sultan River Project
Eastern Washington Market
Clark Fork of Columbia Projects
Cascades Site
Umatilla Site
Cherry Ck. Site
Walla Walla
Coon Hollow Site
Mountain Sheep Site
Salmon Tunnel Site
Austin Site
The Dalles Site
Heppner
Deschutes Projects
Metolius Projects
Alternatives
Salmon River Sites
Salmon
Council
Weiser
Vale
Emmett
Boise
Prineville
Madras
Albany
Eugene
Portland Market
Tillamook
Clackamas Projects
Salem
Dallas
Umpqua Sites
Rogue River Sites
Jacksonville
Klamath Falls
Upper Klamath River Projects
Lower Klamath Projects
Eureka
Pit & McCloud River Projects
Winnemucca
Elko
Red Bluff
Feather River Projects
Yuba River Projects
Auburn
Sacramento
Reno
Carson
Fallon
S.F. American River Projects
Mokelumne River Projects
Stanislaus River Projects
Stockton
Tuolumne River Projects
Oakland
San Francisco
San Jose
Santa Cruz
Goldfield
Tonopah
Ely
Pioche
Bowers Valley Projects
San Joaquin River Projects
Kings River Projects
Merced
Hollister
Fresno
Kern River Projects
Bakersfield
San Luis Obispo
Southern California Market
Los Angeles
San Diego
Yuma
Colorado R.
Parker Project
Bulls Head Project
Las Vegas
Lower Boulder Canyon Project
Diamond Creek Projects
Lee Ferry Project
Marble Canyon Project
Flagstaff
Phoenix
Central Arizona Market
Gallup
Socorro
Silver City
Deming
El Paso
Tombstone
Nogales
Douglas
Idaho-Utah Market
Great Salt Lake
Salt Lake City
Ogden
Provo
Middle Snake River Sites
Shoshone
Blackfoot
Milad
Rigby
Driggs
Henrys Fork Projects
White
Lander
Rawlings
Green River
Evanston
Coalville
Flaming Gorge Project
Green River Projects
Gore Project
Sulphur Springs
Grand Jc.
Leadville
Delta
Buena Vista
Montrose
Ouray
Durango
Rattlesnake Project
Junction Project

FIG. 28.

Puget Sound Market.—This market includes the territory on the shores of Puget Sound and eastward to the crest of the Cascade Range, from the Canadian line southward to the Columbia water-shed, with Seattle, Wash., as the principal load center. The combined output of the most important municipal and public utility systems serving this territory shows an increase from 238 600 000 kw-hr., in 1911, to 716 400 000 kw-hr. in 1921 and, approximately, 750 000 000 kw-hr. for 1922; the rate of growth from 1911 to 1922, therefore, was 11% compounded annually. Future growth has been forecasted on the following decreasing rates compounded annually: 1922 to 1925, 11%; 1926 to 1930, 10%; 1931 to 1935, 9%; and 1936 to 1940, 8 per cent. The following actual load for 1921, and the estimated loads for 1930 and 1940, are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	716 400 000	109 600
1930	1 656 700 000	253 400
1940	3 745 500 000	572 900

Eastern Washington Market.—This market includes the central part of the Columbia Basin and the territory eastward to the State boundary, with the principal load center at Spokane, Wash. The combined output of the most important public utility systems shows an increase from 150 600 000 kw-hr. in 1910 to 430 200 000 kw-hr. in 1921 and about 442 600 000 kw-hr. in 1922; the rate of growth from 1910 to 1922, therefore, was 9.5% compounded. Further growth has been forecasted on the following decreasing rates compounded annually: 1922 to 1925, 9%; 1926 to 1930, 8%; 1931 to 1935, 7%; and 1936 to 1940, 6 per cent. The following actual load for 1921 and the estimated loads for 1930 and 1940, are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	430 200 000	65 800
1930	844 600 000	129 200
1940	1 585 400 000	242 500

Idaho-Utah Market.—This territory includes the Snake River Valley in Southern Idaho from the western nearly to the eastern boundary of the State, and the Ogden and Salt Lake City regions in Utah. To date, there has been a break in the center of the territory (between two divisions of a single system), and the inclusion of the entire territory in one market with a load center north of Ogden is purely arbitrary; lines that will actually consolidate the system are now contemplated, however, so that the arbitrary classification seems to be warranted. The combined output of the most important public utility systems serving the market shows an increase from 188 000 000 kw-hr., in 1910 to 316 900 000 kw-hr. in 1914, 670 200 000 in 1921, and about 641 600 000 in 1922. The growth from 1914 to 1921 was at the rate of 11% compounded, with wide variation from the curve in certain years, due probably to fluctuations in mining demand. Future growth has been forecasted, starting with the actual output of 1921, on the following decreasing rates compounded annually: 1921 to 1925, 10%; 1926 to 1930, 9%; 1931 to 1935, 8%; and 1936 to 1940, 7 per

cent. The following actual load for 1921 and the estimated loads for 1930 and 1940, are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	670 200 000	102 500
1930	1 509 800 000	230 900
1940	3 112 300 000	476 100

Portland Market.—This market includes the territory in Northwestern Oregon and Southwestern Washington having a definite commercial and load center at Portland, Ore. The combined output of the most important public utility systems shows an increase from 152 200 000 kw-hr. in 1911 to 411-500 000 kw-hr. in 1921, the rate of growth during the period, therefore, was, 10.5% compounded annually. Further growth has been forecasted on the following decreasing rates compounded annually: 1921 to 1925, 10%; 1926 to 1930, 9%; 1931 to 1935, 8%; and 1936 to 1940, 7 per cent. The following actual load for 1921, and the estimated loads for 1930 and 1940, are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	411 500 000	62 900
1930	927 000 000	141 800
1940	1 910 400 000	292 200

Northern California Market.—This market includes the populated territory in California north of Monterey on the coast and Merced in the Central Valley, and, in addition, the territory in and around Reno in Western Nevada, which is to be joined to the California systems. The networks of the operating companies extend over a wide area, but a combined load center may be fixed in the vicinity of Port Costa on the east side of San Francisco Bay.

The combined output of practically all public utilities and municipal systems shows a uniform growth from 1 241 400 000 kw-hr. in 1914 to 1 999 800 000 kw-hr. in 1920, and a falling off to 1 986 600 000 kw-hr. in 1921; the rate of growth from 1914 to 1921 was 7% compounded. Future growth has been forecasted on the following decreasing rates compounded annually: 1921 to 1925, 7%; 1926 to 1930, 6%; and 1931 to 1940, 5 per cent. It will be noted that the estimated percentage was not decreased in 1936 as in other estimates; this is because the rate was already very low. The following actual load for 1921 and estimated loads for 1930 and 1940 are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	1 986 600 000	303 900
1930	3 485 400 000	533 200
1940	5 677 500 000	868 500

Southern California Market.—This market includes the populated territory in California south of Monterey on the coast, Merced in the interior valley, and Mono Lake Basin east of the Sierra, as well as the mining region of Southwestern Nevada. The chief load center is in the vicinity of Los Angeles, but

a secondary center of great importance is in San Joaquin Valley; nevertheless, on the map (Fig. 28), the entire load for the market is centered at Los Angeles.

The combined output of practically all public utility and municipal systems shows a steady growth from 854 100 000 kw-hr. in 1913 to 1 025 800 000 kw-hr. in 1916, and then a steady growth, at an increased rate, to 2 002 600 000 kw-hr. in 1921. The rate of growth from 1916 to 1921, 14.4% compounded annually, is phenomenal. Future growth has been forecasted on the following decreasing rates compounded annually: 1921 to 1925, 13%; 1926 to 1930, 12%; 1931 to 1935, 10%; and 1936 to 1940, 8 per cent. Objection may be raised to the fairness of decreasing the rate in this estimate by 2% in each of the last three 5-year periods, instead of 1% every 5 years, as in other estimates (or even maintaining a flat rate for the last 10 years, as in the Northern California market). Several factors demand a more rapid rate of decrease: (*a*) the astonishing rate of growth has been maintained only since 1916; (*b*) the cumulative effect of compounding at so high a rate would presuppose a rate of growth in population and industry that has never been equalled for any very long period; and (*c*) the rapidly developing territory in the Central San Joaquin Valley, that might well be classified with the Northern California market (and has actually transmitted power to the northern market) has contributed to the high rate of growth recorded for the Southern California market. For these reasons, the speaker believes the sliding scale adopted is, if anything, more than ample. The following actual load for 1921, and estimated loads for 1930 and 1940 are plotted on Fig. 28:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	2 002 600 000	306 300
1930	5 754 300 000	880 200
1940	13 616 800 000	2 083 000

Central Arizona Market.—This market includes the mining and agricultural territory in Arizona, extending from Prescott and Jerome southward to Phœnix, and thence eastward to Miami, Globe, and Clifton. Part of the mining load in this district is not now carried by electric power. The heaviest general load is in and near Phœnix, but on account of a prospective increase in the mining region eastward and northward the load center has been fixed arbitrarily at Miami. The future growth of load is problematical and depends largely on the mining industry. An estimate of the future growth of load by Col. William Kelly, Corps of Engineers, U. S. A., M. Am. Soc. C. E., Chief Engineer of the Federal Power Commission, has been published in the Second Annual Report of the Commission. This estimate is based on more detailed data concerning mining load than are available to the speaker, and, therefore, has been adopted as a whole.

Col. Kelly estimates a public utility mean load of 30 000 kw. and a mining load of 56 000 kw. in 1926. He assumes the public utility load will increase at 9% until 1925, 8% until 1930, 7% until 1935, and 6% until 1940; also, that the mining load will increase at 5.2% until 1935, and at 4% thereafter. On these rates, and with his 1926 figures as a basis, the loads in 1921, 1930, and 1940, have been plotted on Fig. 28, as follows:

Year.	Kilowatt-hours.	Equivalent mean horse-power.
1921	539 000 000	82 400
1930	957 500 000	146 500
1940	1 610 100 000	246 300

The figures for 1921 include the actual load on existing electric systems and also mining load.

Power Resources

The only comprehensive estimate giving the total potential power in all the streams in the Pacific drainage (as well as elsewhere in the United States) is that prepared by the U. S. Geological Survey in 1908. This general estimate has been republished several times and with certain revisions. It was made on a theoretical program with total fall and estimated stream flow as its principal basis without reference to the engineering features of the power sites along the streams. The plant efficiency adopted in the original estimate was much higher than is attained under present practice. A revision published later* and on a more conservative plant efficiency, credits the seven States entirely within the Pacific drainage with a maximum of 31 437 000 h.p., or 58.32% of the total water power of the entire country. The figures by States are given in Table 18.

TABLE 18.—Estimates of the Minimum and Maximum Potential Water Power Resources of the Seven States Entirely Within the Pacific Drainage

State.	Total Potential Water Power.			
	Minimum.		Maximum.	
	Horse-power.	Percentage of total United States.	Horse-power.	Percentage of total United States.
Arizona	893 000	3.20	1 698 000	3.15
California	3 424 000	12.25	7 818 000	14.50
Idaho	2 362 000	8.45	5 067 000	9.40
Nevada	172 000	0.62	276 000	0.51
Oregon	3 148 000	11.27	6 613 000	12.27
Utah	743 000	2.66	1 318 000	2.45
Washington	4 932 000	17.65	8 647 000	16.04
Total	15 674 000	56.10	31 437 000	58.32

In addition to many large developments already under Government permit at the date of the Federal Power Act, the Federal Power Commission has received applications aggregating millions of horse-power throughout the country. As some of these applications have not been finally reported on (and estimates of their power capacities are, therefore, somewhat indefinite) and there are conflicts between others, it is difficult to ascertain exactly the net

* Table 2, Senate Doc. 316, 64th Cong., Second Session.

horse-power they represent. A rough estimate with conflicts or duplications eliminated gives for the projects in the seven States in the Pacific drainage the following approximate power, available 90% of the time:

Arizona	3 650 000 h. p.
California	2 600 000 "
Idaho	100 000 "
Nevada	....
Oregon	700 000 h. p.
Utah	1 400 000 "
Washington	1 050 000 "
Total	9 500 000 h. p.

The most important divisions of the 800 000 sq. miles of the United States west of the Rocky Mountains, as regards water power are:

(*a*) The Columbia River drainage, 259 000 sq. miles.

(*b*) The Colorado River drainage, 244 000 sq. miles.

(*c*) The coast drainages, comprising in order southward:

(1) The area tributary to Puget Sound, 14 300 sq. miles.

(2) The coast areas in Washington and Oregon, and in California north of San Francisco Bay, tributary directly to the Pacific Ocean, 44 700 sq. miles.

(3) The Sierra Nevada streams draining into San Francisco Bay, or into the closed basins of Tulare and Kern Lakes, the entire area in the Great Valley embracing 59 800 sq. miles; also Owens Lake Basin.

Few important power sites are to be found in the 200 000 sq. miles embraced within the arid mountain ranges and desert valleys of the Great Basin extending eastward from the base of the Sierra Nevada through Nevada and Utah to the base of the Wasatch Range, and thence southeastward through the plateau region north of the Grand Canyon. Fortunately, this region is as destitute of power demand as it is of power resources.

Few power sites are found on the coast streams emptying into the Pacific Ocean south of San Francisco Bay, and such as there are have been almost completely utilized by existing plants.

For the reasons just stated, little study need be given to the Great Basin and South Pacific drainages, and attention may be concentrated on the Columbia, the Colorado, and the Coast drainages south to San Francisco Bay.

Columbia River Drainage.—The drainage area of the Columbia above its mouth (including 34 700 sq. miles in Canada on the main stream and several of its tributaries) is 259 000 sq. miles. Of this total, 237 000 sq. miles are above The Dalles, where the stream breaks through the Cascade Range, and 32 000 sq. miles are embraced within the drainage of the main stream below The Dalles, and several tributaries, of which the most important—the Willamette and

Cowlitz—drain large valleys lying between the Coast Range and the Cascades, south and north of the Columbia.*

The Columbia River rises in Canada, flowing first nearly northward and then southwestward and crossing the International Boundary in the extreme northeastern corner of the State of Washington. Near the mouth of Spokane River, the Columbia turns west along the base of a secondary mountain system extending eastward from the Cascades. At the mouth of the Okanogan, the stream again turns southward, and then eastward to the mouth of Snake River, where it again turns abruptly to the west, finally to break through the Cascades and discharge into the Pacific Ocean. Eastward and northward from the great loop of the river (extending from the mouth of the Spokane to the mouth of the Snake) is a series of great lava flows that cover the floor of the basin eastward to the mountains of Idaho and northward nearly to the Canadian Boundary. Much of this lava is covered, in the lower parts of the main basin and on the tributaries, with deep sedimentary soils. Clark Fork (the principal head-water tributary of the Columbia in the United States) encircles the north, and Snake River encircles the south, of the mountain ranges of Central Idaho, which are a western extension of the Rockies. Rainfall in the Columbia Basin ranges from 8 to 80 in. or more; the head-waters of the main stream and many of its principal tributaries are well forested.

The most important tributaries of the Columbia in total potential water power, taken in order down stream, are Clark Fork, Snake River, and the Deschutes. Clark Fork should be listed after Snake River if the streams are arranged in the order of magnitude of potential power. Tributaries with smaller power possibilities will be mentioned subsequently.

The Rockies on the eastern boundary of the Columbia drainage reach elevations of 10 000 ft.; elevations in the detached ranges of Idaho are rarely more than 8 000 ft.; elevations in the mountains north of the basin (along the Canadian Boundary) are rarely more than 5 000 ft., and elevations to the southward near the heads of the Owyhee, John Day, and Deschutes are from 6 000 to 7 000 ft. The highest elevations in the Columbia drainage are along its western boundary in the Cascade Range through which the Columbia has broken its way to the sea. The backbone of the Cascades in Oregon is, in general, more than 6 000 ft. in elevation, with peaks from 8 000 to 11 225 ft.; in Washington, the general elevation of the ridge is from 4 000 to 6 000 ft. or more, with peaks from 10 000 to 14 363 ft. As a result of these topographic features, the tributaries of the Columbia draining the Cascades have, in general, heavier gradients and smaller drainage areas than its eastern tributaries.

Where the Columbia crosses the International Boundary the elevation is about 1 340 ft. above sea level, and the distance from the mouth of the river is about 685 miles; at the "Cascades", the lowest power site on the main stream, the elevation is 6 ft. above sea level, and the distance is 143 miles from the mouth; the fall in the 542 miles between these two points, therefore, averages only 2.5 ft. per mile. The gradient of the stream, however, is by no means uniform, and most of the drop is concentrated at rapids or low falls, few of

* See *Proceedings*, Am. Soc. C. E., January, 1923, p. 115, for a map showing the areas of various divisions of the Columbia drainage.

which are very pronounced. The great potential power of the stream is due to its high discharge rather than its rapid fall.

The power sites that have been most carefully investigated to date are, in order down stream: Kettle Falls, Priest Rapids, Grand Coulee, Umatilla Rapids, The Dalles, and the Cascades.

The site at Kettle Falls, now under permit from the Federal Power Commission to Washington Water Power Company, is unusual in that it can be developed serially in six stages that will bring the power capacity, successively, to about 13 000, 25 000, 49 000, 74 000, 98 000 and 123 000 h. p. The investment in the project, in a measure, will be proportionate to the successive steps, which should be of great advantage.

A dam has been planned near the head of Grand Coulee to furnish power for the so-called Columbia Basin pumping project (an alternative for the development of the Clark Fork storage and gravity canal) to irrigate 1 753 000 acres of land in that part of the basin bounded on the west by the Columbia and on the south by the Snake.

The important site at Priest Rapids is now under permit from the Federal Power Commission to Washington Irrigation and Development Company. The rapids at this point fall 90 ft. in 9 miles, and the dam and power plant are to be built at the foot of the rapids. The Company states that the average flow during the low-water period is 50 000 sec-ft. and during the summer months the discharge is more than 400 000 sec-ft. The dam will be 90 ft. high and about 2 miles long. Primary power at the site has been variously estimated to a maximum of 400 000 h. p., with an additional 300 000 secondary h. p. available chiefly during the irrigation season. The supporters of the project have made extensive investigations of various natural resources of the region, that may be developed through the construction of the plant.

The State Engineer of Oregon made preliminary investigation of the site at Umatilla Rapids and found that a head of 35 ft. with a minimum flow of 40 000 sec-ft. was available and estimated that 120 000 h. p. could be developed continuously, and 320 000 h. p. for the irrigation season, at a cost estimated, in 1916, at $20 500 000 based on the 320 000 h. p. capacity. This estimate, however, would have to be adjusted to present prices.

A much more detailed study of The Dalles by the State of Oregon showed an available head ranging from 45 to 105 ft., depending on the stage of the river, with a minimum flow of 50 000 sec-ft., 480 000 h. p. available continuously, 120 000 h. p. additional for 11 months, 100 000 h. p. further additional for 10 months, and 100 000 h. p. still further for 8 months, giving a total of 800 000 h. p. available for 8 months or more, at a total estimated cost of $50-000 000, with prices as in 1916.

A rather careful preliminary study of a site at the Cascades, 4 miles below Cascade Locks, was also made by the State and showed a head of 44 ft., at minimum flow of 50 000 sec-ft., with an estimated capacity of 200 000 h. p. continuous, 90 000 additional h. p. for 11 months, 120 000 h. p. additional for 10 months, and 220 000 h. p. additional for 8 months, making a total of

420 000 h. p. available for 8 months or more at a cost of $11 000 000 exclusive of the dam, for which there were no trustworthy data.

Snake River.—This river, the most important tributary of the Columbia, both in area drained and in potential power possibilities, rises in the mountains north of Jackson Lake in Northwestern Wyoming, flows first in a southerly and then in a westerly direction to a junction with the Columbia River in Central Washington, 300 miles above its mouth. The drainage area of Snake River is 108 600 sq. miles. Several power sites in Idaho have already been developed, and the stream is used extensively for irrigation. Further development of storage in the head-waters for irrigation and power are contemplated, and power development in that part of the stream in Central Idaho will be largely dependent on the flow available after irrigation needs are supplied. The discharge of springs and important tributaries in Western Idaho below the largest irrigation diversions increase the supply available for power purposes, and the most important future power developments will probably be on that section of the Snake along the boundary between Oregon and Idaho and thence down to its mouth.

Profiles of the central section of the Snake River from Minidoka Dam to Enterprise, Idaho (southwest of Boise), show a total fall from Elevation 4 195 to Elevation 2 232, or 1 963 ft. in 242 miles, or a mean of 8 ft. per mile. The mean, however, has little significance as long stretches of moderate grade are followed by shorter stretches with rapid fall. The most rapid drops (certain of which are partly developed) are at Minidoka, 55 ft. in ½ mile; Milner, 200 ft. in 4 miles below the crest of Milner Dam; from the crest of Dry Creek Falls to the crest of Twin Falls, 410 ft. in 14.2 miles; from the crest of Twin Falls to the crest of Shoshone Falls, 155 ft. in 2.9 miles; from the crest of Shoshone Falls to the foot of Pilar Falls, 210 ft. in 1.7 miles; in the 2.8 miles above Rock Creek, a fall of 140 ft.; from the crest of Upper Salmon Falls to the foot of Dolman Rapids, 92 ft. in 2.2 miles; and at the Lower Salmon Falls, 35 ft. in 0.1 mile.

Lower Salmon Falls is 141 miles above Enterprise and, therefore, about 100 miles below Minidoka. The entire fall from Minidoka to Lower Salmon Falls is 1 450 ft. which leaves only a fall of 513 ft. in the 141 miles of the section of the river extending down to Enterprise. Power plants are in operation at Lower Salmon Falls, Shoshone Falls, and Minidoka, as well as at American Falls and Idaho Falls up stream from Minidoka.

Salmon River, the most important tributary of the Snake, has a comparatively regular profile on a grade broken by a series of small drops rather than long flat sections with rapid drops, as on the Snake. Profiles show that from Salmon, Idaho, to Riggins at the mouth of Little Salmon River, the stream falls from Elevation 3 935 to Elevation 1 705, or 2 230 ft. in 173 miles. The fall in the Salmon below Riggins to the junction with the Snake is 815 ft. in 86.6 miles.

Power investigations made by E. W. Kramer, Hydro-Electric Engineer, U. S. Forest Service, showed three sites with a combined capacity of more than 100 000 h. p., on the section of the Salmon between the mouths of the Middle Fork and Little Salmon.

Rather careful preliminary investigation of six power sites on a part of Snake River were made by the State Engineer of Oregon.* These projects, taken in order down stream, are Salmon River Tunnel, Mountain Sheep, Cherry Creek, Coon Hollow, Asotin, and Palouse Rapids.

The Coon Hollow and Cherry Creek sites may be considered as alternative locations to utilize nearly the same fall as that available at the Mountain Sheep site. The maximum water surface elevation above the dams, and the mininum water surface elevation for the tail-races of the three developments are, as follows: Mountain Sheep, 1 135 and 915 ft.; Coon Hollow, 1 104 and 884 ft.; Cherry Creek lies between the other two. The tail-water of the Salmon Tunnel site is Elevation 1 160.

The Salmon River Tunnel project contemplates the diversion of Salmon River through a narrow ridge to a drop on Snake River about 80 miles above Lewiston, Idaho. The net head available would range from 505 to 530 ft. The normal minimum flow would be 4 000 sec-ft. producing 200 000 continuous h. p., estimated, at 1916 costs, to total $19 215 000, or an average of $96.08 per h. p. The installation could be increased to 300 000 h. p., giving an additional 100 000 h. p. from April to October, with a total cost of $25 320 000, or $84.40 per h. p. This estimate and the following estimates, as in the case of those on the Columbia, would have to be adjusted to present-day prices.

Mountain Sheep site is 55 miles above Lewiston, Idaho, on Snake River, and 3 miles above the mouth of Salmon River. The net head would range from 200 to 220 ft., the flow from 6 000 sec-ft. minimum to 10 000 sec-ft., with the addition of Salmon River through the diversion tunnel mentioned previously, or by storage as mentioned subsequently for the Coon Hollow project. The discharge could be increased to 15 000 sec-ft. with both the Salmon River diversion and the Coon Hollow storage. It was estimated that 100 000 h. p. could be made available continuously, 200 000 h. p. with either of the alternative supplementary water supplies, and 300 000 h. p. with both the supplementary supplies. The estimated cost for the development of 200 000 h. p., exclusive of the cost of supplementary supplies, was $14 000 000, or $70 per h. p., and the cost of 300 000 h. p., on the same basis, was $16 400 000, or $54.70 per h. p.

Cherry Creek site is on the Snake River, 51 miles above Lewiston, 1 mile below the mouth of Salmon River. The head is 200 ft., the minimum discharge 10 000 sec-ft., or 15 000 sec-ft. for an average of 11 months; it was estimated that 200 000 continuous h. p. could be developed, at a cost of $13 600 000, or $68 per h. p.; and that 300 000 h. p. could be developed with storage at an estimated cost of $18 100 000, or $60.35 per h. p.

Coon Hollow site is 45 miles above Lewiston; the available head is from 200 to 220 ft. and the water supply the same as that for the Cherry Creek project. It was estimated that 200 000 continuous h. p. would cost $15 200 000, or $76 per h. p.; and that 300 000 h. p. would have been available for 11 months, or 300 000 h. p. continuous with the possible use of the existing storage.

* "Oregon's Opportunity in National Preparedness", *Bulletin 5*, Office of State Engineer, Salem, Ore., February, 1916.

The Asotin site is above Asotin, Wash., the head being 60 ft. and the minimum stream flow, 10 000 sec-ft. With storage, 15 000 sec-ft. could be obtained. On the basis of the minimum flow, 54 000 h. p. could be developed at an estimated cost of $5 400 000, or $100 per h. p., and with storage, 80 000 h. p. could be developed at an estimated cost of $6 600 000, or $82.50 per h. p.

The Palouse Rapids site is 7 miles below Riparia, Wash. The head is 32 ft., the minimum flow 12 000 sec-ft. and the flow with storage, 17 000 sec-ft. It was estimated that 50 000 h. p. could be developed, corresponding to a greater flow than the minimum, because the irrigation load comes during the high water. The estimated cost was $4 500 000, or $90 per h. p.

Clark Fork of the Columbia.—Clark Fork, the most important head-water branch of the Columbia in the United States, rises on the western slope of the Continental Divide just west of Butte, Mont., and flows in a westerly and then a northerly direction, finally emptying into the Columbia a short distance north of the Canadian boundary. The total length of the stream is 420 miles, and its drainage area at the mouth is 25 800 sq. miles. Altitudes within its basin range from 1 346 ft. above sea level at the mouth to more than 8 000 ft. at the summit of some of the highest peaks. From the boundary to the mouth of the river, the fall is about 400 ft. in 16 miles, so that where it crosses into Canada the elevation is 1 746 ft. above sea level.

The flow of Clark Fork is subject to the natural regulative effect of several large lakes that may in the future be greatly increased. These lakes are Flathead Lake (Elevation 2 881) on Flathead River, the principal tributary, Lake Pend Oreille (Elevation 2 059) on the main stream, and Priest Lake (Elevation 2 439) on the head-waters of Priest River, a tributary. There is only one important power site on Clark Fork above the mouth of Flathead River (Elevation 2 482)—the so-called "Fish Creek Site" of about 20 000 h. p. Mr. Kramer has made a preliminary study for the development of Flathead River and Clark Fork down to Lake Pend Oreille, for power purposes, and finds that, in addition to the existing developments, 621 000 h. p. are available in eight sites, with complete regulation of Flathead Lake. Between Lake Pend Oreille and the border, the most important sites are at Albany Falls, Metaline Falls, and Z Canyon. The Federal Power Commission has received application for a project at Z Canyon that will develop 195 000 h. p. The construction of a high dam at Z Canyon would cause back water to Albany Falls.

The completion of the so-called Columbia Basin Gravity Project would develop two sources of power: (*a*) the power at the canal drops on the project itself; and (*b*) the power on the Flathead and Clark Fork Rivers regulated by storage in Flathead Lake for irrigation in the Columbia Basin. The former would be seasonal power available in varying amounts from April to October of each year and reaching a maximum of 189 955 h. p. in August, with a mean of 132 889 h. p. for the seven months, of which a mean of 122 650 h. p. would be available for general sale and the remainder would be used in irrigation pumping on the project itself. The latter would give year-round power in the 820-ft. fall between Flathead Lake and Pend Oreille Lake, varying, according to past records, between a maximum of 333 800 h. p. (1910) and a minimum of 132 970 h. p. (1920). The mean for the period, 1908 to 1920, was

257 205 h. p.; about 190 000 h. p. would have been available for 90% of the time. As shown, such a development in connection with irrigation would reduce the total output below that available with the stream developed for power alone.

Investigations made by the Commission of Conservation of Canada show an important site on Clark Fork, immediately above its junction with the Columbia, where a head of 100 ft. and a continuous output of 73 000 h. p. may be secured.

Deschutes River.—Deschutes River and one of its principal tributaries, Metolius River, draining the east side of the Cascades in Central Oregon and emptying into the Columbia near The Dalles, are very important streams in total power available and for the reason that they are comparatively near existing markets and the sites are such that development can be made in moderate sized units, following market demand. Sites on both streams have been investigated by the State Engineer of Oregon and by the Federal Power Commission. The report of the Commission shows fifteen sites in a stretch of 111 miles above the mouth of the Deschutes, with heads ranging from a minimum of 32 ft. to a maximum of 270 ft. and, in general, from approximately 70 to 140 ft. The capacities of the sites ranged from 11 300 to 83 500 h. p., but were in general from 25 000 to 50 000 h. p. The costs ranged from $92 to $205 per h. p., as estimated, for 85% load factor and delivered to the Columbia. The costs of many of the projects range from $95 to $175 per h. p.

Estimates for four sites on Metolius River showed heads of 75, 260, 400, and 300 ft., respectively, power capacities from 9 100 to 35 500 h. p., and unit costs from $116 to $252 per h. p.

Willamette River.—The most important undeveloped power sites in the Willamette Basin are on its tributaries, the Clackamas, Santiam, and McKenzie. On the Clackamas, Portland Railway, Light and Power Company has begun a progressive development and is operating two power plants. A continuous capacity of about 143 000 h. p. may be developed at six other sites for utilization in Portland and vicinity as the demand increases; the development of other tributaries will probably be completed later.

Other Tributaries of the Columbia.—The estimates just given in the Columbia drainage are based on rather careful investigations of specific sites. Theoretical computations of horse-power available on other tributaries* are as follows: Yakima River and tributaries, 34 000 h. p.; Natches River, 57 530 h. p; Klickitat River, 154 000 h. p.; White Salmon River, 105 000 h. p.; Little White Salmon River, 8 550 h. p.; Lewis River, 78 000 h. p.; Toutle River, 44 000 h. p.; and Cowlitz River, 175 000 h. p. On account of topographic or other difficulties, it will probably be found that not all the capacity of these streams can be developed.

Coast Drainages

Puget Sound.—The streams running from the Cascades westward into the east side of Puget Sound and from the Olympic Mountains eastward into the Sound and northward into the Strait of Juan de Fuca, afford many

* "Water Powers of the Cascade Range", U. S. Geological Survey, *Water Supply Papers Nos. 253, 313,* and *369.*

opportunities for economical power development in units of moderate size. The drainage basins of these streams are comparatively small, the largest being that of the Skagit River with about 2 500 sq. miles at a point below all important power sites; the total drainage area tributary to the Sound is only 14 700 sq. miles. The elevations in the Sound drainages range from sea level to a maximum of 14 363 ft. at the crest of Mount Ranier. The rainfall ranges from 40 or 50 in. along the shore of the Sound to a maximum of about 150 in. near the crest of the Olympic Mountains. The geological structure is volcanic. Lakes are numerous, and the stream flow is comparatively abundant and regular. The gradients of the streams are far steeper than in large rivers like the Snake and Columbia. Most of the existing and proposed developments are of the low or moderate head type.

The City of Tacoma is about to start active construction of a large development on the Skokomish, and its system should have an ultimate flat load capacity of 80 000 h. p.

On the north end of the Olympic Peninsula, applications have been filed for several sites on the Soleduc and Elwha Rivers. These developments should have a flat load capacity of 70 000 h. p., or more.

Skagit River has claimed much attention since the City of Seattle determined on its utilization for municipal power purposes. An excellent storage site at the head of a series of three projects will permit the development of 158 000 continuous h. p., and 10 000 h. p. additional is available on the Cascade River, a tributary in the vicinity of the system on the main stream. Additional important sites are available on other tributaries.

Other streams flowing into the east side of the Sound are important sources of power in various stages of development. Theoretical computations* give the following power capacities: Nisqually River, 35 700 h. p.; Puyallup River, 40 400 h. p.; White River, 102 000 h. p.; and Cedar River, 37 870 h. p.

North Pacific.—The coast streams in Washington, and in Oregon and California from the Columbia southward to San Francisco Bay, drain an area of about 44 700 sq. miles lying entirely on the west side of the Olympic, Coast, and Cascade Ranges, with the exception of the head-waters of Klamath River which drain some territory in, and to the east of, the Cascades. The largest single drainage basin is that of the Klamath, which has an area of about 16 000 sq. miles above the mouth. The next streams in size and importance are Rogue River and Umpqua River. Elevations within the Klamath Basin vary from sea level to about 9 000 or 10 000 ft. at the summit of several peaks in the head-waters basin and a maximum of more than 14 000 ft. on Mt. Shasta; elevations in the Rogue River basin range up to 10 000 ft. The highest point in the Umpqua Basin is about 8 000 ft. All the basins are, in general, well forested and have mean annual precipitations of from 40 to 100 in., or more, except the inland parts of the Klamath Basin, which are cut off from the ocean by intervening ridges that reduce the rainfall to 10 in., or even less, in the extreme eastern part of the drainage.

* U. S. Geological Survey, *Water Supply Paper No. 313.*

Although both Klamath and Rogue Rivers are far from the important load centers of Central California and the Portland District in Oregon, plants of The California Oregon Power Company on both streams are now transmitting power north and south to these markets, in addition to serving local demands. The future use of the Klamath River may be considerably affected by irrigation development.

Umpqua River, draining into the ocean in Central Oregon, although an important power stream, is as yet practically untouched. Careful estimates by Mr. Kramer show that with head-waters storage, a continuous output of 220 000 h. p. may be obtained at fifteen different sites, seven of which, with a combined capacity of 161 000 h. p., are known to be very desirable. The sites are of low or medium head.

On Rogue River, two sites controlled by The California Oregon Power Company were reported by the State Engineer of Oregon to have heads of about 100 ft., and continuous capacities of 12 000 and 13 000 h. p. The unit costs were $70 and $130, respectively, at prices prevailing in 1916. These two sites and the partly developed sites do not comprise the total power on the stream.

Klamath River affords many opportunities for the development of power in the 255 miles between Klamath Lake in the head-waters basin and the ocean. The total fall in this distance is about 4 100 ft., and the gradient of the stream is comparatively regular, most of the drop being in rapids rather than falls. Two low-head plants are already in operation below the outlet of Klamath Lakes Basin.

There are many dam sites throughout the canyon and the height to which future dams are built, or the number of projects to be developed, will depend largely on market conditions. Of the power capacities plotted on Fig. 28, the square at the mouth of the Salmon represents 67 500 h. p. in three projects; and the square above the mouth of Shasta River represents three proposed developments and one site with a combined capacity of 42 000 h. p. There are other opportunities on the middle river for the development of at least 70 000 h. p.

Central California Streams.—The Great Valley of California is drained by the Sacramento and San Joaquin Rivers (the San Joaquin joining the Sacramento just above the point where the latter empties into San Francisco Bay), and by several streams discharging into Tulare and Kern Lake Basins. The total area above the outlet into the Bay (including the Kern and Tulare Lake Basins at the south end of the San Joaquin Valley, but not naturally tributary to this river) is 59 500 sq. miles. In this area only the streams draining the western slope of the Sierra Nevada, and the extreme southern end of the Cascade Range, have important power possibilities. The most important of these in order southward are Pit River, Feather River, Yuba River, and American River, all tributaries of the Sacramento, and Mokelumne River, Tuolumne River, Stanislaus River, Merced River, San Joaquin River, Kings River, and Kern River, all tributaries of the San Joaquin, or draining into the Tulare and Kern Basins. Power sites of minor importance are found

on the head-waters of the Sacramento River, Deer Creek, Mill Creek, Kaweah River, and Tule River.

The power possibilities of nearly all the streams mentioned have been at least partly developed. The elevation of the canyon mouths of most of the streams is between 500 and 750 ft. above sea level and the elevation of the head-waters becomes progressively greater to the south. The only one of these streams that drains the territory east of the Cordillera is Pit River, which heads in the interior basin in the Warner Mountains in Northeastern California and breaks through the southern end of the Cascades between Mount Shasta and Lassen Peak. Feather River drains a large mountain valley that can hardly be classed, however, as part of the interior basin. The Feather River drainage, although usually assumed to be wholly in the Sierra Nevada, is really the dividing line between the Cascades and the Sierra. The difference in structure between the two ranges is well marked in the northern and southern parts of the basin, Lassen Peak on the northern boundary being a typical volcanic cone of the Cascades and the peaks of the southern part of the basin are granite, the distinguishing formation of the backbone of the Sierra Nevada. This granite backbone extends south through the entire length of the range to the spurs south of Mount Whitney, and is flanked in the foothill belt to the west by andesites and shales; at only a few points, as in the Mono Lake region, is there any volcanic formation such as is typical of the Cascades.

The stream flow characteristics change toward the south, being affected not only by geology, but by topography and by changes in rainfall. Pit River and the North Fork of Feather River are spring-fed streams of the general character found in the Cascades. The Middle Fork of Feather River is more nearly typical of the streams of the Sierra in its flow than those of the Cascades, and the general tendency to the south is toward well marked low-water and flood seasons. The yield per square mile also has a general tendency to decrease toward the south, particularly from Kings River to the end of the range. The heaviest rainfall is in the head-waters of the North Fork of Yuba River, where the annual mean is about 80 in. Toward the south, the rate of rainfall decreases, first rather rapidly and then more slowly to a mean of not more than 15 or 20 in. in the head-waters of the South Fork of the Kern and 10 in. or less in the foothills of the Kern.

The flow of streams that head on the main ridge of the Sierra is sustained by melting snows at the higher elevations, but those streams that do not penetrate to the higher regions (or touch the main ridge only at one or two points, such as the Merced) must depend to a greater extent on storage. The most flashy of the Sierran streams are probably those heading on the west face of some of the secondary ridges to the west of the main ridge. Tule River, heading on the west side of the Great Western Divide is an example of this class. The rapid gradient, the large areas of exposed granite, and the relatively small size of its drainage area combine to give it a particularly rapid run-off, with only little high country to maintain the low flow from melting snow.

In few sections of this country are the benefits of water storage so well exemplified as in some of the secondary drainages of the Southern Sierra, Big Creek of the San Joaquin drainage may be cited as an example where construction of storage has made possible a series of plants now partly completed. Before the storage was constructed the minimum flow of Big Creek was less than 3 sec-ft.; corresponding to about 1 000 h. p. with 4 000-ft. fall. With storage, two plants with a combined head of nearly 4 000 ft. out of a total of 5 000 ft. ultimately available, and reservoirs only partly developed have an average annual output of nearly 40 000 h. p.

Pit River and its principal tributary, the McCloud, offer excellent opportunity for the development of power. Both streams are spring-fed and power plants may be operated successfully without storage. Most of the sites are of the low-head type, but the largest has a head of about 800 ft. Within the past two years two plants have been built on Hat Creek and one on the main Pit by Mount Shasta Power Corporation, a subsidiary of Pacific Gas and Electric Company, as the first steps in a series of seven plants, five of which are to be on the main river. Two additional projects are under permit from the Federal Power Commission to the Pit River Power Company and three sites on the Pit and McCloud near the junction of the two streams have been filed on or are under permit to individual applicants. The total continuous power of the undeveloped sites is more than 465 000 h. p.

Feather River shares with the Pit a pre-eminent value for the development of water power. Two plants and a large reservoir on the North Fork, a spring-fed stream, form the first steps in a serial development of that branch by Great Western Power Company of California. As planned, this series will consist of six plants. Surveys are already under way for the construction of a combined power and irrigation storage and a series of power plants on the Middle Fork of the Feather River. It is uncertain, however, what the final arrangement of this system will be. In addition to the development of the main branches of Feather River, there are attractive opportunities for the development of their principal tributaries, and 500 000 continuous h. p. is a conservative estimate of the undeveloped energy in the basin.

Of the three branches of Yuba River, North, Middle, and South Forks, the South Fork is utilized largely by the existing developments of Pacific Gas and Electric Company, which company also owns one plant diverting from North Fork. Undeveloped sites on North and Middle Forks have been applied for by two power companies and an irrigation district. When the present conflicts between the various applicants have been disposed of, and an allowance has been made for sites on South Fork, the total capacity of undeveloped sites in the basin will be about 265 000 h. p.

The canyon of the North Fork of American River is cut too deeply to afford attractive power sites without storage. From time to time, the development of Middle Fork has been proposed by various companies, but at present this project seems to be dormant. On South Fork and its tributaries, operating companies and individuals have filed on three projects with a combined

capacity of about 80 000 h. p. and on one of the developments construction is under way. Minor developments have been in operation on this stream for many years.

Mokelumne River has been utilized for many years by one plant, and above this plant and below the reservoirs used in its operation, are undeveloped sites. The three plants proposed by present applicants would have a continuous combined capacity of 50 500 h. p.

Tuolumne River is widely known as the source of the Hetch Hetchy power and water supply system of the City of San Francisco. Only three very small water-power developments have hitherto been in operation on the stream or its tributaries. In the near future, the city will complete its Moccasin Creek plant with a continuous capacity of 70 000 h. p. and, in subsequent years, will develop a plant between Hetch Hetchy Reservoir and Early Intake of the aqueduct and another plant utilizing the water supply diverted from Cherry Creek and Eleanor Creek to a permanent plant on the north bank of the river near Early Intake. A power company has applied to the Federal and State authorities for permission to develop storage and power on the South Fork of the Tuolumne. The combined continuous capacity of all these plants, when completed, will be 170 000 h. p., in addition to which Turlock and Modesto Irrigation Districts will develop considerable seasonal power at their Don Pedro Irrigation Dam on the lower river. The power here available has not been included on Fig. 28.

San Joaquin River is the most important power stream in the south central part of the Sierra Nevada. The lowest power development will probably be at a proposed irrigation dam at about Elevation 450; the tail water of the only existing plant on the main river is at Elevation 650 and the total drainage area above the lowest power plant will be 1 640 sq. miles. Elevations range from 11 000 to more than 13 000 ft. along the crest of the Sierra in the head-waters. Much of the basin is well forested. With one exception, the developments to date have been on secondary streams such as the North Fork of the San Joaquin, which is occupied by a series of plants of San Joaquin Light and Power Corporation, and Big Creek, which is utilized by another series belonging to Southern California Edison Company. The latter company proposes large extensions of its present system both on the main stream and on the principal tributaries. Although the locations now proposed are not final, it appears that projects of the series, as yet undeveloped, will furnish a continuous output of 500 000 h. p.

Kings River, second only to the San Joaquin for potential power in the southern part of the Sierra, is as yet undeveloped. As on the San Joaquin, the lowest power development will probably be at an irrigation dam where the stream leaves the foot-hills. The drainage area above that point is about 1 700 sq. miles, and elevations range up to about those found in the head-waters of the San Joaquin. The drainage area is much more rugged and good forest cover is found chiefly in the secondary basins such as that of the North Fork. This branch of the river is of the same character as Big Creek of the San Joaquin, the stream dropping at a tremendous gradient

from a plateau basin to the main river and offering, as on Big Creek, feasible heads of 2 000 ft. The series of plants will depend largely on storage. A complete plan of development for the North Fork has been completed by San Joaquin Light and Power Corporation. On the two principal branches of the stream, Middle and South Forks, the City of Los Angeles has made preliminary plans for a series of developments; minor branches of these two streams would probably furnish additional power. A conservative estimate for all contemplated developments on Kings River shows a continuous output of not less than 400 000 h. p., providing certain upper storage sites prove to be feasible.

Kaweah River, south of the Kings, drains the western slope of secondary ridges of the Sierra, cut off almost entirely from the main Divide by the head-waters of South Fork of Kings and the North Fork of Kern River. The stream, therefore, does not receive so great a benefit from melting snows and natural storage in small lakes. Much of the Middle Fork area is well forested. The few attractive power sites are developed at present almost entirely by existing systems of Southern California Edison Company.

On Tule River, the only potential powers still undeveloped are extensions of an existing plant owned by San Joaquin Light and Power Corporation. The stream, therefore, is of little future potential value.

Kern River, the southernmost of the important streams draining from the Sierra Nevada, has a total area of more than 2 000 sq. miles above the mouth of the canyon, ranging in elevation from 850 to 14 500 ft. Much of the drainage area is dry mountain country lying in the foot-hills, or in the comparatively arid ridges in the South Fork Basin. The most important power sites are on North Fork and on the main river below the junction of North and South Forks. Southern California Edison Company has under contemplation and construction a complete system for the utilization of North Fork and the main river. Three of its plants and one belonging to San Joaquin Light and Power Corporation utilize a considerable part of the total fall. The continuous power capacity of the remaining sites of the series and two projects applied for by the City of Los Angeles on South Fork amount to approximately 62 500 h. p. Future development of the stream will probably be delayed pending the completion of more attractive developments on other drainages, and the final program may be affected by the proposed construction of a large storage reservoir, primarily for use in irrigation, at the junction of the South and Middle Forks.

Owens River drains part of the eastern slope of the Sierra. The development of the principal sites on Owens River and its tributaries has been planned by the City of Los Angeles in connection with its aqueduct and municipal water and power system. The most important sites are at Owens River Gorge below the proposed Long Valley storage reservoir of 340 000 acre-ft. capacity. The total fall in the gorge, not all of which is controlled by the city, is 2 300 ft. With complete regulation, about 56 000 h. p. could be developed in a series of plants utilizing the entire fall. Other developments

on minor streams (of small drainage area, but very rapid gradients) would bring the total continuous output for the Owens Valley region to 80 000 h. p.

On the east side of the Sierra, there are only a few streams, aside from Owens River, that are of great power importance. Truckee River is already well developed, Leevining and Rush Creeks emptying into Mono Lake are in process of development and proposals have been made to develop the minor sites on Carson and Walker Rivers.

Colorado River Basin.—Few streams have received so much recent publicity as the Colorado River, which drains parts of Colorado, Wyoming, Utah, Arizona, New Mexico, Nevada, and California. The stream is extremely valuable as a source of supply for both irrigation and power. It is naturally divided into three sections: The head-waters basin, above the mouth of the San Juan; the canyon region, from that point down to the mouth of Virgin River where the Colorado, after flowing in a general southwesterly direction, turns almost due south; and the valley region, from the mouth of Black Canyon southward to the Gulf of California. The entire drainage area above Yuma is 244 000 sq. miles. Of the entire flow at Laguna Dam which is above the mouth of the Gila (the principal tributary in Arizona), more than 90% originates above the Grand Canyon. The most important potential powers are between the mouth of the San Juan and the mouth of Black Canyon, the total fall between the two points being about 2 500 ft. There are, however, large water powers in the upper basin on the two branches, particularly on Green River.

Estimates of the proposed capacity of the various power projects on the Colorado River are greatly complicated by conflicts and by different programs for river regulation. At certain points as many as three projects are proposed for the utilization of a given fall. In showing, on Fig. 28, the power available at some of the most important sites, the capacities in the upper basin are those reported by the Federal Power Commission; the capacities from the mouth of the Green to Diamond Creek, inclusive, are on an assumed ultimate regulation of 10 000 sec-ft. through the approximate heads, and the power capacities from Lower Boulder Canyon to the Parker site, inclusive, are on the assumed final power regulation to 15 000 sec-ft. through the available heads.

These assumptions should be kept in mind in comparing the power of these sites with that made available by any other program of regulation or development.

The Colorado River Problem and the California Power Supply.—The Colorado River problem has a marked bearing on the power situation throughout California.

Early in their history the operating companies in Southern California had to go far north into the Southern Sierra Nevada for their hydro-electric supply; similar conditions, although not so acute, also caused the companies of Central California to develop plants north of their market. The general trend of transmission in the State is, therefore, from north to south. This condition may be changed by the building of a single plant on the

Colorado, by the Federal Government, municipality, or public utility. A large block of power will be made available in a single step, and the utilities and mines of Arizona will be able to absorb only a small part of this power. The utilities and municipal systems of Southern California will be the only other prospective purchasers. The companies and the City of Los Angeles have already acquired rights on the streams of the Sierra and have outlined programs that are in various stages of development. Some programs, in their initial stages, are complicated by conflicts with established rights, and promise high unit cost; these developments may well be dropped with little loss, and with no thought of taking them up in the future. Others are well toward completion (at least, in part) and must be placed in operation at the earliest possible date in order to avoid enormous financial loss; once in operation their output must be absorbed into the market. Others, as yet only in initial stages, are to be completed under State or Federal grants that will lapse unless the systems are placed in operation within a limited time. With the lapse of such grants adverse titles (particularly for the use of water) will become established, and will further complicate the plans for power development if resumed at a later date. In short, a hydro-electric system operating under conditions that now exist in California must plan for years in advance not only its construction program but also its campaign to secure and maintain rights for the use of water and public lands; it is not in the relatively simple position of the steam-generating utility, that has only to add a new unit to its plant when the load increases. It appears, therefore, that even if Colorado River power is made available in Southern California at a lower switchboard cost than that immediately available from the Sierra the problems just mentioned will affect the rate at which it is absorbed by the existing systems. Most of the questions may be solved on some practical basis, and doubtless their solution will be accelerated by the recognized benefit that will result from the final control of the Lower Colorado River.

It is to be anticipated that on the introduction of Colorado River power in the southern part of the State the amount of power transmitted southward from the Sierra will be greatly reduced and that some of the systems now transmitting both north and south from the San Joaquin drainage will send all their power to the north; the flow of power southward will certainly be reduced, and, under certain conditions, may be completely reversed.

General Summary

Granting that the previously discussed estimates of increase in power demand of various markets are approximately correct, it is certain that within the next 20 years there will be no shortage of power in the Pacific States. The only territory in which the power demands will exceed the local power resources is in Southern California and there the local shortage can be remedied by power transmitted from the central part of the State, unless in the meantime part of the great potential power of the Colorado is developed.

Colorado River power will not be absorbed into the California market merely because of a slightly lower market price. The price will have to be reduced sufficiently to justify the operating systems in abandoning exten-

sive development programs now under way. Until such time as Colorado River power is made available in Southern California, the trend of power transmission in that State will be (as in the past) toward the south, but thereafter, the resources of the Southern Sierra may be utilized in markets lying to the northward.

Early construction at the major sites on the Columbia River itself is dependent chiefly on development of new industries, as the initial block of power that would be brought in by any of these projects (except that at Kettle Falls), is greater than would be warranted by ordinary growth of the present market; all ordinary demands in the Columbia drainage can be met through construction of projects on its tributaries (such as those shown on Fig. 28, or at smaller sites too numerous to be covered in this paper).

Although there is ample potential power throughout the West to supply the needs for many years, complications between power and irrigation are constantly increasing, and close study of the various problems on the most important streams should be undertaken at the earliest possible date if the best plans for their ultimate development are to be secured.

Acknowledgments

The writer wishes to express his thanks to George C. Tenney, Associate Editor, *Journal of Electricity*, San Francisco, Calif., and to E. W. Kramer, Hydro-Electric Engineer, U. S. Forest Service, for certain information concerning market conditions and potential powers of the Pacific Northwest.

TENTATIVE PLAN FOR THE CONSTRUCTION OF A 780-FOOT ROCK-FILL DAM, ON THE COLORADO RIVER, AT LEES FERRY, ARIZONA

Discussion*

By E. C. La Rue, M. Am. Soc. C. E.†

E. C. La Rue,‡ M. Am. Soc. C. E. (by letter).§—The writer's paper was prepared before detailed surveys had been made of the Lees Ferry project. It was necessary, therefore, to estimate the capacity of the reservoir, the depth to bed-rock, distance between canyon walls at the dam site, length of spillway tunnel, etc. The results of detailed surveys and diamond drill borings are now available, and it is believed that definite information relating to the Lees Ferry project will be of interest. Table 5 gives the figures disclosed by the survey in comparison with those estimated by the writer.

TABLE 5.—Data Relating to the Lees Ferry Project.
(Preliminary Estimates and Data Disclosed by Surveys.)

	Preliminary estimate.	Data disclosed by surveys.
Width of canyon at water surface, in feet	450	480
Width of canyon at Elevation 720 above water surface at dam site, in feet	1 000	1 600
Width of canyon at top, in feet	1 800	1 900
Length of spillway at Elevation 200 above water surface, in feet	3 000	2 900
Storage capacity of reservoir at Elevation 700 above low-water level, in acre-feet	50 000 000	54 000 000
Capacity of reservoir between Elevations 600 and 700 above low water, in acre-feet	20 000 000	21 000 000
Depth of bed-rock in river channel at dam site, in feet.	100	Less than 100

The greatest error was made in estimating the width of the canyon at Elevation 720 above the water surface at the dam site, the writer's estimate being 1 000 ft., whereas the correct distance is 1 600 ft. The writer's estimate of the width of the canyon at the top was nearly correct, which indicates that the upper face of the canyon walls is more nearly vertical than was assumed. The volume of the rock-fill dam would be greater, but the conditions for blasting the material from the upper part of the canyon walls are more favorable owing to the fact that the upper face of the wall is nearly vertical.

Those who have studied the paper have gained the impression that the writer was advocating, a 700-ft. dam at Lees Ferry to store 50 000 000 acre-ft.

* Discussion on the paper by E. C. La Rue, M. Am. Soc. C. E., continued from December, 1922, *Proceedings*.

† Author's closure.

‡ Hydr. Engr., U. S. Geological Survey, Pasadena, Calif.

§ Received by the Secretary, March 28, 1923.

of water as a project feasible of construction at this time. The primary purpose of the paper was to bring out engineering opinion on the practicability of constructing a dam such as that described. The Lees Ferry site was selected in order to illustrate the methods of construction suggested, because it was well known to the writer and seemed to offer unusually favorable conditions for the building of such a dam. The writer was not and is not advocating the Lees Ferry project or any other specific project on the Colorado River as the proper one to be developed at this time. During the past seven years, he has stated that before any money is spent on the construction of a high dam on the Lower Colorado, an investigation of all possibilities should be made, and a report should be prepared by competent engineers, outlining a comprehensive plan for the development of the Colorado River Basin from Wyoming to the Gulf of California. He has also explained that the first large project to be undertaken on the lower river, should be a unit of a carefully prepared comprehensive plan for the development of the whole basin. In the writer's opinion, sufficient information is not available to form the basis for such a comprehensive report, and he will state further that no report based on detailed engineering data has ever been prepared to show the most economical plan for the development of the Colorado River below the mouth of Green River.

When complete data are available, an intelligent conclusion can be reached as to whether it will prove most economical to develop the first large flood-control reservoir at Lees Ferry or elsewhere. It will then be possible to evaluate properly the advantages of creating a storage reservoir where it will control this great river for comprehensive power development through a fall of more than 2 000 ft. and simplify the spillway plans and construction problems of eight or ten dams, and where it will afford adequate regulation for irrigation and the prevention of flood damage, and weigh them against the advantages of a reservoir that may control 10% more water at the foot of the power section of the river and only half as far from the principal area of irrigable and flood-threatened lands. The real value of creating a storage reservoir at the head of the fall at Elevation 3 100, which would make possible the development of between 3 000 000 and 4 000 000 h. p. between Lees Ferry and Black Canyon, and result in a saving of millions of dollars in the construction cost of all dams built in the river below, would be disclosed.

Mr. Coutlee* has called attention to a number of rock-fill dams in Canada. It is interesting to note that a rockslide 2 miles long and 2 miles wide on Old Man River did not act as a water-tight barrier, it being stated that without gathering much head the small river was able to seep through the dam. Mr. Coutlee appears, however, to hold the opinion that satisfactory results can be obtained by the construction of dams of the rock-fill type.

Mr. Muckleston† called attention to a natural rockslide that occurred in 1893 on the Birahi-Ganga, a small stream in India. Although this natural dam was 900 ft. high, 11 000 ft. wide at the base, and 2 000 ft. wide at the top, Mr. Muckleston states: "Nevertheless, the dam went out as soon as it was

* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1605.
† *Loc. cit.*, p. 1607.

topped." He failed to state that the natural dam had no spillway and that due to this fact it was known that the dam would fail.

The writer believes that the paper describing the failure of the Gohna Lake Dam contains information of great value to engineers interested in determining the feasibility of constructing high rock-fill dams. The following quotation was taken from a paper entitled "The Great Landslip at Gohna, in Garhwal, and the Measures Adopted to Prevent Serious Loss of Life",* by J. H. Glass, Chief Engineer and Secretary of the Department of Public Works of Bengal:

"On the 19th of November, 1893, the first intimation of the Gohna landslip reached the Government. * * * The district authorities had heard from a native source that early in the previous September an enormous mass of material from the Maithana hill had subsided into the Bireh Ganga river, completely blocking it, and that a large lake was under course of formation. For some time afterwards material continued to fall, huge blocks thundering down the hill, and crashing into the mass below, but there was no further fall of any size till about the end of October, when a great slice of the hill moved forward into the valley, making an effective barrier of enormous proportions, and completely obstructing the flow of the river. The falling mass was accompanied by a deafening noise, and the neighborhood for miles around was filled by clouds of dust, which whitened the ground and tree branches like snow. Gohna is situated away from any road route, and is most difficult of access, so that it was not possible to obtain readily any accurate account of what had really happened. * * *

"The result of Colonel Pulford's inspection was made known to Government in a semi-official letter, dated the 21st December. He reported that the mass of earth and rock composing the landslip was about 900 ft. in height, 2 000 ft. across at top, 11 000 ft. at the base along the valley, and 3 000 ft. at top and 600 ft. at bottom across the valley. These figures give an idea of the magnitude of the slip. It was largely composed of enormous masses of rock, some of which, lying on the surface, were calculated to be many tons in weight, with an admixture of detritus and a thick layer of white impalpable powder over all. The slip had taken place from a precipitous hill bordering the river on the right bank. The hill, which is called Maithana, has an elevation at the summit of about 11 000 ft. above sea-level, and the part from which the fall occurred is about 4 000 ft. above the bed of the stream. The force of the fall from so great a height as 4 000 ft. carried the rocks and *débris* right across the valley of the river, and for some distance up the steeply scarped hill on the left bank; then, its energy expended, the mass slipped back into the valley, forming a dam, with a big slope up against the hill on the left bank. The further great slip in October filled up the dam on the right bank against the hill on that side, in such a manner that the top of the dam had a large depression some 150 ft. below two sloping mounds of rock and *débris*.

"The bed of the Bireh Ganga river slopes at about 250 ft. per mile at Gohna, and is the bottom of a narrow gorge, with steep, and sometimes precipitous sides. The gentler slopes of the hills are grass-covered at the base, and, higher up, are clothed with evergreen, oak, fir, and rhododendrons. Immediately below the slip, the course of the river turns nearly at right angles, and passes through a narrow gorge with precipitous sides of rock. In the more open parts of the valley, the slopes are terraced, with great labour, for cultivation; but the configuration of the ground and the nature of the soil are such as to prohibit any large amount of cultivation. The river basin at Gohna, draining into the lake formed by the landslip, has an area of about 90 miles, and is bounded on

* *Journal*, Soc. of Arts (Lond.), March, 1896, p. 431.

the north and east by a snow-clad ridge of the Himalayas, rising to an altitude of 21 200 ft. above sea-level. A large portion, therefore, of the water of the river is derived from the melting of the snows, and this supply is, of course, at its greatest during the hot season. * * *

"From the information then before him, Colonel Pulford fixed the date on which the water would rise to the top of the dam as the middle of May. Later on, however, when data in regard to the area enclosed by the landslip, and in respect to the rainfall and snowfall during the year over the catchment area of the river had been collected by the Public Works Officer who was sent to Gohna for that purpose, Colonel Pulford reported that the lake would probably overflow about the middle of August; and the 15th of August was accordingly adopted as the date by which all arrangements for dealing with the flood should be in working order. * * *

"Before the end of July all the arrangements which had been ordered for the protection of life and property were completed. A good deal of percolation was noticeable at the toe of the dam. * * * The area of the lake when the water was level with the top of the dam was calculated from the contoured map prepared by Lieutenant Crookshank, R. E., to be about 37 000 000 sq. ft. in round numbers, or say 1⅓ sq. miles, and the total contents were roughly estimated at 16 650 000 000 cu. ft. * * *

"The first symptom of the approaching collapse occurred on the 10th August. For some time past the water had been rising very steadily, owing to the rains which had commenced early in June and the melting of the snows, and on the 4th August it had reached within 82 ft. of the lowest part of the dam. On the 9th August there was a very heavy rain which caused a serious slip in the down-stream face of the dam, leaving an almost perpendicular section some 400 ft. high. The section thus exposed disclosed a layer of stones at the top, below which only pulverized rock was visible. Percolation, which was now very heavy, was running in a considerable stream over the boulders of the former river-bed laid bare by the slip referred to. This slip caused the death of a man called the Gohna Fakir, his wife and three children. * * * On the 11th August the first warning was telegraphed to all concerned, intimating that the lake would escape within fifteen days' time; and this was followed by a notice that the lake was 50 ft. from the top of the dam. In all 87 warning messages were despatched from Gohna. Percolation was still steadily increasing. On the 10th July it was calculated to be 50 cu. ft. per sec., on the 17th July, 150 cu. ft., and on the 12th August, 350 cu. ft. per sec. On the 22nd August, Lieut. Crookshank issued the warning message that the lake was expected to escape within 48 hours. By cutting a very narrow channel 2 ft. 6 in. deep in the crest of the dam he expected to bring in the flood early on the morning of the 24th August, but owing to increased percolation and smaller rainfall than on previous days, the lake rose so slowly that it did not reach the bottom of the cutting until the afternoon, and as it was considered important that the flood should occur during day-light, the channel was filled up so that the over-topping might take place on the morning of the 25th idem. The lake water commenced to trickle over the dam through the cutting at 6.35 A. M. on the 25th August. At first the destructive action of the water was very slow, and the volume escaping from the lake over the dam did not exceed 200 cu. ft. per sec. up to 3.30 P. M., at which time a thick mist descended and heavy rain fell preventing Lieut. Crookshank from taking further observations. But at 2 P. M., the percolation which had now increased to an alarming extent, caused a sudden cutting back of the dam on the down-stream side to within 1 000 ft. of the overflow point. At 2.30 P. M., a message had been despatched down the line to the effect that the dam was cutting back, and that the lake was expected to commence falling during the night. The thick mist and heavy rain still continued to obscure the slip, and it was quite impossible to observe the action of the water on the dam. At 11.30 P. M., a loud crash was heard,

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and in spite of the mist and rain the air was filled with a fine dust which rose from the dam. On observing this, Lieut. Crookshank went with the civil functionary, Palwari Mali Dasauli, down to the dam, and made a final inspection of its condition. It was then midnight. As far as he could ascertain the dam was completely breached, and the lake water was escaping through a rapidly increasing channel. The effect on the dam had already been very marked, and the upper surface where Lieut. Crookshank and the Palwari were standing suddenly sank some feet. At twenty minutes after midnight another warning message was sent down the line to the effect that a very great rush of water was being discharged from the lake. It was not possible to gauge the discharge owing to the thick mist which prevailed and the torrents of rain which were falling, nor could it be ascertained at what rate the lake was emptying. At 5 A. M., although the valley was full of mist and the light very scanty, it became evident that a great fall had taken place in the level of the lake, and the irrigation officers and civil authorities at Hardwar were warned to expect a great flood. The sides of the lake, after it had emptied below high-water mark, were waste and desolate. They were cracked and had slipped in many places, and were devoid of trees and vegetation, and an offensive smell rose from the ground which had been so long submerged. Below the dam, the valley of the Bireh Ganga was filled up with the huge blocks of rock, washed away from the landslip by the torrent, and it was found that at the gorge below Gohna, the level of the bed of the stream had been raised about 234 ft. The discharge from the lake was still going on, but was confined to the natural drainage of the river basin. In emptying itself of so large a part of its contents, the lake had made a strong and substantial weir, composed of huge blocks of rock, tightly packed with boulders, and having a long gentle slope stretching far down the valley. The first rush of the impounded water commenced about 11.30 on the night of 25th August, and by 4 o'clock on the following morning the lake level had fallen 390 ft., equivalent, in round numbers, to a discharge of about ten thousand millions of cubic feet in the space of four and a half hours. At Chamoli, a village some 13 miles from Gohna, the flood first began to rise at 11.55 on the 25th August, and the maximum height attained was about 160 ft. above ordinary flood level. The abutments of the permanent bridge, a temple, the Dharmsala, Bazar, and Dispensary, were completely washed away, and the bed of the river was raised about 50 ft. with the *débris* deposited by the flood. * * *

"The large rocks and *débris* of the landslip have been so consolidated and packed by the flood that they now form a massive and solid dam, which is not likely ever to be disturbed, and a permanent lake has probably been established at Gohna. The dimensions of this lake are as follows: Greatest length about 3 900 yd., average width, 400 yd., and greatest depth 300 ft. It may be that in time Gohna will become a pleasant resort for the weary traveller from the plains, and that boats will sail upon the lake. But it will be long before then!"

Data regarding the Gohna Lake Dam are given in Fig. 14. Fig. 15 shows the variation in the seepage through the dam as the head of water on the dam increased. In the paper describing the Gohna Lake landslip, a landslip on the lower face of the dam is mentioned, which it was stated was caused by a heavy rain. When this slip occurred, the head of water on the dam was about 840 ft. It is the writer's opinion that the slip was what is commonly called a "blow out." No doubt the down-stream section of the natural dam was too tight to permit free drainage of seepage water. However, it should be noted that even after the blow-out occurred the dam did not fail until it was actually overtopped. This dam would be standing to-day if there had been adequate

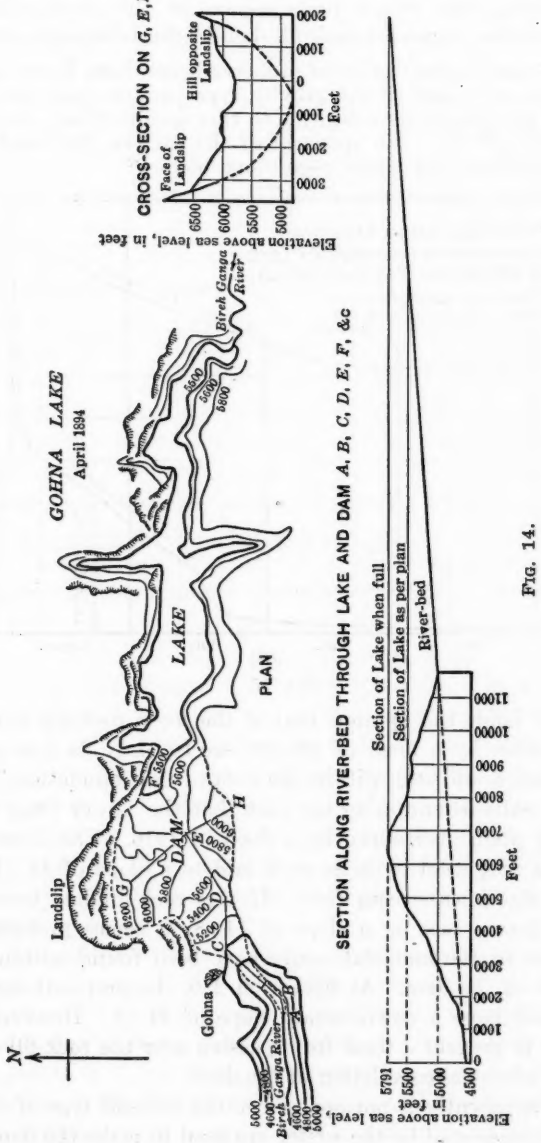
CROSS-SECTION ON G, E, H
Hill opposite Landslip
Face of Landslip
Feet
Elevation above sea level, in feet
Birah Ganga River
GOHNA LAKE
April 1894
LAKE
PLAN
DAM
Landslip
Gohna
N
SECTION ALONG RIVER-BED THROUGH LAKE AND DAM A, B, C, D, E, F, &c
Section of Lake when full
Section of Lake as per plan
River-bed
Feet
Elevation above sea level, in feet

FIG. 14.

spillway, and conditions favorable for free drainage in the down-stream part of the dam.

Although regretting that Mr. Warner,* instead of presenting serious discussion based on his many years of experience, has limited himself largely to facetious criticism, the writer finds solace in the apparently unqualified approval given to the suggested rock-fill dam in the following sentence:

"The present and future value of property below Lees Ferry makes imperative the adoption of a dam of the gravity type [and it must be admitted that a rock-fill dam pre-eminently conforms to this specification], because it is the only type which will have the approval of the farmer, the bond-buyer's engineer, and the Federal and State regulatory bodies."

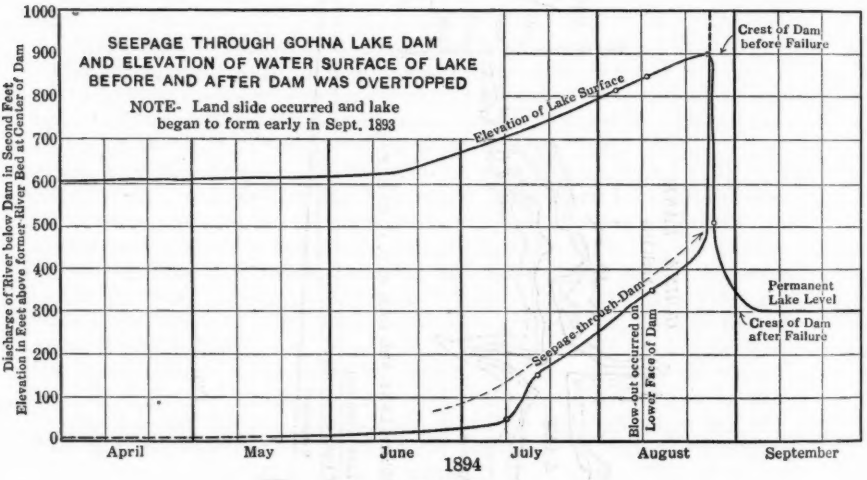

FIG. 15.

Mr. Noetzli† holds the opinion that if the loose rock-fill were overtopped during construction by a flood of 100 000 sec-ft., only "a few of the largest blocks of the dam would stay within the area of the foundation." To support his opinion, he calls attention to the fact that the Lower Otay rock-fill dam was swept away when overtopped by a flood in 1916. The Lower Otay Dam had a steel core supported by loose rock laid to a slope of $1\frac{1}{2}$: 1 on both the up-stream and the down-stream face. If this rock-fill had been constructed with its down-stream face on a slope of 14 : 1, it seems probable that, after the failure, some of the material would have been found within the original area of the base of the dam. At Elevation 250, the proposed rock-fill dam at Lees Ferry would have a down-stream slope of 14 : 1. However, precaution should be taken to prevent a flood from passing over the rock-fill either during construction or after the completion of the dam.

Mr. Noetzli apparently is not opposed to the rock-fill type of dam if means other than those suggested by the writer are used to make the dam water-tight. As Chairman of the Arizona Engineering Commission, the writer visited the site of the Parker Dam on the Colorado River and believes that a safe dam

* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1607.
† *Loc. cit.*, p. 1609.

of the height proposed by Mr. Noetzli can be built at that point at reasonable cost. The combination concrete rock-fill dam proposed by Mr. Noetzli for the Parker site may be the proper solution of the problem.

The writer wishes to express his appreciation of the discussion prepared by Mr. Bryan* giving the geological setting of the Lees Ferry dam site. The data submitted by Mr. Bryan will prove of value to the engineers who are finally called on to determine the most feasible type of dam to build at that site.

In proposing to build the dam to a height of 250 ft. with the first blast, the writer was of the opinion that the rock-fill would displace the fine sand and silt and settle to the layer of sand and boulders. It might be safer as a first step, to blast a smaller quantity of material into the river, allowing the flood water to wash out the fine material which would cause the rock-fill to settle to a firm foundation. This change of plan would meet one objection mentioned by Mr. Davis,† who suggests a concrete pavement cemented to the abutment walls to prevent leakage between the rock-fill and the canyon walls. Owing to vertical jointing, the canyon walls at the dam site are not smooth. If leakage should occur between the rock-fill and the abutment walls, the water must travel a comparatively long distance, perhaps 1½ miles near the base of the dam. It does not seem reasonable to believe that such leakage would take place if the voids in the loose rock can be filled with a fine material.

Unfortunately, Mr. Davis has assumed that the writer holds the opinion that a dam 780 ft. high should be built as a first step in the plan to control the flood flow. A dam to raise the water 700 ft. was chosen, because it is what may be considered the maximum development feasible at the Lees Ferry site. A higher dam would affect the dam proposed below the mouth of Green River. The writer's opinion on this matter is, that no engineer is justified in presenting a project for flood control as being the most feasible, until he has first collected the necessary engineering data to show the possibilities for development at each dam site on the Colorado River below the mouth of the Green River. Sufficient data are now available to show that the Lees Ferry project has considerable merit. A flood-control dam at Lees Ferry, about 400 ft. high, to store 8 000 000 acre-ft. of water, with a power plant in the Grand Canyon at the mouth of Diamond Creek, may be the proper plan for a start toward the development of the 4 000 000 h. p. available on the lower river. A report on such a plan would be of interest.

Mr. Davis has called attention to the great damage that would result through suddenly releasing 50 000 000 acre-ft. of water. This is obvious, the same quantity of water suddenly released at Boulder Canyon would be even more disastrous as this project is nearer the property most likely to be damaged.

Mr. Davis states that the writer is in error in claiming that the floods of the Colorado (excepting the Gila) may be controlled by building a dam at Lees Ferry and calls attention to the 50 000 sq. miles of drainage area between Lees Ferry and the Gila River. We are concerned with the drainage area tributary to the Colorado between Lees Ferry and Boulder Canyon. It is the

* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1615.
† *Loc. cit.*, p. 1628.

flood water contributed by this drainage and not the size of the basin that is important. Mr. Davis has presented no data to support his contention that dangerous floods originate in the basin between Lees Ferry and Boulder Canyon. The Little Colorado is by far the most important tributary to the Colorado in this section. Intermittent records are available, showing the flow of this stream since 1905. The largest flood recorded is 20 200 sec-ft. In January, 1916, when the Gila discharged a flood of 200 000 sec-ft., the flow of the Little Colorado was 16 000 sec-ft. There is a feasible site on the Little Colorado where a dam to raise the water 50 ft. would make possible the regulation of the flow of this stream to 600 sec-ft., or less. Furthermore, a power dam on the Colorado at the mouth of Diamond Creek would smooth out any flood that may reasonably be expected from the Paria, Little Colorado, or Kanab Creek. There is no record of a flood from the Virgin River, that would endanger property on the Lower Colorado.

For a given height of dam at Lees Ferry and at Boulder Canyon, the greater storage capacity may be had at Lees Ferry. In Table 3,* Mr. Davis gives, under the heading, "Boulder Canyon Reservoir", a capacity table which applies at the dam site in Black Canyon. The dam site in Black Canyon is 18 miles below the site in Boulder Canyon. The writer understands that Mr. Davis recommends the construction of a dam at the Boulder Canyon site.

In regard to the loss of water by evaporation, Mr. Davis might have gone one step further in order to show the evaporation that would take place if the Boulder Canyon site were to be developed. Evaporation at Boulder Canyon would be greater than at Lees Ferry for a given area of water surface. With equal storage capacity at Lees Ferry and Boulder Canyon, the loss due to evaporation would be practically the same.

Mr. Davis correctly states that the underground conditions at the Lees Ferry dam site are unknown and that these conditions at Boulder Canyon have been carefully investigated. More than $200 000 has been made available for the investigation of the Lower Colorado, nearly all which has been spent at Boulder and Black Canyons.

As the depth to bed-rock at the Lees Ferry dam site is considerably less than 100 ft., the suggestion made by Messrs. Stevens† and Fortier‡ are worthy of further consideration. With bed-rock comparatively near the surface and the condition for unwatering the site favorable, the writer has changed his opinion with reference to the plan for making the rock-fill water-tight. It seems probable that a feasible plan for a combination concrete rock-fill dam can be devised.

Mr. Fortier has submitted valuable data regarding the silt problem on the Lower Colorado, but the writer cannot accept his conclusion that, in order to store a greater quantity of silt, the first flood-control dam should be built at Boulder Canyon.

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* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1632.

† *Loc. cit.*, p. 1633.

‡ *Proceedings*, Am. Soc. C. E., October, 1922, p. 1711.

The constructive criticism submitted by so many engineers is greatly appreciated by the writer, and his objective has been realized. However, he wishes to admit that it was a mistake to present for discussion a proposition to build a rock-fill dam 780 ft. in height above its foundation. The magnitude of the project seemed to stagger the imagination of the engineers who discussed the paper. A more valuable discussion would have resulted if the writer had designed a dam about 400 ft. high. The principles, however, would be the same.

If a thorough study of the physical condition on the Lower Colorado should show that the first flood-control dam should be at Lees Ferry, the time will have arrived for careful study to determine the type of dam most feasible for this site.

No doubt a board of competent engineers would give due consideration to the rock-fill type of dam as well as the concrete dam of the gravity, arch, and multiple-arch type. As the depth to bed-rock at the Lees Ferry dam site is less than 100 ft., some engineers believe that a concrete dam of the multiple-arch type would be safe and also the most economical.

LOCOMOTIVE LOADINGS FOR RAILWAY BRIDGES

Discussion*

By D. B. Steinman, M. Am. Soc. C. E.†

D. B. Steinman,‡ M. Am. Soc. C. E. (by letter).§—The writer gave his best efforts to the preparation of this paper and was agreeably surprised by its generous reception. The interest shown and the many expressions of appreciation have more than rewarded him for his work.

The paper was intended to be suggestive rather than final, to evoke discussion, to outline the questions at issue, and to indicate methods of analysis, solution, and application. The proposed loading specification, in its three different forms, was merely tentative, and was submitted to the members of the Society as a basis for discussion or modification. It is, therefore, most gratifying to the writer to see his labors rewarded by the endorsement of the Special Committee on Specifications for Bridge Design and Construction, of the Society, in its recommended specifications for steel railway bridges.‖

The three forms of loading specification proposed in the paper may be designated as follows:

M-Loading No. 1—Typical engine diagram.
M-Loading No. 2—Simplified loading diagram.
M-Loading No. 3—Equivalent uniform loading formula.

The discussions disclosed varying preferences between these alternative loading specifications. The preponderance of choice appears to have been for M-Loading No. 1, the typical engine diagram. This also proved to be the choice of the Special Committee on Specifications for Bridge Design and Construction, which adopted M-Loading No. 1 (with a slight modification) for inclusion in the recommended specifications.¶

The modification referred to was the change of the spacing of the heavy driver axles from 5½ ft. to 5 ft., so that all wheel spacings are multiples of 5 ft., which makes a more convenient and simpler loading diagram. Although not as correct theoretically as the original 5½-ft. spacing, it is practically justified by various considerations; it departs somewhat (on the side of safety) from the Composite loading at the shorter spans, yielding a margin of stress-producing effect where it is most likely to be needed under future intensification of axle loads. The Cooper loading and the modified M-10 loading

* Discussion on the paper by D. B. Steinman, M. Am. Soc. C. E., continued from January, 1923, *Proceedings*.

† Author's closure.

‡ Cons. Engr., New York City.

§ Received by the Secretary, January 9, 1923.

‖ *Proceedings*, Am. Soc. C. E., January, 1923, p. 54.

¶ *Loc. cit.*, p. 59.

are shown on Figs. 16 and 17, respectively. The greater simplicity and convenience of the M-loading, as thus graphically presented, should require no further discussion.

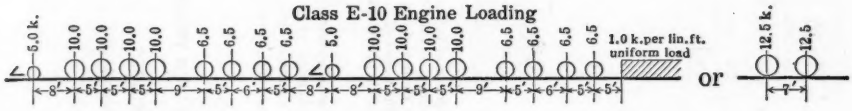


FIG. 16.

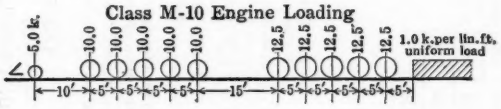


FIG. 17.

A wheel-load moment table for this M-loading is presented in Table 16,* of the Final Report on Specifications for Steel Railway Bridge Superstructure of the Special Committee on Specifications for Bridge Design and Construction. The corresponding table for the Cooper loading (Table 10†), does not compare with Table 16 in brevity and simplicity.

A diagram of equivalent uniform loads for the M-loading is presented in Fig. 18. A comparison of this diagram with Fig. 2‡ (which it supersedes) shows that the change from a 5½-ft. to a 5-ft. spacing has increased the stresses at the shorter spans about 5 per cent. A reference to Fig. 4§, or Fig. 6,‖ will show that this change is desirable inasmuch as the previous M-loading was slightly deficient at those span lengths.

A chart of equivalent uniform loads for this M-loading is presented in Plate I (superseding Plate XVII).‖ Plate I is extremely accurate and key diagrams have been added to facilitate its application. A table of wheel placements for maximum stress is also included. Plate I constitutes a complete and self-contained stress manual, and the writer recommends it as preferable to other tables or diagrams for obtaining stresses.

A graphic comparison of the approved M-loading with the Cooper loading and the Composite loading is presented in Fig. 19. This diagram is similar to Fig. 6,‖ which it supersedes. A comparison of Figs. 6 and 19 shows that the adopted modification of wheel spacing yields a desirable improvement of stress margin at short spans, at a slight sacrifice of scientific accuracy.

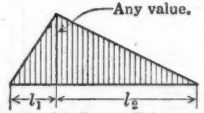
Fig. 20 presents a graphic comparison of the adopted M-loading with a modified loading formula which may be used as a "short-cut" for approximate results. This equation, for the M-10 loading, takes the simple form:

$$q = 700 + \frac{11\,000}{\sqrt{2\,l_1 + l_2}} \qquad (12)$$

* *Proceedings*, Am. Soc. C. E., January, 1923, p. 97.
† *Loc. cit.*, p. 81.
‡ *Proceedings*, Am. Soc. C. E., May, 1922, p. 1053.
§ *Loc. cit.*, p. 1055.
‖ *Loc. cit.*, p. 1059.

Shorter Segment l_1

Longer Segment l_2	0	10	20	30	40	50	60	70	80	90	100	120	140	160	180	200	220	240	260	280	300	400	500	600	700	800	900	1000
1000	6897	6878	6860	6840	6823	6807	6789	6772	6757	6740	6724	6694	6664	6635	6608	6580	6555	6530	6506	6481	6459	6355	6264	6185	6117	6053	5998	5948
900	7000	6978	6956	6934	6915	6894	6875	6855	6836	6818	6800	6763	6730	6698	6666	6636	6608	6579	6551	6525	6500	6385	6285	6200	6125	6059	6000	
800	7120	7095	7070	7045	7020	6997	6973	6950	6928	6907	6885	6845	6806	6768	6731	6697	6663	6632	6600	6570	6543	6415	6306	6212	6132	6060		
700	7268	7238	7205	7175	7145	7115	7090	7060	7030	7010	6985	6935	6890	6845	6805	6765	6725	6690	6655	6620	6590	6440	6325	6220	6135			
600	7450	7410	7370	7330	7295	7260	7225	7190	7160	7130	7100	7040	6985	6930	6885	6830	6790	6750	6710	6670	6630	6470	6335	6225				
500	7680	7630	7580	7530	7480	7440	7395	7350	7310	7270	7235	7160	7095	7030	6970	6915	6860	6810	6765	6720	6675	6490	6340					
400	8000	7925	7860	7790	7725	7665	7610	7550	7500	7450	7400	7310	7220	7140	7070	7000	6935	6875	6820	6765	6715	6500						
300	8460	8350	8245	8150	8055	7970	7885	7810	7735	7660	7600	7475	7360	7260	7165	7080	7000	6925	6855	6790	6730							
280	8580	8460	8340	8235	8135	8040	7950	7865	7785	7710	7640	7510	7390	7280	7180	7090	7010	6930	6860	6790								
260	8720	8580	8455	8335	8225	8120	8025	7930	7845	7765	7690	7545	7420	7305	7200	7100	7015	6935	6860									
240	8875	8720	8575	8440	8320	8200	8100	8000	7905	7815	7735	7580	7445	7325	7210	7110	7020	6935										
220	9045	8870	8705	8560	8420	8295	8180	8070	7965	7870	7780	7620	7470	7340	7225	7120	7020											
200	9240	9040	8860	8690	8535	8395	8260	8140	8030	7925	7830	7650	7495	7355	7230	7120												
180	9475	9240	9030	8835	8660	8500	8355	8220	8100	7980	7875	7685	7520	7370	7240													
160	9740	9460	9215	8995	8795	8615	8450	8300	8160	8035	7920	7710	7530	7370														
140	10 070	9730	9440	9175	8940	8735	8550	8380	8225	8085	7955	7730	7535															
120	10 480	10 055	9695	9380	9110	8865	8650	8460	8285	8130	7990	7740																
100	11 000	10 450	10 000	9615	9290	9000	8750	8530	8335	8160	8000																	
90	11 325	10 695	10 175	9745	9380	9065	8795	8560	8350	8160																		
80	11 710	10 960	10 365	9875	9470	9130	8830	8575	8350																			
70	12 170	11 270	10 575	10 020	9560	9130	8860	8585																				
60	12 740	11 640	10 800	10 160	9650	9220	8870																					
50	13 480	12 070	11 050	10 300	9710	9240																						
40	14 490	12 590	11 325	10 420	9745																							
30	15 960	13 220	11 570	10 475																								
20	18 420	13 940	11 710																									
10	23 970	14 480																										



l_1 = Shorter Segment of Influence Triangle.
l_2 = Longer Segment of Influence Triangle.
q = Equivalent Uniform Load taken from Chart for the values of l_1 and l_2 Values given in pounds per linear foot per track.

This table can be used for figuring the maximum stress in any member or section of any span. It gives stresses equal to (or slightly greater than) the maximum stresses producible by the heaviest existing locomotive loadings.

TABLE OF EQUIVALENT UNIFORM LOADS (PER TRACK) FOR DESIGNING RAILWAY BRIDGES

Based on Proposed Loading Formula M-60

$$q = 5000 + 60\,000\,\frac{\sqrt{l_2}}{l_1 + l_2}$$

For Maximum Bending Moment at any section of any span. l_1 and l_2 = the two segments of the span.

$$M = \tfrac{1}{2} \cdot q \cdot l_1 \cdot l_2$$

For Maximum End Reaction or End Shear, $l_1 = 0$ and l_2 = the span.

$$V = \tfrac{1}{2} \cdot q \cdot l_2$$

For Maximum Shear at any section of a span, $l_1 = 0$. l_2 = the longer segment of the span and l = the span.

$$V = \tfrac{1}{2} \cdot q \cdot l_2 \cdot \frac{l_2}{l}$$

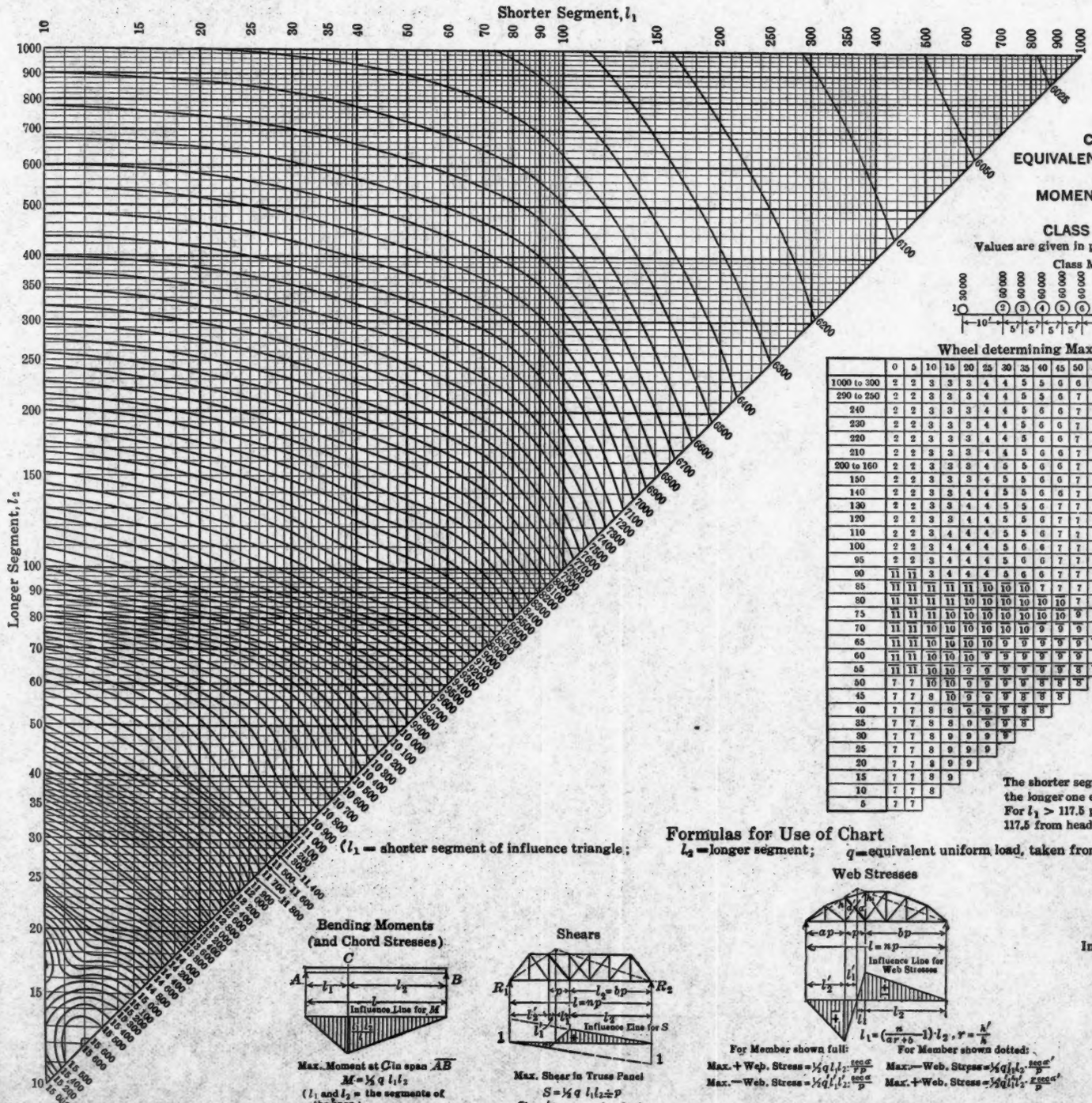
For Maximum Floorbeam Reaction or stress in Hanger, l_1 and l_2 = the adjoining panel lengths

$$R = \tfrac{1}{2} \cdot q \cdot (l_1 + l_2)$$

FIG. 18.

Longer Segment, l_2

Shorter Segment l_1	1	2	3	4	5	6	7	8	9	10
1000	6897	6878	6860	6840	6823	6807	6789	6772	6757	6740
900	7000	6978	6956	6934	6915	6894	6875	6855	6836	6818
800	7120	7095	7070	7045	7020	6997	6973	6950	6928	6907
700	7268	7238	7205	7175	7145	7115	7090	7060	7030	7010
600	7450	7410	7370	7330	7295	7260	7225	7190	7160	7130
500	7680	7630	7580	7530	7480	7440	7395	7350	7310	7270
400	8000	7925	7860	7790	7725	7665	7610	7550	7500	7450
300	8460	8350	8245	8150	8055	7970	7885	7810	7735	7660
280	8580	8460	8340	8235	8135	8040	7950	7865	7785	7710
260	8720	8580	8455	8335	8225	8120	8025	7930	7845	7765
240	8875	8720	8575	8440	8320	8200	8100	8000	7905	7815
220	9045	8870	8705	8560	8420	8295	8180	8070	7965	7870
200	9240	9040	8860	8690	8535	8395	8260	8140	8030	7925
180	9475	9240	9030	8835	8660	8500	8355	8220	8100	7980
160	9740	9460	9215	8995	8795	8615	8450	8300	8160	8035
140	10 070	9730	9440	9175	8940	8735	8550	8380	8225	8085
120	10 480	10 055	9695	9380	9110	8865	8650	8460	8285	8130
100	11 000	10 450	10 000	9615	9290	9000	8750	8530	8335	8160
90	11 325	10 695	10 175	9745	9380	9065	8795	8560	8350	8160
80	11 710	10 960	10 365	9875	9470	9130	8830	8575	8350	
70	12 170	11 270	10 575	10 020	9560	9130	8860	8585		
60	12 740	11 640	10 800	10 160	9650	9220	8870			
50	13 480	12 070	11 050	10 300	9710	9240				
40	14 490	12 590	11 325	10 420	9745					
30	15 960	13 220	11 570	10 475						
20	18 420	13 940	11 710							
10	23 970	14 480								

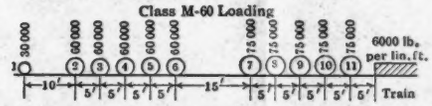


(l_1 = shorter segment of influence triangle; l_2 = longer segment; q = equivalent uniform load, taken from chart for l_1 and l_2)

CURVES OF EQUIVALENT UNIFORM LOADS FOR MOMENTS AND SHEARS FOR CLASS M-60 LOADING

Values are given in pounds per linear foot per track

Class M-60 Loading

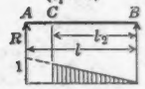


Wheel determining Maximum Stress

	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	100	110
1000 to 300	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	10	11	
290 to 250	2	2	3	3	4	4	5	5	6	7	7	7	8	8	9	9	9	10	10	11	
240	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	9	10	10	11		
230	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	9	10	10	11		
220	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	9	10	10	11		
210	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	9	10	10	11		
200 to 160	2	2	3	3	4	5	5	6	6	7	7	8	8	8	9	9	9	10	10	11	
150	2	2	3	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	11		
140	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	10	11	
130	2	2	3	3	4	4	5	5	6	7	7	7	8	8	9	9	10	10	10	11	
120	2	2	3	3	4	4	5	5	6	7	7	8	8	8	9	9	10	10	10	11	
110	2	2	3	4	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
100	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	11
95	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	
90	11	11	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10		
85	11	11	11	11	11	10	10	10	7	7	7	8	8	9	9	9	9	10			
80	11	11	11	11	10	10	10	10	10	10	7	8	8	9	9	9	10				
75	11	11	11	10	10	10	10	10	10	10	9	8	8	9	9	9					
70	11	11	10	10	10	10	10	10	9	9	9	9	8	8	9	9					
65	11	11	10	10	10	10	9	9	9	9	9	9	8	9	9						
60	11	11	10	10	10	9	9	9	9	9	9	9	8	8							
55	11	11	10	10	9	9	9	9	9	9	8	8									
50	7	7	10	10	9	9	9	9	8	8	8										
45	7	7	9	10	9	9	9	8	8	8											
40	7	7	9	8	9	9	9	8	8												
35	7	7	8	8	9	9	9	8													
30	7	7	8	9	9	9	9														
25	7	7	8	9	9	9															
20	7	7	8	9	9																
15	7	7	8	9																	
10	7	7	8																		
5	7	7																			

The shorter segment is shead followed by the longer one except where wheel is overlined For $l_1 > 117.5$ place head of uniform load 117.5 from head of span.

Equivalent Uniform Loads for Shears and Reactions ($l_1 = 0$)



Max. Shear at C $S = \frac{1}{4} q l_2 \frac{l_2}{l}$

Max. Reaction at A ($l_2 = l$) $R = \frac{1}{4} q l$

l_1	q
1000	6612
975	6627
950	6643
925	6660
900	6678
875	6696
850	6716
825	6737
800	6759
775	6783
750	6808
725	6834
700	6863
675	6894
650	6926
625	6962
600	7000
575	7041
550	7085
525	7134
500	7187
475	7246
450	7316
425	7382
400	7462
375	7551
350	7652
325	7767
300	7898
275	8051
250	8230
225	8441
200	8696
175	9007
160	9229
150	9393
140	9574
130	9772
120	9990
110	10 227
100	10 485
95	10 620
90	10 926
85	11 273
80	11 625
75	11 973
70	12 306
65	12 604
60	12 917
55	13 140
50	13 500
45	14 148
40	14 906
35	15 796
30	16 833
25	18 000
20	18 750
15	20 000
10	22 500
5	30 000

Formulas for Use of Chart

Bending Moments (and Chord Stresses)

Max. Moment at C in span $\overline{AB}$ $M = \frac{1}{8} q\, l_1 l_2$

(l_1 and l_2 = the segments of the span)

Shears

Max. Shear in Truss Panel $S = \frac{1}{4} q\, l_1 l_2 \div p$

($l_1 = \frac{l_2}{n-1}$. $l_2 = bp$)

Web Stresses

$l_1 = \left(\frac{n}{ar+b} - 1\right) l_2$, $r = \frac{h'}{h}$

For Member shown full:

Max. + Web. Stress $= \frac{1}{2} q\, l_1 l_2 \frac{\sec \alpha}{rp}$

Max. — Web. Stress $= \frac{1}{2} q\, l_1 l_2 \frac{\sec \alpha}{p}$

For Member shown dotted:

Max. — Web. Stress $= \frac{1}{2} q\, l_1 l_2 \frac{\sec \alpha'}{p}$

Max. + Web. Stress $= \frac{1}{2} q\, l_1 l_2 \frac{r \sec \alpha'}{p}$

Floorbeam Reactions Intermediate Pier Reactions

Max. Intermediate Reaction $R = \frac{1}{2} q\, (l_1 + l_2)$

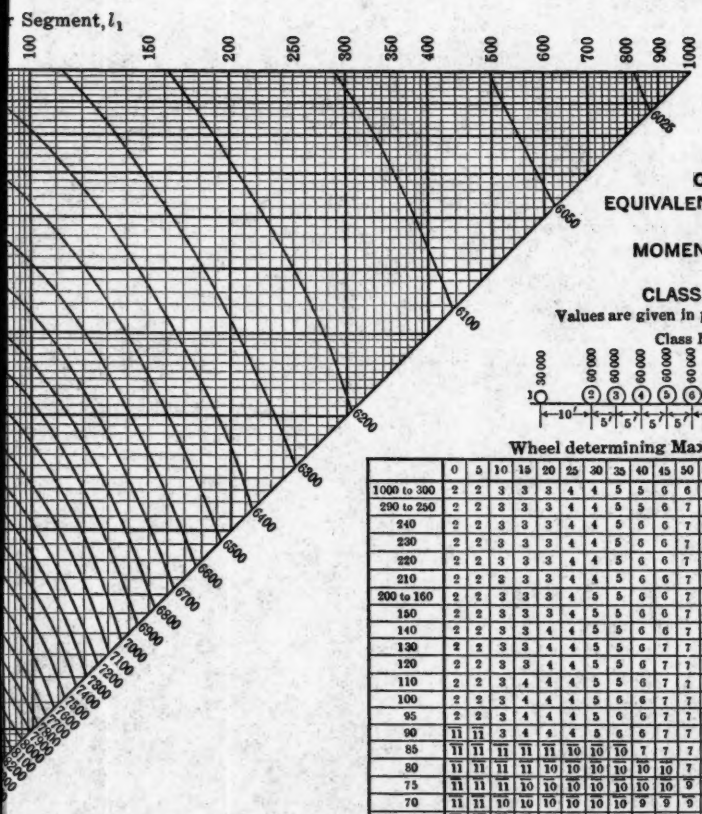
(l_1 and l_2 = the two adjoining spans.)

Fig. 18.

l_1 and l_2 = the adjoining panel lengths

$R = \frac{1}{2} \cdot q \cdot (l_1 + l_2)$

r Segment, l_1



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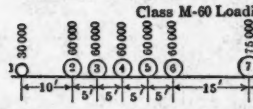
Class M-60 Loadi



Wheel determining Maximum S

	0	5	10	15	20	25	30	35	40	45	50	55	60	65
1000 to 300	2	2	3	3	3	4	4	5	5	6	6	7	7	8
290 to 250	2	2	3	3	3	4	4	5	5	6	7	7	7	8
240	2	2	3	3	3	4	4	5	6	6	7	7	7	8
230	2	2	3	3	3	4	4	5	6	6	7	7	7	8
220	2	2	3	3	3	4	4	5	6	6	7	7	8	8
210	2	2	3	3	3	4	4	5	6	6	7	7	8	8
200 to 160	2	2	3	3	3	4	5	5	6	6	7	7	8	8
150	2	2	3	3	3	4	5	5	6	6	7	7	8	8
140	2	2	3	3	4	4	5	5	6	6	7	7	8	8
130	2	2	3	3	4	4	5	5	6	7	7	7	8	8
120	2	2	3	3	4	4	5	5	6	7	7	8	8	8
110	2	2	3	4	4	4	5	5	6	7	7	8	8	8
100	2	2	3	4	4	4	5	6	6	7	7	8	8	8
95	2	2	3	4	4	4	5	6	6	7	7	8	8	8
90	11	11	3	4	4	4	5	6	6	7	7	8	8	8
85	11	11	11	11	11	10	10	10	7	7	7	8	8	9
80	11	11	11	11	10	10	10	10	10	10	7	8	8	9
75	11	11	11	10	10	10	10	10	10	10	9	8	8	9
70	11	11	10	10	10	10	10	10	9	9	9	8	8	9
65	11	11	10	10	10	10	9	9	9	9	9	9	9	9
60	11	11	10	10	10	9	9	9	9	9	9	9	8	
55	11	11	10	10	9	9	9	9	9	9	8	8		
50	7	7	10	10	9	9	9	9	8	8	8			
45	7	7	8	10	9	9	9	8	8	8				
40	7	7	8	8	9	9	9	8	8					
35	7	7	8	8	9	9	9	8						
30	7	7	8	9	9	9	9							
25	7	7	8	9	9	9								
20	7	7	8	9	9									
15	7	7	8	9										
10	7	7	8											
5	7	7												

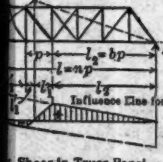
The shorter segment is s
the longer one except wh
For $l_1 > 117.5$ place head
117.5 from head of span.

ence triangle;

Formulas for Use of Chart

l_2 = longer segment; q = equivalent uniform load, taken from chart

Shears

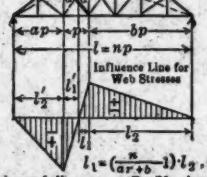


. Shear in Truss Panel

$S = \frac{1}{2} q\, l_1 l_2 \div p$

$= \frac{l_2}{n-1}$. $l_2 = bp$)

Web Stresses



$l_1 = (\frac{n}{ar+b} - 1) \cdot l_2$, $r = \frac{h'}{h}$

For Member shown full:

Max. + Web. Stress $= \frac{1}{2} q\, l_1 l_2 \cdot \frac{\sec \alpha}{p}$

Max. − Web. Stress $= \frac{1}{2} q\, l_1' l_2' \cdot \frac{\sec \alpha}{p}$

For Member shown dotted:

Max. − Web. Stress $= \frac{1}{2} q\, l_1 l_2 \cdot \frac{\sec \alpha'}{p}$

Max. + Web. Stress $= \frac{1}{2} q\, l_1' l_2' \cdot \frac{r \sec \alpha'}{p}$

Floorb
Intermedi

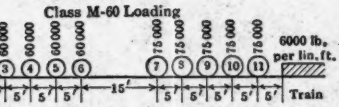
Max. In

(l_1 and
spans.)

CURVES OF EQUIVALENT UNIFORM LOADS FOR MOMENTS AND SHEARS FOR CLASS M-60 LOADING

are given in pounds per linear foot per track

Class M-60 Loading



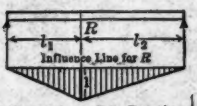
mining Maximum Stress

35	40	45	50	55	60	65	70	75	80	85	90	100	110
5	5	6	6	7	7	8	8	9	9	9	10	10	11
5	5	6	7	7	7	8	8	9	9	9	10	10	11
5	6	6	7	7	7	8	8	9	9	9	10	10	11
5	6	6	7	7	7	8	8	9	9	9	10	10	11
5	6	6	7	7	8	8	8	9	9	9	10	10	11
5	6	6	7	7	8	8	8	9	9	9	10	10	11
5	6	6	7	7	8	8	8	9	9	9	10	10	11
5	6	6	7	7	8	8	8	9	9	9	10	10	11
5	6	7	7	7	8	8	8	9	9	9	10	10	11
5	6	7	7	8	8	8	9	9	9	10	10	10	11
5	6	7	7	8	8	8	9	9	9	10	10	10	11
6	6	7	7	8	8	8	9	9	9	10	10	10	
6	6	7	7	8	8	8	9	9	9	10	10		
10	7	7	7	8	8	9	9	9	9	10			
10	10	10	7	8	8	9	9	9	10				
10	10	10	9	8	8	9	9	9					
10	9	9	9	8	8	9	9						
9	9	9	9	9	9	9							
9	9	9	9	9	9	8							
9	9	9	8	8									
9	8	8	8										
8	8	8											
8	8												
8													

he shorter segment is ahead followed by
he longer one except where wheel is overlined
or $l_1 > 117.5$ place head of uniform load
7.5 from head of span.

d, taken from chart for l_1 and l_2)

Floorbeam Reactions
Intermediate Pier Reactions



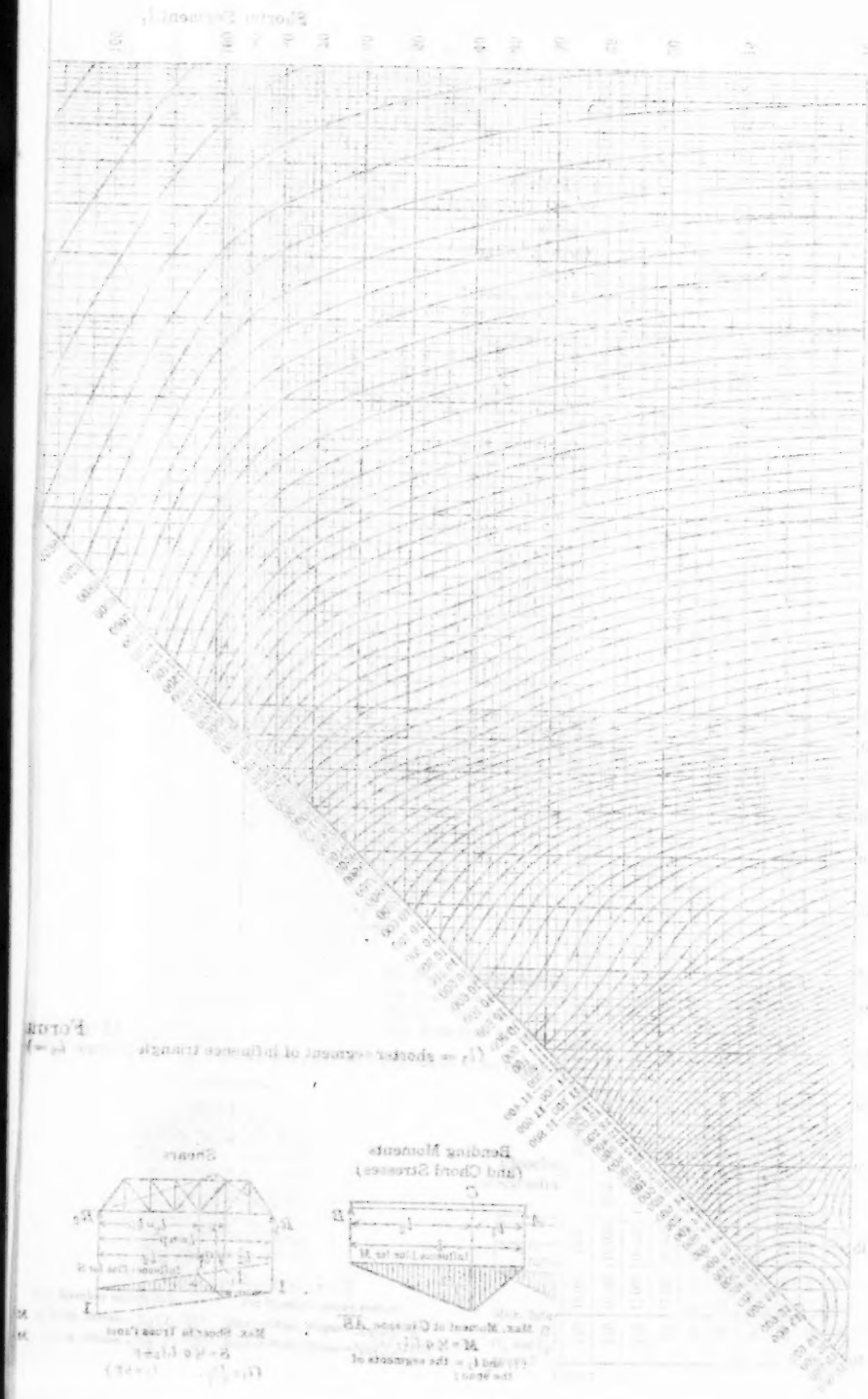
Max. Intermediate Reaction
$R = \frac{1}{2} q (l_1 + l_2)$
(l_1 and l_2 = the two adjoining spans.)

ed:
$\frac{l_1}{l_2} \cdot \frac{\text{less } a'}{P}$
$\frac{l_2}{l_1} \cdot \frac{\text{less } a'}{P}$

Equivalent Uniform Loads for Shears and Reactions
($l_1 = 0$)

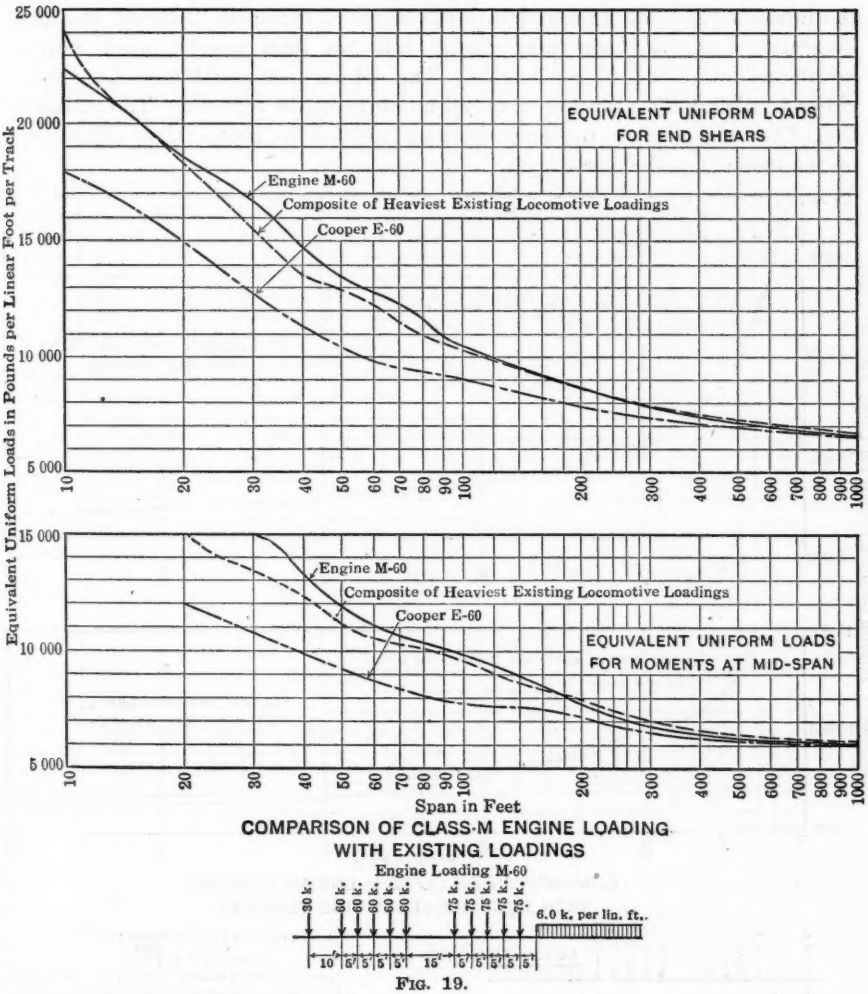
Max. Shear at C
$S = \frac{1}{2} q l_2 \frac{l_2}{l}$
Max. Reaction at A
($l_2 = l$)
$R = \frac{1}{2} q l$

l_1	q
1000	6612
975	6627
950	6643
925	6660
900	6678
875	6696
850	6716
825	6737
800	6759
775	6783
750	6808
725	6834
700	6863
675	6894
650	6926
625	6962
600	7000
575	7041
550	7085
525	7134
500	7187
475	7246
450	7310
425	7382
400	7462
375	7551
350	7652
325	7767
300	7898
275	8051
250	8230
225	8441
200	8696
175	9007
160	9229
150	9393
140	9574
130	9772
120	9990
110	10 227
100	10 485
95	10 620
90	10 926
85	11 273
80	11 625
75	11 973
70	12 306
65	12 664
60	12 917
55	13 140
50	13 500
45	14 148
40	14 906
35	15 796
30	16 833
25	18 000
20	18 750
15	20 000
10	22 500
5	30 000


Equivalent Uniform Loads in Pounds per Linear Foot per Track

in which,

q = the equivalent uniform load, in pounds per foot.
l_1 = the shorter loaded segment, in feet.
l_2 = the longer loaded segment, in feet.


EQUIVALENT UNIFORM LOADS FOR END SHEARS
Engine M-60
Composite of Heaviest Existing Locomotive Loadings
Cooper E-60
Equivalent Uniform Loads in Pounds per Linear Foot per Track
25 000
20 000
15 000
10 000
5 000
EQUIVALENT UNIFORM LOADS FOR MOMENTS AT MID-SPAN
Engine M-60
Composite of Heaviest Existing Locomotive Loadings
Cooper E-60
10 20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 1000
Span in Feet

COMPARISON OF CLASS-M ENGINE LOADING WITH EXISTING LOADINGS

FIG. 19.

This loading formula reduces to:

$$q = 700 + \frac{11\,000}{\sqrt{\text{span}}}, \text{ for end shears} \quad (13)$$

$$q = 700 + \frac{11\,000}{\sqrt{l_2}}, \text{ for intermediate shears} \quad (14)$$

$$q = 700 + \frac{9\,000}{\sqrt{\text{span}}}, \text{ for mid-span moments} \quad (15)$$

The previous loading formula (Fig. 13)* was developed independently of the typical engine diagram to approximate the Composite loading; consequently, at some points, the respective variations of the two loadings (No. 1 and No. 3) from the Composite loading were additive. In this discussion, the formula has been developed with the object of fitting the adopted M-engine diagram. Fig. 20 shows how closely this has been accomplished without sacrificing simplicity of coefficients. The coincidence could have been made closer by abandoning the use of round numbers in the formula. The introduction of the coefficient in the denominator permitted the independent adjustment of the moment and shear curves.

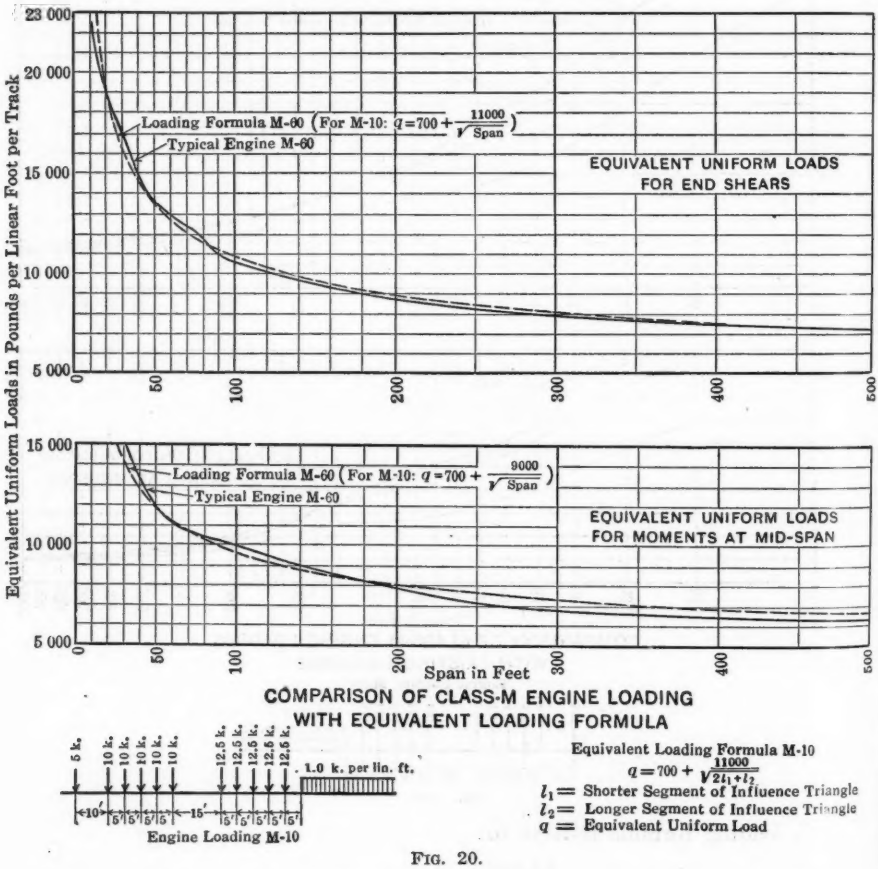


FIG. 20.

A table (Table 18†) of maximum moments, shears, and floor-beam reactions for the adopted M-loading has been presented by the Special Committee on Specifications for Bridge Design and Construction. Several of the discussors suggested the desirability of such a table to match the corresponding table which has been in use for the Cooper loading.

* *Proceedings*, Am. Soc. C. E., May, 1922, p. 1071.
† *Proceedings*, Am. Soc. C. E., January, 1923, p. 99.

Other tables suggested in the discussions have been prepared for the adopted M-loading. Table 19* gives the maximum bending moments at the various panel points of truss bridges for spans up to 450 ft., and Table 20† gives the shears in the various panels of truss bridges of spans up to 450 ft.

The argument that tables and diagrams are available for the application of the Cooper loading is nullified by the publication of the tables and diagrams for the M-loading just described. All tables and diagrams for the Cooper loading known to the writer are matched in form and scope by tables and diagrams for the M-loading. In addition, tables and diagrams which permit quick conversion of stresses or ratings between the two loading systems have also been prepared.

Table 15‡ as presented in the report of the Special Committee on Specifications for Bridge Design and Construction, gives conversion coefficients between E and M values for each type of stress (moment, shear, or floor-beam reaction) for span lengths up to 250 ft. By doubling the span lengths, the coefficients in the last column may be used for center moments for spans up to 500 ft.

Tables 11 and 12 (superseding Tables 3 and 4§) give a summary of the conversion values of the E and M-loadings for various spans up to 1 000 ft.

TABLE 11.—EQUIVALENT COOPER RATINGS OF THE M-50 ENGINE LOADING.

Span.	E-rating for maximum moment.	E-rating for end shear.
10	E-62.5	E-62.5
20	E-62.5	E-62.5
30	E-68.5	E-66.8
40	E-67.8	E-65.9
50	E-65.4	E-64.5
60	E-64.2	E-66.1
70	E-63.6	E-65.0
80	E-64.4	E-62.4
90	E-64.6	E-59.7
100	E-64.4	E-58.3
120	E-61.5	E-57.5
140	E-59.2	E-56.9
160	E-57.3	E-56.4
180	E-56.1	E-56.0
200	E-55.2	E-55.4
250	E-53.3	E-54.7
300	E-52.5	E-54.2
400	E-51.4	E-53.3
500	E-51.0	E-52.7
600	E-50.6	E-52.4
700	E-50.5	E-52.0
800	E-50.4	E-51.8
900	E-50.3	E-51.6
1000	E-50.2	E-51.5

A conversion chart for changing E-ratings into equivalent M-ratings, for any point in any span up to 1 000 ft., is presented in Fig. 21. This diagram shows that E-60 is equivalent to M-45 at short spans and to M-60 at long spans.

The inverse chart for changing M-ratings into E-ratings is presented in Fig. 22. This diagram shows that M-60 is equivalent to E-81 at short spans, and to E-60 at long spans. For smaller indices, reduce the values in propor-

* *Proceedings*, Am. Soc. C. E., January, 1923, p. 101.
† *Loc. cit.*, p. 107.
‡ *Loc. cit.*, p. 96.
§ *Proceedings*, Am. Soc. C. E., May, 1922, p. 1061.

tion; thus, the equivalent of M-50 ranges from E-68 to E-50. With these tables and diagrams, the use of the M-loading is greatly facilitated.

Because of the great number of discussions received, it is practically impossible to answer them in detail, and it becomes necessary to confine this discussion to a review of the salient points that have been raised.

Although nearly all the discussions express a total or partial endorsement of the writer's conclusions, a number of dissenting views have been advanced.

TABLE 12.—EQUIVALENT M-RATINGS OF THE COOPER E-60 LOADING.

Span.	M-rating for maximum moment.	M-rating for end shear.
10	M-48.0	M-48.0
20	M-48.0	M-48.0
30	M-43.8	M-44.4
40	M-44.2	M-45.5
50	M-45.9	M-46.5
60	M-46.7	M-45.4
70	M-47.1	M-46.2
80	M-46.6	M-48.1
90	M-46.4	M-50.2
100	M-46.6	M-51.5
120	M-48.8	M-52.2
140	M-50.6	M-52.7
160	M-51.5	M-53.2
180	M-53.5	M-53.6
200	M-54.3	M-54.1
250	M-56.3	M-54.8
300	M-57.1	M-55.3
400	M-58.4	M-56.3
500	M-58.8	M-56.9
600	M-59.4	M-57.2
700	M-59.5	M-57.7
800	M-59.6	M-57.9
900	M-59.7	M-58.1
1 000	M-59.8	M-58.3

The discussions of this group make frequent reference, as authority for their position, to the decision of the American Railway Engineering Association in 1920 to retain the Cooper loading, based on studies presented in a Sub-Committee report by Messrs. Welty and Bouton.* Those studies were based on comparisons of modern locomotives and train loads with the Cooper standard, from which the authors arrived at the conclusion that the Cooper loading adequately covered the locomotives assumed.

The writer, assuming substantially the same locomotives and train loads, and following the reasoning outlined in his paper, arrives at the conclusion that the Cooper loading is unsatisfactory as a present-day specification and that a better loading can be devised.

In attacking this conclusion, the principal argument of the dissenting group is that the writer's study is based on extra heavy locomotives, not in common use. In reply, it may be pointed out that, with one exception, the locomotives considered by the writer are identical with the heavy locomotives considered in the report of the Sub-Committee of the A. R. E. A., referred to previously. The assumptions are the same, but the conclusions are different. Those who endorse the report mentioned cannot consistently attack the writer's loading assumptions.

* *Proceedings*, Am. Ry. Eng. Assoc., January, 1920, pp. 511–520.

The writer would defend his loading assumptions, even if there had been no precedent to guide or justify him. He hoped he had made the logic of his argument clear. He based his studies on the heaviest existing locomotives, because they are the best index available of probable future development. History, in this respect, has repeated itself so often that it seems a lack of foresight to regard the present locomotive development as the limit of growth. Mr. Loweth, in his discussion,* recalls the frequent arguments in the past that the limit of locomotive development had been reached, and he writes: "Each

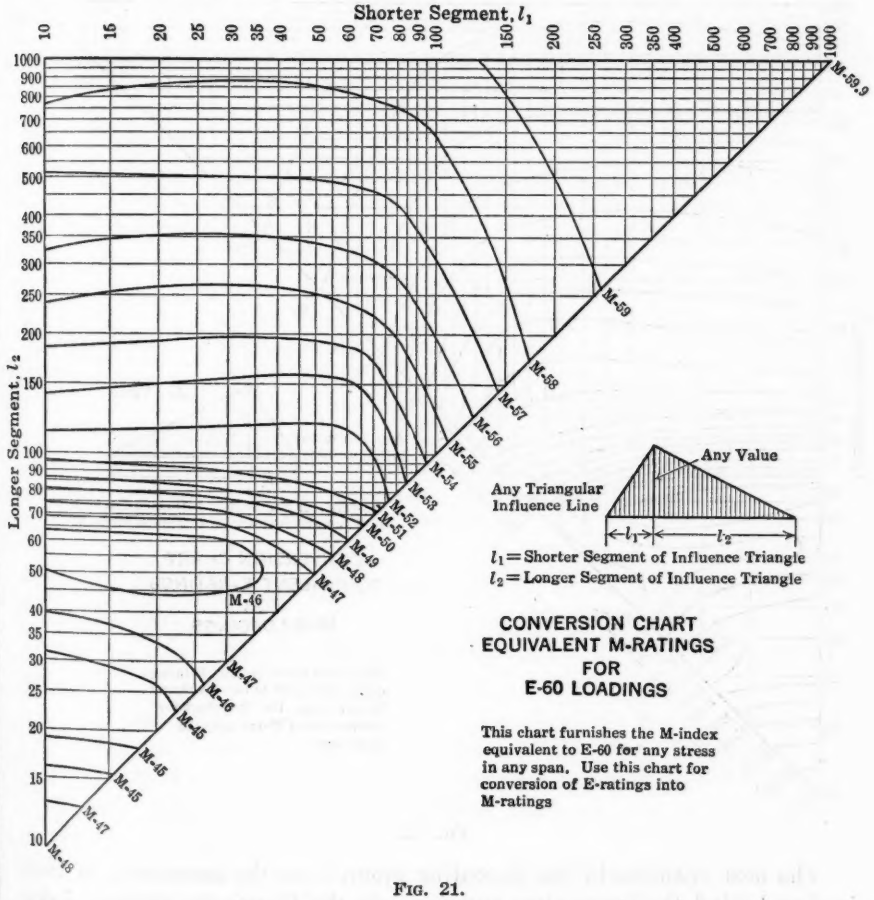


FIG. 21.

new locomotive seemed to be about the maximum that could be designed and would be practicable; yet, as the years passed, there was a continuing succession of heavier locomotives, and the end is not yet in sight." The extra heavy locomotives of yesterday have become the average locomotives of to-day; and the extra heavy locomotives of to-day will be the average locomotives of to-morrow. In the past, the life of bridges has been curtailed by loading augmenta-

* *Proceedings*, Am. Soc. C. E., January, 1923, p. 133.

tion; engineers should not be guilty of repeating that mistake. Bridges should be proportioned to the loads of 25 or 30 years hence, not to the loads prevailing to-day. Loading diagrams should conform, as far as possible, to the expected future loads. Specifications should be progressive, not short-sighted. Good engineering in this field demands vision, based on recognition of the lessons of the past.

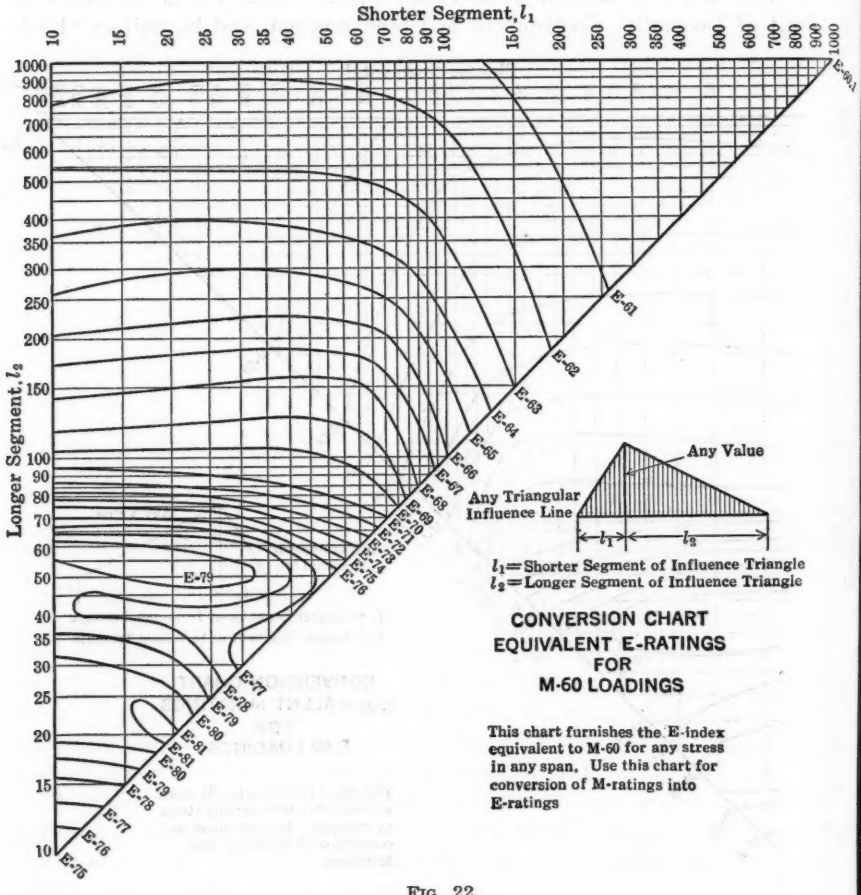

Shorter Segment, l_1
Longer Segment, l_2
Any Value
Any Triangular Influence Line
l_1
l_2
l_1=Shorter Segment of Influence Triangle
l_2=Longer Segment of Influence Triangle
CONVERSION CHART
EQUIVALENT E-RATINGS
FOR
M-60 LOADINGS
This chart furnishes the E-index equivalent to M-60 for any stress in any span. Use this chart for conversion of M-ratings into E-ratings

FIG. 22.

The next criticism by the dissenting group is on the assumption of train loading behind the locomotives entering into the Composite loading. Influenced by their use of the Cooper loading, they contend that 6 000 lb. per lin. ft. is too light a uniform load to follow these locomotives. The answer is simple: The actual train loads following these locomotives scarcely equal and certainly do not exceed 6 000 lb. per lin. ft. In the report of the Sub-Committee of the A. R. E. A., previously referred to, the train load assumed behind the same locomotives consisted of 210 000-lb. cars, amounting to only 4 770 lb. per lin. ft. The writer has been more generous in this regard than the authors of the

report which is being cited as an authority. On consulting the railroads in question, it will be found that the heaviest locomotives are used for hauling trains of cars weighing less than 6 000 lb. per lin. ft. Trains weighing 4 770 lb. per lin. ft. are assumed by the Pennsylvania Railroad Company behind its heaviest locomotives. The trains behind the heaviest locomotives of the Michigan Central Railroad weigh only 4 000 lb. per lin. ft. The locomotives are made heavy in order to haul longer trains, not heavier cars. The increase in car weights has not kept pace with the increase in locomotive weights. This point has received repeated endorsement in the discussions. As 6 000 lb. per lin. ft. generously represents the actual loading behind the heaviest existing locomotives, there appears to be no reason to assume a heavier loading behind those locomotives in the present investigation.

Others of the dissenting group, admitting the correctness of the writer's ratio of train load to locomotive loads, propose to solve the problem by adopting a modification of the Cooper loading instead of the proposed M-loading. They suggest a one-sixth reduction of the uniform train load behind the Cooper locomotive diagram, in order to change E-60 (with a uniform load of 6 000 lb.) into E-60 + 5 000, and E-72 into E-72 + 6 000.

Careful consideration shows, however, that this proposal would be an incomplete and unsatisfactory solution of the problem. Although correcting the disproportion in ratings between short spans and long spans, it will not correct the disparity in ratings between different members of the same span. Applying this proposed solution to the truss analyzed for purposes of illustration in Fig. 1,* this modification of the Cooper system changes the disparity of ratings from 37% to 36%—an insignificant improvement. This is shown on Plate II for different loadings on the same span. For the Pennsylvania N-1-S locomotives, the variations in rating within this truss are:

Cooper system	E -54.7 to E -75.0 = 37%
Modified Cooper system	E′-55.0 to E′-75.0 = 36%
M-system	M-46.2 to M-55.6 = 20%

For the Erie 2-10-2 locomotives, the variations in rating for this truss are:

Cooper system	E -53.9 to E -68.4 = 27%
Modified Cooper system	E′-54.2 to E′-68.4 = 26%
M-system	M-45.6 to M-50.6 = 11%

For the Pennsylvania I-1-S locomotives, the variations in rating within the 140-ft. span are:

Cooper system	E -53.5 to E -67.6 = 26%
Modified Cooper system	E′-53.9 to E′-67.6 = 25%
M-system	M-45.4 to M-50.4 = 11%

Averaging the results for the three locomotives, the following values are obtained:

Cooper system	Average discrepancy = 30%
Modified Cooper system	Average discrepancy = 29%
M-system	Average discrepancy = 14%

* *Proceedings*, Am. Soc. C. E., May, 1922, p. 1049.

In other words, the proposed modification of the Cooper loading reduces the disparity of ratings within a given span by only 1%, whereas the adoption of the M-loading would reduce the error to less than one-half. These comparisons (Plate II) show that the suggested modification of the E-system does not solve the problem of disparity of ratings within a span. Neither will it correct the disparity of rating between two short spans, for example, between 40 ft. and 140 ft. In addition to this defect, the proposed modification of the Cooper loading would not avoid a revision of existing stress tables and ratings; and this revision would be more likely to cause error and confusion than a clean-cut change to a new system.

Once the disproportion of the standard Cooper system is recognized, the remedy is not to "patch up" that system with inadequate modifications, but to make a fresh start with a simpler one founded on modern heavy loadings and, therefore, better designed to cover future developments. If the M-system is admitted to be simpler and better than the E-system, engineers should not stop half way in correcting their standards, but should make a clean-cut change and thereby secure all the advantages of the new standard.

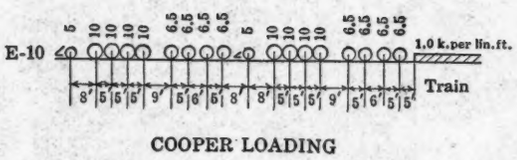
Another argument advanced against the proposed loading is that the majority of existing bridges are of spans of less than 100 or 120 ft., and that for these short spans the Cooper system yields a sufficiently close approximation to the stress ratios of modern loadings. This argument might have weight if the specifications were limited to spans of this length. It is true that if all spans were less than 80 or 100 ft., it would make comparatively little difference what system of loading specifications was used; but specifications are not limited to plate-girder spans. The A. R. E. A. Specifications include 300-ft. spans; the new specifications of the Special Committee on Specifications for Bridge Design and Construction, of the Society, are intended to apply to bridges of any span. If the Cooper system is satisfactory for plate-girder spans, but gives rise to serious discrepancies in longer spans, it should certainly be superseded by a loading system that gives consistent and harmonious results at all spans.

With the exception of the dissenting group, the arguments of which have just been reviewed, the general reaction to the writer's investigation and conclusions has been that of confirmation and endorsement. It is well expressed by the following quotation from the discussion by Mr. Strachan*:

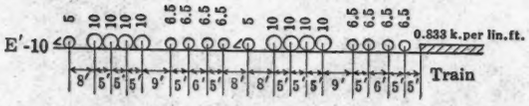
"* * * the author's determination of M-60 after careful study of the stress-producing effects of the heaviest existing locomotives, and his subsequent building up of a series of M-diagrams bearing simple ratios to M-60, in the manner of the Cooper loading, is an important step in the direction of introducing a scientific and modern standard load system for the economical design of railroad bridges and for the rating or comparison of their carrying capacities; a system of which the need is great in consequence of the changes in locomotive construction which have taken place since the Cooper system of loading was devised."

The individual discussions will now be reviewed in the order of their publication.

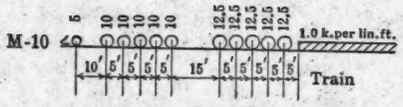
* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1677.



COOPER LOADING

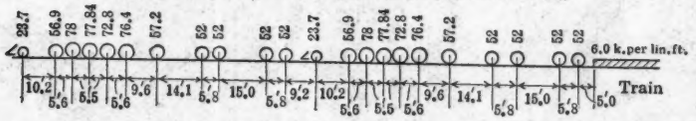


MODIFIED COOPER LOADING



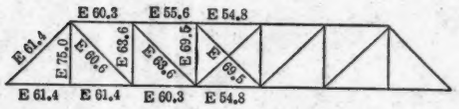
CLASS M LOADING

RATINGS FOR TRUSS DESIGNED FOR



2 PENNSYLVANIA N1S ENGINES
(SANTA FÉ)

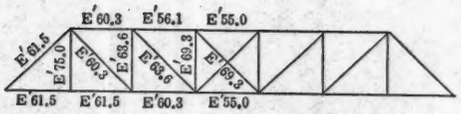
E-RATING



Max. Rating = E 75.0 Min. Rating = E 54.7

Variation = 37%

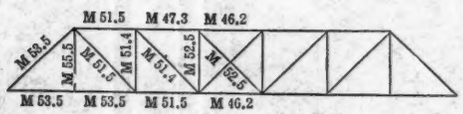
E'-RATING



Max. Rating = E'75.0 Min. Rating = E'55.0

Variation = 36%

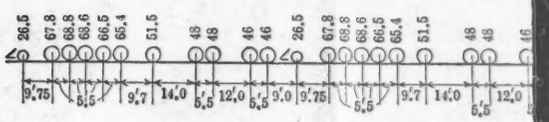
M-RATING



Max. Rating = M 55.6 Min. Rating = M 46.2

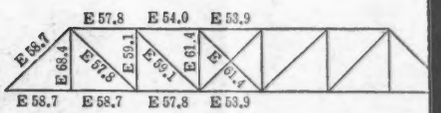
Variation = 20%

RATINGS FOR TRUSS DESIGNED FO



2 ERIE ENGINES 2-10-2
(SANTA FÉ)

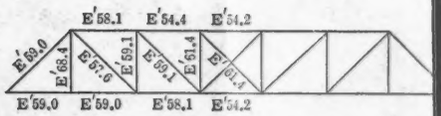
E-RATING



Max. Rating = E 68.4 Min. Rating = E 5

Variation = 27%

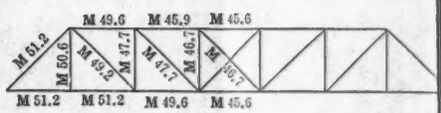
E'-RATING



Max. Rating = E'68.4 Min. Rating = E'5

Variation = 26%

M-RATING

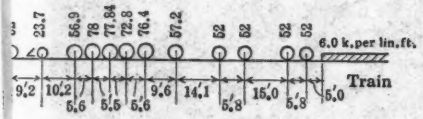


Max. Rating = M 50.6 Min. Rating = M 45

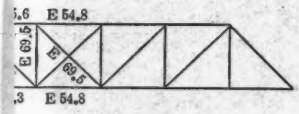
Variation = 11%

COMPARISON OF VARIATIONS OF RATING IN A 140-FT. SPAN FOR DIFFERENT LOADING SYSTEMS

TRUSS DESIGNED FOR

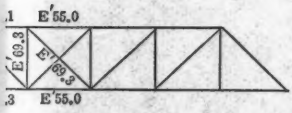


VANIA N1S ENGINES
(SANTA FÉ)



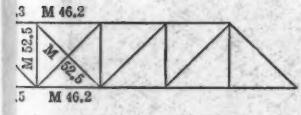
5.0 Min. Rating = E 54.7

ariation = 37%



5.0 Min. Rating = E'55.0

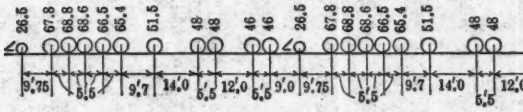
ariation = 36%



5.6 Min. Rating = M 46.2

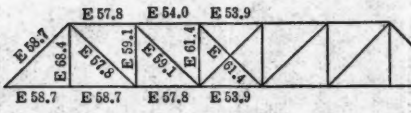
ariation = 20%

RATINGS FOR TRUSS DESIGNED F



2 ERIE ENGINES 2-10-2
(SANTA FÉ)

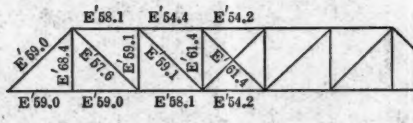
E-RATING



Max. Rating = E 68.4 Min. Rating = E

Variation = 27%

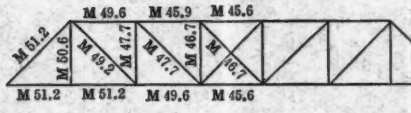
E'-RATING



Max. Rating = E'68.4 Min. Rating = E

Variation = 26%

M-RATING

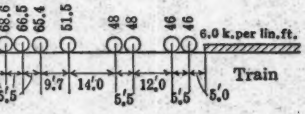


Max. Rating = M 50.6 Min. Rating = M

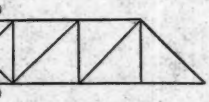
Variation = 11%

COMPARISON OF VARIATIONS OF RATING IN A 140-FT. SPAN FOR DIFFERENT LOADING SYSTEMS

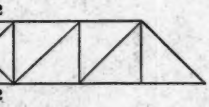
S DESIGNED FOR



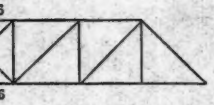
NES 2-10-2
FÉ)



Min. Rating = E 53.9
= 27%



Min. Rating = E′54.2
= 26%



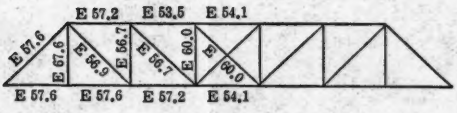
Min. Rating = M 45.6
= 11%

T. SPAN

RATINGS FOR TRUSS DESIGNED FOR

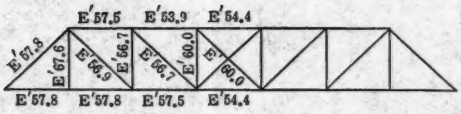
2 PENNSYLVANIA I1S ENGINES
(DECAPOD)

E-RATING



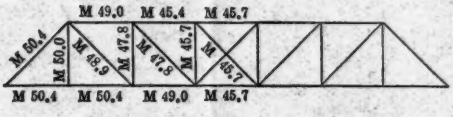
Max. Rating = E 67.6 Min. Rating = E 53.5
Variation = 26%

E′-RATING



Max. Rating = E′67.6 Min. Rating = E′53.9
Variation = 25%

M-RATING



Max. Rating = M 50.4 Min. Rating = M 45.4
Variation = 11%

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The criticism of Mr. Purdon,* that the engines considered in the investigation are "enormous" or "exceptional" has already been answered. Even if all existing locomotives had been included in the analysis, the results would have been the same, as the object of the analysis was to determine the enveloping or maximum stress effects. No specification can be regarded as scientific or adequate unless it takes into account maximum stress-producing effects. Mr. Purdon's suggestion that bridges should provide for possible small engines "as a minimum" does not appear to be logical.

Mr. Purdon makes the point that the proposed typical engine (M-Loading No. 1) "does not resemble an engine". That, of course, is a matter of personal opinion. If the M-diagram does not resemble any particular engine, that is because it is a composite of different types, conventionalized and simplified. It combines the governing features of the Santa Fé or Decapod with those of the Mallet locomotive. As stated by Mr. Cochrane†:

"Modern locomotives differ so greatly in the arrangement and spacing of axle loads that no single type can be found to represent all of them, unless it is a more or less conventional system, such as the author's first alternative loading. A composite loading derived from a number of different types of heavy locomotives will undoubtedly be more nearly representative of future loadings than any single loading series, and, consequently, will make more adequate provisions for future developments in locomotive design."

The point is also clearly covered by Professor Bird.‡

The proposed M-loading diagram may be regarded as an outline of a Mallet engine with different weights on the two sets of axles (for which there are precedents); or, it may be regarded as a Santa Fé or Decapod locomotive with the engine placed next to the train (for maximum stress). Resemblance to any particular type could easily be secured by minor modifications; but it would be at the expense of simplicity and symmetry.

Mr. Purdon attempts to demonstrate the adequacy of the E-loading by showing that a certain span designed for the Cooper loading gives satisfactory uniformity of stress under an E-42 double-headed Consolidation engine. This was naturally to be expected, as the Cooper loading diagram is a double-headed Consolidation engine. It is for types other than Consolidation engines, and for loads exceeding E-50, that the Cooper loading reveals its inadequacy.

Mr. Purdon gives figures showing the rapidly growing use of the Santa Fé type of locomotive, although its efficiency only slightly exceeds that of the Consolidation engine. He also shows the much greater efficiency of the Decapod or "ten-wheeler". In these conclusions, he is confirmed by Mr. Bland;§ but they serve only to strengthen the writer's position. Whether the Santa Fé or the Decapod locomotive represents the governing type in the future is immaterial in this discussion, as their stress-producing effects are practically identical; the proposed M-loading will provide for them, whereas the E-loading does not fit either. Plate II shows how the M-loading gives much better results for Santa Fé and Decapod locomotives than the Cooper

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1675.
† *Loc. cit.*, p. 1678.
‡ *Loc. cit.*, p. 1699.
§ *Loc. cit.*, p. 1696.

loading. For the Decapod locomotive, the E-rating variation in a given span is 26%, whereas the M-rating variation is only 11 per cent.

Mr. Strachan* points out the inconsistent accuracy of extreme precision in bridge designing when the fundamental loading assumptions are wrong. He characterizes the Cooper system as "an ancient generalization, which, although excellent in its day, has outlived its usefulness." He endorses the proposed M-loading, and states that "its adoption would tend toward augmented economy and the elimination of many unprofitable complications inseparable from the use of an obsolete load specification." Although preferring the typical engine diagram (M-Loading No. 1) as a load specification, Mr. Strachan suggests the use of the alternative loadings as "short-cuts" under suitable circumstances. This idea has been applied in Fig. 20 in which is presented a simple loading formula that may be used as an approximate substitute for the M-engine loading specification.

Mr. Cochrane† supports the writer in his claim that the Cooper loading suffers from lack of balance between the locomotive axle concentrations and the uniform train load. He points out that, in the Cooper series, the maximum axle load is ten times as great as the train load per linear foot, whereas, in the case of the heaviest loadings now in use, this ratio is about 13. (This ratio for the Pennsylvania Railroad H-C-1-S locomotive, reaches 14 or 15). Mr. Cochrane, therefore, agrees with the writer in the desirability of making the axle loads heavier than those of the Cooper series in proportion to the uniform train load.

For specification purposes, Mr. Cochrane expresses a preference for the loading formula (M-Loading No. 3) as being the simplest and most direct. He suggests, however, the use of an engine diagram (M-Loading No. 1) in conjunction with the formula as an approximate depicture of the loading. To reduce the discrepancy between the two alternative loadings, he proposes some modifications of the formula; but the writer submits Fig. 20 as a simpler solution of the problem.

Mr. Lucius‡ predicts that the tendency in future bridge loadings will not be toward increased driver axle loads, but rather toward an increase in the number of drivers. This is an argument for the adoption of the M-system in preference to the E-system, as the increase in the number of drivers means the abandonment of the Consolidation type of locomotive for the more modern types on which the M-loading is based.

Mr. Lucius also raises the question of electrification and the new motor diagrams which it may develop. This is a question in regard to which it is difficult to come to any conclusion or to make any predictions at this time, as the standardization and development of electric motor diagrams is still in its infancy. Nevertheless, a study of the tendency in recent heavy electric locomotive diagrams indicates that the M-system will be better fitted than the E-system to the electric locomotives of the future.

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1677.
† *Loc. cit.*, p. 1678.
‡ *Loc. cit.*, p. 1681.

Mr. Worthington* emphasizes the fact that the present locomotive types have outgrown the Cooper loading diagram, and endorses the writer's conclusion that a material revision of this loading diagram is necessary in order to secure uniformity of stress and economy of metal. After a careful study in conjunction with H. F. Passel, M. Am. Soc. C. E., he expresses a preference for the typical engine diagram (M-Loading No. 1).

Mr. Gilbert† also confirms the writer in his claim that the Cooper loading does not suitably provide for modern types of engines. From his own investigations, Mr. Gilbert believes that a diagram of the Santa Fé type would properly represent modern locomotive loadings. He points out the increasing use of the Santa Fé type, and states, as his conclusion, that the Santa Fé is a permanent type which should be given full consideration in selecting a proper bridge loading. This conclusion appears to be incontrovertible. The writer would point out that the M-loading diagram contains the essential elements of the Santa Fé axle diagram, conventionalized and simplified; and he would call attention to the comparison previously mentioned, which shows how the M-loading is superior to the Cooper loading in representing the stress-producing effects of Santa Fé locomotives.

Mr. Gilbert characterizes the proposed M-60 loading as "equal to or greater than that of any locomotive now in service", and questions whether railroad executives will approve of such a heavy loading. The writer is convinced that the adoption of the present maximum (M-60) for the design of new bridges on main lines will be amply justified by the inevitable increase in loading during the life of those structures. However, if any railroad officials regard this loading as excessive, there is nothing to prevent them from using M-50 or M-55 instead; or, preferably, the unit stresses may be augmented so as to give the same weight of structure with M-60 loading.

In the proposed M-60 loading, each of the five heavy axle loads are 75 000 lb. Mr. Gilbert shows by tabulated data that average axle loads exceeding 60 000 lb. are common, and that axle loads of 78 000 lb. are in use. To this may be added the Pennsylvania H-C-1-S engine, which has axle loads of 86 400 lb. Mr. Gilbert admits that the tendency of driver axle loads is to increase (on account of the increasing demand for tractive power). In view of these facts, a loading specification (M-60) with 75 000 lb. on the driver axles, does not seem to be excessive if bridges are to be designed with any regard to future requirements.

Mr. Gilbert shows by diagrams (Fig. 13‡) the inadequacy of E-60 loading to provide for actual engines; he finds the resulting over-stress produced by Santa Fé locomotives in the shorter spans to be approximately 25 per cent. This would correspond to E-75 which is equivalent, at these spans, to M-60. Accordingly, Mr. Gilbert's diagrams demonstrate that M-60 is not too high a loading to protect new designs against present-day locomotives.

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1681.
† *Loc. cit.*, p. 1682.
‡ *Loc. cit.*, p. 1685.

Mr. Gilbert emphasizes the importance of a loading specification which will give ample section to short spans, so as to provide the needed protection against corrosion to floor systems, hangers, etc. This requirement is fulfilled by the M-system as it corrects a relative deficiency of 25% in the Cooper loading for short spans.

Mr. Waddell* generously endorses the writer's thesis. He discounts the argument for retaining the Cooper loading as a rating standard, declaring that the M-loading system provides just as convenient a rating standard.

Mr. Waddell expresses a preference for M-Loading No. 3 (the loading formula) as the most scientific and convenient; and he endorses the suggestion made by Mr. Cochrane that M-Loading No. 1 (the typical engine diagram) be used in conjunction with the formula as an approximate representation.

Mr. Lindenthal† properly characterizes the subject as one in which meticulous accuracy is out of place. He expresses his preference for M-Loading No. 1 (the typical engine diagram), as it would represent the smallest departure from present methods, merely substituting a new and simpler wheel-load diagram for the Cooper diagram, and as it is sufficiently inclusive of the maximum effects of loads from any of the seven locomotive wheel diagrams illustrated in the paper.

Mr. Lindenthal points out that the floor construction, consisting of short spans, suffers more than the trusses from an increase of locomotive weights. That is one of the writer's reasons for advocating a loading specification which will be relatively more generous to short spans than the Cooper specification. Mr. Lindenthal mentions the greater allowance for impact in floor members, but he confuses the issue by suggesting that this impact allowance covers to a certain extent unforeseen increases in wheel loads. The writer is of the opinion that the impact provision for short spans in present specifications is inadequate and cannot be considered as including a margin for load increase.

Mr. Lindenthal raises the question of relative impact (due to difference in speed) of freight and passenger locomotives. This point must await further experiment and, when determined, should be covered in the specification for impact. The writer does not believe, however, that light passenger locomotives at full speed will produce greater total stresses than the heavy axle loads at freight-train speed; in addition, the possibility of the heavy locomotives running over a bridge at full speed when relieved of train load, must be considered as well as the possibility of passenger trains being limited in speed in going over the structure. The subject of impact provision is a large problem in itself, and should not be permitted to obscure the issue in the present investigation. Moreover, even if Mr. Lindenthal is correct in his contention that passenger locomotives should govern in design, it does not affect the choice between the M-loading and the Cooper loading, as the trains behind passenger locomotives are much lighter than those behind freight locomotives, and the ratio of train weight to locomotive weight in passenger

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1687.

† *Loc. cit.*, p. 1689.

engines conforms to the M-loading and not to the E-loading. For example, the K-4-S passenger locomotive of the Pennsylvania Lines, with its following train load, is found to be closely and consistently represented by M-49.

Professor Caughey* endorses the writer's thesis that the Cooper loading should be replaced by a loading more in agreement with the weights and axle groupings of modern railway rolling stock. He records his preference for M-Loading No. 1 (the typical engine diagram) and gives some cogent reasons for this choice.

Professor Caughey raises the question of train loads in exceptional cases being heavier than the engine, when light locomotives are used with short trains of heavy cars. Any such exceptional case should receive special study in adopting a design load; if a suitable index is used, either the M-loading or the E-loading will give safe stresses for this case.

The writer is grateful to Mr. Ralston,† for his expressive review of the questions under consideration, and for his encouraging words of appreciation. Mr. Ralston endorses the writer's conclusions as to the erratics of the Cooper loading system, and advocates the adoption of the M-system as consistent and logical. He gives a striking review of the history of locomotive development, and records his conviction that the M-loading system will remain applicable for a long time to come.

Mr. Bland, in his discussion previously mentioned, contributes an interesting account of the introduction of the E-loading system. He recalls that the late Theodore Cooper, M. Am. Soc. C. E., gave the E-25 loading as a minimum, and the E-40 as a maximum, which latter he regarded as the heaviest loading possible. Among the disadvantages of the Cooper loading, Mr. Bland cites the incorrect axle-load ratios and distances, the fixing of a ratio between train load and axle load which was not applicable to loads above E-40 or E-50, and the inapplicability of the E-standard to engines other than those of the Consolidation type.

From considerations of locomotive development, Mr. Bland concludes that the E-loading is not applicable beyond Cooper's upper limit of E-40, and that, for many years, the Cooper loading has ceased to be a proper standard.

After condemning the inconsistencies resulting from the use of the Cooper loading (such as the necessity for different E-ratings for different parts of the same bridge), and recording his hope that the M-loading in one of its three forms will soon be adopted by the Profession, Mr. Bland expresses his own preference for M-Loading No. 3 (the loading formula) for bridge design. The writer is in sympathy with this preference for the loading formula, but regrets that conditions do not appear ripe for this innovation in loading specifications. If M-Loading No. 1 (the typical engine diagram) is adopted as a general specification, the individual designer may use a loading formula (either that of Fig. 13‡ or Fig. 20) as a sufficiently close equivalent.

Mr. Bland endorses the writer's underlying principle of a Composite loading, and recommends that each railroad "consider the group of its engines

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1689.
† *Loc. cit.*, p. 1691.
‡ *Proceedings*, Am. Soc. C. E., May, 1922, p. 1071.

for which it wishes to provide, form a 'composite' of such engines, and then choose the M-loading (taking into account future development) that best fits its case, and use such loading in its specifications." According to his correspondence with the writer, Mr. Bland has done this for the engines of the Pennsylvania System, and has established a series of M-loadings which give consistent protection to the various types of locomotives in use.

Based on considerations of comparative efficiency, Mr. Bland makes a prognosis of future engine development. He shows that the development of the Mikado type (2-8-2) is a step in the wrong direction, but that the Decapod type (2-10-0) offers the greatest advantages. The I-1-S engine (Decapod) yields 69% more tractive power than the H-10-S (Consolidation), with an increase of only 45% in the weight of the engine. Accordingly, he concludes that the Decapod type is more likely to be increased in the future than other types. As the stress effects of the Decapod and Sante Fé types are practically the same, either may be considered as a governing type to which a correct loading specification should conform in order to provide for the most probable future development.

Mr. Johnson* admits the defects of the Cooper loading as applied to roads using the heavy locomotives of the present day. He points out, however, the great mileage of roads on which lighter locomotives prevail, and he considers the Cooper loading satisfactory for such lines. In reply, the writer would state that the M-loading (with the index reduced from M-60) would be just as satisfactory as the E-loading for these lines, and a loading standard (like the M-series) which can be used on all lines, both heavy and light, is obviously preferable to one which is limited in application to second-class roads.

Furthermore, Mr. Johnson admits that Santa Fé locomotives are a governing type on these lighter roads, and the writer has previously shown that the M-loading gives more consistent protection than the E-loading to engines of the Santa Fé type. It may also be noted that the secondary lines referred to generally have steeper grades and sharper curves, requiring heavier locomotives in proportion to the weight of the train; and the M-system, with its correction of ratio of engine weight to train weight, will fit this loading better than the E-system.

Mr. Johnson admits that the proposed M-loading is in the right direction, but thinks its application to the lighter lines of the country should be deferred. As opposing this view may be urged the desirability of a uniform standard for all lines. If the advantages of the M-system for the main lines are admitted, considerations of uniformity would argue for the adoption of that loading system for all lines. M-50 or M-60 can be used on the main lines and M-40 or M-50 on the lighter lines, depending on the governing types and weights of locomotives.

Mr. Rogers† admits that the M-60 loading covers all stresses in all spans from the five heaviest present-day types of engines. He adds that the proposed conventional loading would conform admirably to the requirements if

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1697.
† *Loc. cit.*, p. 1698.

all these five types of locomotives were in use on any one railroad; otherwise, he argues, there will be a waste of material put into bridge structures to suit the requirements of engines of foreign roads. In this argument, Mr. Rogers misses the underlying principle of the writer's paper, namely, that a composite loading is the most scientific solution of the problem of devising a single standard to fit all engine types with a view to future development. Such comparisons as those illustrated previously show that the M-loading (based on the Composite) yields sufficiently close agreement with individual engines entering into the Composite; certainly a much closer agreement than that provided by the Cooper loading. Inasmuch as disproportion of stress means waste of material, the comparisons given demonstrate that designs made with the M-loading will involve less waste than if the E-loading is used.

Mr. Rogers also raises the point that lighter engines are not reduced models of the heavy locomotives which entered into the Composite. The fact is, however, that although the M-loading may not be a correct pictorial representation of these lighter locomotives, it will be found to yield correctly proportioned designs. Mr. Bland, who has made special studies along this line, has found the M-loadings to give uniform and consistent results for the lighter engines on the Pennsylvania Lines; for instance, he found that the Mikado and Consolidation locomotives are correctly and consistently represented by M-50 as a design load. (According to Mr. Bland's graphs and conclusions, the L-1-S Mikado is economically protected by M-50; the K-4-S passenger locomotive by M-49; the H-10-S Consolidation by M-47; the H-8a-S by M-45; and the H-6b-S by M-42.)

Mr. Rogers adds the argument that if the locomotives on any line reasonably conform to the Cooper loading, the use of the M-loading would involve a waste of metal at short spans. The answer to this is the fact that the disproportion between long and short spans arises mainly from the incorrect ratio of train load to locomotive loads in the Cooper system. A change in type of locomotive does not alter this condition. With proper reduction of the index, the M-loading will give consistent results for the conditions assumed by Mr. Rogers. However, if there is any excess of metal in short spans resulting from the use of the M-loading, that extra metal is in a good place. A loading that adds extra metal to floor systems and short spans is unquestionably preferable to one which skimps the short spans and wastes metal on the long spans.

Mr. Rogers recommends that instead of adopting the M-loading with varying index, each railroad should devise its own special loading for design. The writer submits that this would result in confusion of ratings and comparisons, and would be far from a solution of the problem.

Mr. Stone* recommends the new loading system as a step in the direction of filling a recognized need. He expresses a preference for M-loading No. 1 (the typical engine diagram), because, with suitable tables, it would be as simple to apply as any other method. He suggests the appointment of a committee to consider the question of endorsing the proposed M-loading. This

* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1699.

has already been done by the Special Committee on Specifications for Bridge Design and Construction.*

Professor Bird, in his discussion previously mentioned, endorses the writer's conclusions and reports that he has calculated a number of designs using the M-loading, and has been pleased to note how well the values fitted the maximum moments and shears producible by actual locomotives.

He refers to the charts of equivalent uniform loads devised by the writer (Plates XI,† XIII,† XVIII,† and I) and suggests that it would be well to explain and illustrate the use of these charts with typical problems. Instead of taking space in this discussion, the writer would refer the readers to the explanation and illustration of these charts published elsewhere.‡

Professor Bird also suggests that tables of moments and shears for the M-loading would be a helpful addition to the paper. Such tables were also prepared by the writer at the request of the Special Committee on Specifications for Bridge Design and Construction and are incorporated in the report of that Committee.§

Professor Bird endorses the idea suggested by Henry B. Seaman,‖ M. Am. Soc. C. E., to include both the E-loading and the M-loading in the specifications, in order to make the change optional during the transition period.

Professor Fuller, in his discussion,¶ endorses the writer's conclusions. "The Cooper loading", he states, "no longer represents even to an approximate degree the modern locomotives and trains."

Professor Fuller declares his preference for M-Loading No. 1 (the typical engine diagram), and raises the interesting question of the flexibility of the loading formula (M-Loading No. 3), suggesting that re-adjustment may be found necessary after the M-loading has been in use half a century or more. In reply, the writer would point out that the loading formula (Equation (4))** contains two numerical coefficients; by modifying one or both of these coefficients, the curves of equivalent uniform loads in Fig. 13†† may be raised or changed in slope to conform to any requirements. In the modified form of the loading formula (Equation (12)), there are three numerical coefficients, giving still greater flexibility. By modifying the coefficient of l_1 in the denominator, the curve for center moments may be adjusted independently of the curve for shears. Fig. 20 is a graphic demonstration of the flexibility of this loading formula, as curves have been obtained to coincide so closely with the arbitrary curves of a typical engine diagram. The coincidence could have been made still closer by deviating from the round numbers chosen for the three coefficients in the formula.

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* *Proceedings*, Am. Soc. C. E., January, 1923, pp. 54, 59.

† *Proceedings*, Am. Soc. C. E., May, 1922, pp. 1047, 1049, 1069.

‡ *Engineering News*, April 22, 1915; "American Civil Engineer's Handbook," 1920, pp. 849, 853, 854, 884, 904, 906 and 908; and *Lefax*, September, 1920, No. 10–348.

§ *Proceedings*, Am. Soc. C. E., January, 1923, pp. 96–111.

‖ *Proceedings*, Am. Soc. C. E., April, 1922, p. 948.

¶ *Proceedings*, Am. Soc. C. E., October, 1922, p. 1700.

** *Proceedings*, Am. Soc. C. E., May, 1922, p. 1065.

†† *Loc. cit.*, p. 1071.

Professor Bishop* emphasizes the conclusion that the Cooper loading has outlived its usefulness and should be replaced. After calling attention to the disproportion and lack of simplicity of the Cooper loading, Professor Bishop expresses a fundamental principle when he states: "Any new loading should give approximately the same results as the actual loads which are likely to be used in the near or distant future." In this connection, he strongly recommends the writer's loading formula (M-Loading No. 3), as overcoming the objections heretofore advanced against equivalent uniform loads, and as yielding stresses which "provide for any of the critical engines of to-day, and, probably, by proportion, those for years to come."

Mr. MacCornack* also endorses the writer's paper, and recommends the proposed wheel-load diagram (M-Loading No. 1) as fairly representative of the actual loads in use. He would adopt such a system of wheel concentrations for examining existing structures and for designing bridge floors and short spans, and would use a loading formula equivalent to M-Loading No. 3 for designing all other bridge members. If preferred, he suggests, the load formula (M-Loading No. 3) might be modified to coincide more closely with the engine loading, and such a modification is submitted by the writer in Fig. 20. Mr. MacCornack rightly emphasizes the point that the important consideration is not the form of the loading specification, whether it is an engine diagram or an equivalent uniform load formula, or any modification of these, but that the loading adopted should represent correctly conditions in all parts of the bridge structure.

Mr. Disney† endorses the writer's thesis. He regards all three of the alternative proposals as having distinctive advantages over the present methods, but considers M-Loading No. 3 as ideal, as "it is a formula which can be memorized and used, without tables or charts, to find the moments and shears at any point of a span by a simple slide-rule operation."

Mr. Disney, however, questions the applicability of the M-loading to Canadian railways on which, he states, the Santa Fé type is the present limit of locomotive development. In reply, the writer would direct attention to Plate II, which shows that the M-loading is better adapted than the Cooper loading to give economical protection to Santa Fé locomotives.

Mr. Ammann‡ admits that unquestionably the proposed M-loadings have the advantage over the Cooper loading of greater simplicity. He points out, however, that the Cooper loading has been widely adopted as a standard, and that tables and diagrams are available to simplify the calculation of stresses for that loading. In reply, the writer submits that the mere fact of any standard being already in use is insufficient justification for retaining it when it is found to yield disproportionate and uneconomical results, and that the argument of tables and diagrams being available for applying the Cooper loading is cancelled by the publication of a complete set of similar tables and charts for the M-loading.

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* *Proceedings*, Am. Soc. C. E., October, 1922, p. 1702.
† *Loc. cit.*, p. 1703.
‡ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1789.

Mr. Ammann makes the statement that the proposed M-engine bears no greater similarity than the Cooper engine to any of the seven heaviest existing engines and those likely to be evolved in the future. The writer submits that this statement is refuted by a glance at the diagrams of the engines under consideration. The 5-axle grouping, which is the essential feature of the Santa Fé and Decapod engines, and the double set of drivers, which is the essential feature of the Mallet engines, are both included in the M-engine diagram, and neither of these features is represented in the Cooper engine diagram. What is more important, the M-loading is a more correct representation than the Cooper loading of the stress-producing effects of Santa Fé and Decapod engines, which represent the leading and most significant types of modern and prospective future engine development.

Mr. Ammann claims that the Cooper loading agrees as closely as the M-loading with the Composite of the heaviest existing loadings; but, in his attempt to prove this, he does not take the Cooper loading as ordinarily specified and understood, but devises instead an arbitrary modification of the form E-72 + 6 000. This does not meet the issue, as this is not the Cooper standard under consideration, but a new loading devised to overcome one of the deficiencies of the Cooper standard. The writer's Fig. 6* shows the correct comparison, which clearly refutes Mr. Ammann's contention.

Mr. Ammann also claims that there are large discrepancies between the M-60 loading and individual heaviest existing engines. If such discrepancies exist, they are merely the inevitable variations between a maximum composite and its lesser components, variations which are on the safe side and which provide a desired margin for future increase of stress at temporary low spots in stress-producing effects or in locomotive development; they are variations which will be reduced by future development toward the maximum composite instead of being (as in the case of the Cooper loading) variations which will become worse with the progressive departure of locomotive design from an outgrown standard.

Moreover, as shown by Plate II, for individual engines of the dominant types, the discrepancies of the M-loading are much smaller than those of the Cooper loading.

The suggestion of correcting the disproportion in the Cooper loading by changing the ratio of train weight to locomotive weight, made by Mr. Ammann, has already been answered. The proposed modification will upset existing ratings and lead to errors, and will only partly solve the problems under consideration. It will not correct the disproportion of stress within a given span, nor the disparity of ratings between different short spans, and it will not obviate the need of revising existing stress tables and ratings. If any change is to be made, no half-way measure should be used in correcting the standards; a fresh start, based on the latest developments, will prove more satisfactory in the long run than attempts to tinker with an outgrown standard.

Mr. Ammann suggests the possibility that the weights of locomotives and cars may continue to increase at different rates. As it is impossible to fore-

* *Proceedings*, Am. Soc. C. E., May, 1922, p. 1059.

tell future relative development, the writer submits that the present-day ratio of driver weights to car weights constitutes the best basis available for establishing a new loading specification. It is unquestionably a more logical basis than the outgrown ratio which obtained a generation or more ago. Mr. Ammann's suggestion of a flexible specification in which the individual engineer may arbitrarily vary the ratio of the uniform train load to the engine concentrations would only lead to chaos in standards, ratings, and stress tables.

Mr. Seaman* endorses the writer's paper and conclusions and predicts that in the future bridges will be designed for the M-loading. He mentions the conversion tables which will facilitate the altering of recorded ratings of the railroads; with this object in view, the writer has added the conversion tables (Tables 11 and 12), and the conversion charts (Figs. 21 and 22) to this discussion, and has prepared the conversion table (Table 15†) published in the report of the Special Committee on Specifications for Bridge Design and Construction.

Mr. Mead‡ also endorses the writer's paper. He declares that the necessity for revision is apparent, and points out that the present loading system gives unbalanced designs.

Mr. Mead strongly endorses the writer's conclusion that train loads and locomotive loads have not increased in the same ratio. The contention that the writer assumed an insufficient train load behind the modern locomotives considered in this paper, is answered by the following quotation from Mr. Mead's discussion:

"At present, train loads do not exceed 6 000 lb. per lin. ft. of single track, except in a few instances which will not affect the conclusions reached. Furthermore, the present design of cars must be changed so radically, in order to increase this train loading, that it seems as if this uniform load would represent a maximum train loading for an indefinite period."

Mr. Mead expresses a preference for M-Loading No. 2 (the uniform load plus spot loads) for its simplicity, flexibility, and ease of application.

Mr. Keith§ endorses the writer's paper and adds an interesting review of some early loading specifications. A useful lesson may be derived from Mr. Keith's account of a specification adopted by the New England Railroad Company in 1882, consisting of a live load headed by a Consolidation locomotive—a new type at that time—with four driving axles of 24 000 lb. per axle. This specification was criticized and ridiculed, many railroad men declaring that such a heavy engine would never be built. "Within three years that road had engines in use heavier than those that had been considered as excessive." History repeats itself; and engineers continue to make the mistake of under-rating future development.

Mr. Lang‖ disagrees with the writer's conclusions with an animus which, under the circumstances, it is difficult to understand. The writer has sim-

* *Proceedings*, Am. Soc. C. E., November, 1922, p. 1791.
† *Proceedings*, Am. Soc. C. E., January, 1923, p. 96.
‡ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1791.
§ *Loc. cit.*, p. 1792.
‖ *Loc. cit.*, p. 1794.

ply given his best efforts to the analysis and study of a problem which appears to be of importance and of general interest, and it would be regrettable if such investigations are to be taken out of the field of sincere professional discussion into the domain of factional controversy.

If, as Mr. Lang states, the paper gives the impression of being "primarily an attack on the Cooper system of loading", that was far from being the writer's intention. The writer joins with Mr. Lang and others in a sincere admiration of Mr. Cooper's contribution and in a full appreciation of its past value in bridge design; but, as has been pointed out by others as well as the writer, the Cooper loading diagram has outlived its usefulness. It has served its purpose well, but new conditions have arisen as a result of locomotive developments far exceeding Mr. Cooper's anticipations, and surely engineers may make revisions to suit these new conditions without detracting credit from the pioneer who blazed the trail. The writer's position in this matter is that expressed by the dean of railroad bridge engineers, Mr. Bland, who has stated that, "if Mr. Cooper was still alive, he would share this view and be among the first to devise a new method and one which not only conformed to existing engines and train loads, but looked forward and beyond."

The principal criticisms made by Mr. Lang have been answered at length. The selections of engines and following train loads in the writer's investigation have been fully explained and defended; they have received the confirmation and endorsement of many eminent bridge engineers as the correct and proper basis for a modern loading specification.

Mr. Lang's question why the special engines used by the writer should be followed by a train load of only 6 000 lb. per lin. ft., is answered by the simple fact that that is the maximum weight of train load actually used behind the engines under discussion. In fact, the heaviest cars to be found behind these locomotives are generally 210 000-lb. coal cars, which give a maximum train weight (with all cars carrying full overload) of only 4 770 lb. per lin. ft.

If these modern engines and their actual following train loads are compared with the E-40 loading, confirmation will be found of the statement challenged by Mr. Lang, that "the axle concentrations have increased 100%, whereas the weight of train load per linear foot has increased barely 50 per cent." This disproportion is still further accentuated when the Pennsylvania H-C-1-S engine with axle loads of 86 400 lb. is considered.

Mr. Lang contends that the Cooper E-73 with a train load of 7 300 lb. per lin. ft. "more nearly approximates present-day equipment and rolling stock." With the exception of some special ore cars, trains will not be found exceeding 6 000 lb. per lin. ft. As a rule, the maximum train weight is considerably less than this figure, even behind the engines which Mr. Lang characterizes as "extreme".

Mr. Lang criticizes the M-loading as perpetuating a material difference in the effect of the engine and train load on bridge structures. The same criticism also applies to the E-loading; the essential distinction is that the Cooper loading would perpetuate the ratio that obtained more than thirty years ago, whereas the M-loading makes a fresh start with the ratio that obtains to-day. If there was any practical way, without compensating dis-

advantage, to make the specification flexible in this respect, the writer would welcome it; in the absence of such expedient, it is surely preferable to base a specification on the present ratio than to perpetuate a ratio that has long been outgrown.

Mr. Lang questions the writer's suggestion that design loads should be 25 to 50% heavier than the average present loading. The writer merely made this suggestion in the light of repeated past experience with the obsolescence of bridge structures under unanticipated load increase. This subject is involved with the question of proper unit stresses. Where a unit stress of 20 000 or 24 000 lb. per sq. in. would be justified, the current practice is to use only 16 000 lb. per sq. in. in order to provide for a possible load increase of 25 or 50 per cent. The writer maintains that it would be more rational and would give better designs to increase the assumed live loads instead of reducing the unit stresses. Designing with ultimate unit stresses for ultimate anticipated loads will give the best proportioned structures, with the metal distributed where it will do the most good in insuring their longevity. When this fundamental principle receives recognition in specifications in the form of increased unit stresses, the design load can properly be augmented from M-50 to M-60 or M-75.

Mr. Lang submits a table of E-ratings for M-60 (Table 10*), in which he shows apparent errors in the writer's figures by comparison with his own. The values which he gives are taken from Table 2,† which refers to the Composite loading; whereas, he should have calculated his values from Table 3,‡ which refers to the proposed typical engine loading (M-Loading No. 1). In addition, Mr. Lang has included in his figures the alternative loading of two axle concentrations which are not a part of the Cooper loading, but have been added to it (with variations) to correct some of the deficiencies of that loading at short spans. (In the proposed M-loading, the necessity for such alternative concentrations is obviated.) With these corrections, the apparent errors in the writer's values will be found to vanish.

Mr. Lang attempts to show that the proposed M-loading does not vary appreciably from the Cooper system of loading, but his own values in Table 10* are a sufficient refutation, as they show a variation of 27% in the E-rating for M-60, from a maximum of E-77.3 at a 40-ft. span to a minimum of E-61.0 at a 300-ft. span. A variation of 27% is not one to be ignored. With the modified M-engine diagram adopted by the Special Committee on Specifications for Bridge Design and Construction, the variation from the Cooper loading is still greater, amounting to 38% (see Table 15 §).

Mr. Lang also attempts to prove that the Cooper loading fits the Composite of the heaviest locomotives better than the proposed M-loading. Inspection of Fig. 4‖ and Fig. 6‖ will prove the contrary. Fig. 4 shows that with the exception of a few isolated points, the M-loading agrees with the Composite within

* *Proceedings*, Am. Soc. C. E., November, 1922, p. 1795.
† *Proceedings*, Am. Soc. C. E., May, 1922, p. 1050.
‡ *Loc. cit.*, p. 1061.
§ *Proceedings*, Am. Soc. C. E., January, 1923, p. 96.
‖ *Proceedings*, Am. Soc. C. E., May, 1922, pp. 1055, 1059.

1%, for all points of all spans up to 1 000 ft.; the few larger variations found are confined to very short spans, with extreme values of — 4 and + 6 per cent. Fig. 6,* as well as Plate XVI,† shows that the Cooper loading varies from the Composite by about 25 per cent. If E-60 is used, this variation from the Composite is found at short spans; if E-73 or E-75 is used, the 25% variation is found at long spans. Mr. Lang uses E-73, but omits the long spans from his diagram (Fig. 15),‡ so that the 25% variation from the Composite is not revealed.

With the modification of the M-loading shown in Fig. 17, the variation of M-60 from the Composite has been augmented at short spans, on the side of safety; but even with this increase, M-60 shows less variation from the Composite than E-73. The maximum variation of center moments from the Composite is 21.3% for E-73 and only 11.9% for M-60, and the maximum variation of end shears from the Composite is 18.3% for E-73 and only 9.3% for M-60. Even this comparison is not fair to the M-loading, as these variations of M-60 from the Composite are confined to the shortest spans and represent an excess intentionally introduced for the better protection of these short spans in which stress effects are most likely to be increased by future developments.

Mr. Lang emphasizes the point that it would involve too much work to re-classify the 250 000 railroad bridges in the United States. This would not be necessary. Moreover, it will be found that very few railroads have made much headway in "Cooperizing" their lines; the bridges for which E-ratings have been calculated and recorded constitute a small proportion of the total. For the bridges as yet unclassified, it will be as easy to fix the M-ratings as the E-ratings; and for the others, even if deemed necessary, it requires very little work per structure to translate E-rating to M-rating by means of a conversion table or chart (Table 15, as presented by the Special Committee on Specifications for Bridge Design and Construction, mentioned previously, Tables 11 and 12, and Figs. 20 and 21). This conversion to M-rating will soon be justified in the time saved in judging the capacity of the structure for heavy loadings.

Mr. Lang makes a special point of showing that about 96% of the railroad bridges are less than 120-ft. span, and that at less than that span the differences between the E-loading and the M-loading are unimportant. The spans exceeding 120 ft. may be only 4% of the total number, but they are considerably more important than that small percentage would indicate. A single long span may outweigh, in importance and in cost, a hundred small viaduct spans in its approaches. One would not think of limiting the specifications to 120 ft., and letting all longer spans take care of themselves. Engineers are not writing specifications limited to plate-girder spans. Moreover, a glance at Plate II, or Table 15 of the report of the Special Committee on Specifications for Bridge Design and Construction, reveals a considerable difference between the M-loading and the E-loading even in short spans; Table

* *Proceedings*, Am. Soc. C. E., May, 1922, p. 1059.
† *Loc. cit.*, p. 1050.
‡ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1796.

15 shows that M-60 is equivalent to E-83 at a 35-ft. span and to E-69.8 at a 98-ft. span, or a variation of 19% below a 100-ft. span.

H. T. Welty, M. Am. Soc. C. E., is one of the authors of the Sub-Committee report* which influenced the American Railway Engineering Association to retain the Cooper loading in 1920; and, in his discussion† of the writer's paper, he maintains his preference for the Cooper loading "with possibly a modification in the uniform load following the series of wheel loads."

Mr. Welty's argument that most railroad bridges have span lengths of less than 100 ft. and that, for such spans, it makes little difference what loading is used, has already been answered.

For longer spans, Mr. Welty points out that the difference between the loadings can be corrected by changing the Cooper loading, using E-73 + 6 000 instead of the Cooper standard, E-73 + 7 300. This suggestion has already been discussed at length, and it has been shown that this would not be a satisfactory solution of the problem. Plate II shows one of the reasons. It would not be as good a solution as a change to M-60.

Professor Williams‡ endorses the writer's investigation as promoting economy, adequacy, and consistency in bridge design. He declares that, in a sense, the Cooper loadings have been a deteriment to structural engineering, and that lack of balance in most designs resulting from the application of the E-rating to express modern loadings even exceeds the 37% given by the writer.

Of the three forms of loading specification proposed, Professor Williams expresses a preference for the typical engine diagram (M-Loading No. 1), as he considers equivalent uniform loads insufficiently accurate to represent the actual effects of wheel concentrations. This point, however, is open to considerable difference of opinion.

Professor Williams emphasizes a point which the writer has made elsewhere namely, that the problems of loads and stresses are interrelated. The present working stress of 16 000 lb. per sq. in. is too low. To provide properly for future increase, the factor of safety should be inserted in the loadings rather than in the unit stresses. The correctness of the principle of "ultimate loads with ultimate unit stresses" cannot be denied, and the Profession will have to come to its acceptance. This thought has been the basis of all the writer's studies for his paper, and may help to explain why his recommendations of design loadings are higher than those customarily regarded as adequate.

Mr. Jones§ weighs the difficulties involved in arriving at a uniform loading specification of enduring application. He endorses the indictment of the Cooper system because of the discrepancies between it and present-day developments, and indicates the reasons for the use by so many railroad companies of loadings other than the Cooper loadings. Although a new specification derived on essentially the same assumptions as those of Cooper may prove to be subject to the same local limitations in the light of future developments, Mr. Jones agrees in the desirability of making a fresh start and of having

* *Bulletin*, Am. Ry. Eng. Assoc., January, 1920, p. 511.
† *Proceedings*, Am. Soc. C. E., November, 1922, p. 1797.
‡ *Loc. cit.*, p. 1798.
§ *Loc. cit.*, p. 1800.

a uniform loading specification throughout the United States. He expresses the hope that, "even though the permanency of a new standard adopted to-day may be voted dubious, one may be adopted, if only as a second step toward the ultimate."

Mr. Jones prefers the typical engine diagram (M-Loading No. 1) as an outline which can be easily reviewed as mechanical developments are made. He, therefore, chooses it for publication in the specifications; he points out, however, that the designer will probably forget the wheel-load diagram, paste the table (Fig. 18), or chart of equivalent uniform loads (Plate I), in his handbook, and, perhaps, memorize the equivalent loading formula (Equation (4*) or Equation (12) for emergencies, and thereby save precious hours of computation.

Mr. Woodruff endorses the writer's argument that the Cooper loading does not fit present conditions, and emphasizes the fact that the divergence is likely to be increased by future developments.

Commenting on the proposed M-loading, Mr. Woodruff raises some questions which have already been discussed. His objection on the ground that some of the locomotives considered by the writer are in limited use, is answered by the argument that these locomotives represent the maximum of each of the modern types and were considered merely as the best indication of the trend of future development. He suggests that the M-loading does not provide adequately for short spans (although it does so more generously than the Cooper loading). The writer has anticipated this suggestion by changing the spacing of the heavy drivers in the M-engine diagram from 5½ ft. to 5 ft.

Mr. Woodruff suggests an increase of 10% in the uniform load of the proposed M-loading on the ground that in the future the train load may increase more rapidly than the engine load; but the writer, guided by past experience and supported by others in these discussions, does not agree with this prediction. The economics of railroad operation demand continued development and augmentation of locomotive weights, but not of car weights. The question of relative train load affects only the long spans; and for those spans the lower average weight (per linear foot) of long trains should be taken into account.

Mr. Woodruff declares his preference for M-Loading No. 1, as it facilitates judgment of heavy shipments.

He suggests designing directly for M-75, both for simplicity and economy; but this is exactly what the writer meant when he recommended designing for the M-60 loading with a provision for 25% overload; he expressed it in that form in order to keep before the designing engineer the intent of the 25% margin, and the thought that the unit stress may then be justifiably increased from 16 000 to 20 000 lb. per sq. in.

Mr. Thomson‡ endorses the writer's paper. He points out that there is more uniformity now than in the past in the loadings used by different railways, and emphasizes the importance and value of proper standardization in this field.

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* *Proceedings*, Am. Soc. C. E., May, 1922, p. 1065.

† *Proceedings*, Am. Soc. C. E., November, 1922, p. 1801.

‡ *Loc. cit.*, p. 1802.

Professor Young* characterizes the data presented by the writer as "so convincing as to leave no other course open than to revise the present obsolete locomotive loadings." He adds: "The author has established that, for heavy loadings on first-class railway systems, the Cooper system is inadequate and that the new proposed system would give safe and logical results." Professor Young expresses the hope that the American Railway Engineering Association will reconsider its decision to abide by the Cooper system.

Although Professor Young endorses the M-loadings as "admirably suited to the heavier lines operating with steam locomotives," he questions whether they are as closely applicable to average or second-class lines. Plate II shows that any line on which Santa Fé engines are operated will be better protected by the M-loading than by the E-loading. Mr. Bland (of the Pennsylvania System) has found, from independent studies, that the M-loading is also admirably adapted to protect Consolidation and Mikado locomotives. The M-equivalents found by him are given on page 969. Accordingly, for average or second-class lines, even those on which a Mallet locomotive will never be used, it will be found to be safe and economical to adopt the M-loadings.

Professor Young also mentions the increasing importance of the electric locomotive. Although this type of traction has not developed sufficiently to permit judgment as to future standards, the writer is prepared to state that the more advanced designs of the electric locomotive show features which fully justify the adoption of the M-loading. The new Norfolk and Western type of electric locomotive has eight axle loads of 70 250 lb. each, in two closely spaced groups, and, therefore, resembles a Mallet wheel diagram. The writer has calculated its stresses and finds that it is safely and economically represented by M-52.

Of the alternative forms of the proposed loading, Professor Young favors the loading formula (M-Loading No. 3) as most direct and scientific, and therein agrees with the preference of the writer. It does not seem expedient at this time, however, to introduce such an innovation into general specifications.

Mr. Godfrey† endorses the writer's conclusion that, for the heaviest locomotives and train loads, the Cooper loading does not give consistent stresses nor economic designs. This, however, Mr. Godfrey does not regard as a sufficient reason for making a change. In the writer's opinion, Mr. Godfrey's arguments involve the fallacy of giving undue prominence to the existing ratings of old bridges, and of under-rating the importance of the proper design of new bridges. Sooner or later, all capacity ratings will have to be reviewed or revised for loading development, and, eventually, all bridges will have to be rebuilt. It appears, therefore, to be a short-sighted policy to perpetuate an old loading system which, it is admitted, does not correspond with the latest in modern engine construction, in order to save the temporary (and over-estimated) trouble of making a change. Moreover, even for the rating of bridges, the M-system will be found to be a time saver, for it will facilitate comparison of rated capacities with actual modern loadings.

* *Proceedings*, Am. Soc. C. E., November, 1923, p. 1803.
† *Loc. cit.*, p. 1804.

Apparently forgetting his own arguments against a change of loading standard, Mr. Godfrey proposes to modify the Cooper loading in order to correct its shortcomings. This proposal would not obviate the need of revising existing bridge ratings, the difficulty of which Mr. Godfrey has cited as his principal objection to a change of specification.

The two modifications of the Cooper loading proposed by Mr. Godfrey are a reduction of the uniform train load behind the locomotive, and an increase in weight of the alternative axle concentrations; the proposed M-loading combines these features in a much simpler diagram, and gives the desired results without the use of any alternative loads.

Mr. Robinson* expresses a strong preference for the Cooper loading. He cites, as his authority, the report of the Sub-Committee of the A. R. E. A. in preparing the Specifications of 1920.† The writer, in his investigation, used the same locomotives that were considered in that report, and from them derived the proposed M-loadings. It appears inconsistent to cite that report as an authority and, at the same time, to criticize the writer's loading assumptions.

Mr. Robinson contends that not many Mallet locomotives will be built in the future, because the Santa Fé and Mikado types are more efficient. According to Mr. Bland's findings, the M-loading fits the Mikado stresses consistently, and, as shown by Plate II, the M-loading fits the stress-producing effects of locomotives of the Santa Fé type much better than the Cooper loading. Mr. Robinson admits that it may prove necessary later to revise the Cooper loading; but, if a change is anticipated, there is no time like the present for making it.

Mr. Robinson also argues that M-60 is too heavy for locomotives in general use. If that is true, the railway engineer can specify M-50 instead. The essential point is not the index number, but the adoption of a correct type of loading to produce properly balanced designs for anticipated development.

Mr. Moore‡ endorses the writer's investigations and conclusions. He makes the important point that any change which better meets the tendencies of locomotive design is an advance.

He declares his preference for M-Loading No. 1 (the typical engine diagram), as it permits stresses to be checked by judgment. The requirements of the lighter lines, he points out, can be met by reducing M-60 to a lower index.

Mr. Moore also raises the interesting question of electric locomotives. He mentions the Norfolk and Western freight locomotive that has four axles of 70 250 lb. each, concentrated in a distance of 16 ft. 6 in., with another similar series of loads 22 ft. away.§ The writer agrees with Mr. Moore that the stress-producing effects of this locomotive are nearly as great as those of the series of steam locomotives used in deriving the Composite. Calculations show that this locomotive, followed by 210 000-lb. cars, is closely and con-

* *Proceedings*, Am. Soc. C. E., November, 1922, p. 1805.
† *Bulletin*, Am. Ry. Eng. Assoc., January, 1920, p. 511.
‡ *Proceedings*, Am. Soc. C. E., December, 1922, p. 1901.
§ *Engineering News-Record*, September 14, 1922, p. 437.

sistently represented by the M-52 loading. (In the Cooper system, the equivalent is E-67 at short spans and E-50 at long spans, a variation of 34 per cent.)

Mr. Moore believes that increase in length of locomotive with new wheel arrangements is more probable in future development than further intensification of axle concentrations. That would be another argument for the M-loading, as it provides for more axle loads per locomotive than the Cooper loading.

Mr. Moore makes one suggestion open to question, namely, that the uniform train load behind the locomotive should be kept constant. There are practical obstacles to such a specification. The writer has chosen a middle ground between this suggestion to keep the train load constant and the contentions of those who believe that future train loads will increase more rapidly than locomotive loads. Economic considerations would indicate that Mr. Moore's prediction is the more likely. The middle course adopted by the writer follows the line indicated by past experience, and is dictated by practical expediency.

Mr. Moore concludes that the proposed loadings are progressive and will be justified by future locomotive developments; and he expresses his conviction that the M-loading will soon be generally adopted.

Professor Wing devotes his discussion* principally to a consideration of the choice of form of loading specification. He prefers the actual heavy engine diagrams for rating the capacity of existing bridges, and the typical engine diagram (M-Loading No. 1) for the design of new bridges. For the rating of existing bridges, he will find that it will save time to determine and use the M-equivalent of the governing locomotives on any line, similar to the series of M-equivalents established by Mr. Bland for the different governing locomotives on the Pennsylvania Lines.

Mr. Stewart† regards the writer's proposals as extreme because they are based on maximum instead of average locomotives. This point has already been discussed at length, and the writer's position that a progressive loading system should anticipate future development really agrees with Mr. Stewart's admission that:

"As the tendency has been * * * for the weight of rolling stock to increase slowly, the standard loading should tend to be more nearly normal to the heavier rather than to the lighter existing or probable locomotives."

The writer does not agree with Mr. Stewart's statement that the M-loading cannot be reduced to fit the lighter classes of locomotives. Independent calculations made by the writer and by Mr. Bland, of the Pennsylvania System, show that the M-system furnishes consistent ratings even for Consolidation locomotives. The M-loading is as good as the Cooper system (and simpler) for the lighter locomotive types, and it is better than the Cooper system for the heavier types.

Professor Jacoby‡ endorses the writer's comparisons between the Composite of the heavy modern loadings and the Cooper standard now in use.

* *Proceedings*, Am. Soc. C. E., December, 1922, p. 1903.
† *Loc. cit.*, p. 1904.
‡ *Loc. cit.*, p. 1905.

He points out that it is an easy matter for any railroad to make its own comparison of its governing locomotives with the M-standard, and to determine how closely a reduction of index from M-60 would fit the results. He recommends the M-Loading No. 1 (the typical engine diagram) as the most expedient and practical.

Mr. Fowler* endorses the writer's position that the Cooper system is badly out of date and out of time with modern bridge engineering, and that a Composite loading is the correct solution of the problem. He also agrees with the writer that the Cooper standard perpetuates the wrong ratio of train load to locomotive load, and cites, as an instance, the fact that the weight of the train behind the heaviest locomotive on the Michigan Central Railroad is only 4 000 lb. per lin. ft.

Mr. Loweth† declares himself in sympathy with the movement to simplify and make more practical the typical engine and train loadings used for proportioning railroad bridges. He cites the repeated past experience of designing bridges for the heaviest locomotives known with a margin for possible increase, only to find the assumed loading exceeded by heavier locomotives before the bridge was completed. Surely, this is a sufficient answer to those who criticize the proposed M-loading as being based on locomotives heavier than the average.

Mr. Loweth's statement that electric locomotives produce less impact is a consideration which, in the writer's opinion, should be covered in the impact formula and not in the loading specification. He suggests further that electric locomotives will be less frequently double-headed than steam locomotives, which is an argument in favor of the M-engine diagram, as that diagram consists of a single-headed locomotive.

Mr. Modjeski‡ endorses the writer's conclusion that the Cooper standard loading does not properly represent modern loading conditions. Mr. Modjeski also suggests modifying the Cooper loading and using E-73 + 6 000 instead of M-60, which suggestion has already been discussed.

Mr. Modjeski indicates a perference for the second alternative, namely, a uniform load with floating concentrations (M-Loading No. 2).

Mr. Turner§ presents a theoretical analysis of impact, particularly as affected by speed. From this, he concludes that the impact for E-60 loading at passenger train speed would be nearly equal to that of an E-75 loading at freight train speed. Similar relations can be established, of course, between different indices of the M-system of loads, but the writer does not see how they affect the choice as between the Class-E and the Class-M loadings. The proper place to cover considerations of impact is in the specification for impact, not in the specification for loads. If it is Mr. Turner's idea that a loading specification should be based on passenger locomotives and not on freight locomotives, the writer does not believe that the Profession would share such a view.

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* *Proceedings*, Am. Soc. C. E., January, 1923, p. 131.
† *Loc. cit.*, p. 132.
‡ *Loc. cit.*, p. 134.
§ *Loc. cit.*, p. 135.

In closing, the writer wishes to state that, after reviewing the discussions, he is confirmed in his conclusions that the Cooper engine diagram does not properly represent modern loading conditions; and he submits the M-loading (Fig. 17) as the final result of his studies to determine the best possible loading specification to meet present-day and future loading conditions.

The writer wishes to take this opportunity of recording his gratitude to all those who have helped in connection with this paper: to O. E. Hovey, H. B. Seaman, J. C. Bland, Members, Am. Soc. C. E., and Mr. F. A. Howard, for their helpful encouragement; to H. T. Welty, F. E. Schmitt, C. E. Fowler, and W. G. Grove, Members, Am. Soc. C. E., for valuable suggestions received; to Messrs. G. Johnson and J. London for loyal and generous assistance in preparing the tables and diagrams; to the Secretary of the Society and his staff for their helpful co-operation; and, above all, to those who have so generously enriched the paper by contributing their discussions.

THE WATER POWER PROBLEM
A SYMPOSIUM
Discussion*

By Messrs. G. B. Herington and Gerard H. Matthes.

G. B. Herington,† M. Am. Soc. C. E. (by letter).‡—Mr. Matthes' paper is a valuable contribution to the literature of applied reconnaissance. The use of the aerial photographic method in river and power surveys as described has a peculiar value not only in making available a base map that is directly visible and interpretable as the eye sees the subject, but it also has a great advantage in speed and thoroughness.

Recent articles, in the technical press, on aerial mapping in cities have brought out this same feature, that is, making clearly and naturally visible an infinite amount of related detail, without conventionalizing. Mr. Matthes describes the method and its application on an actual engineering reconnaissance problem of a high order. Further interest would be added if in his closure he would discuss more fully the use of the stereoscope and the manner in which it functions, as this seems to be a very interesting feature of this method of mapping.

One of the greatest difficulties in reconnaissance usually is the necessity of continuous discrimination—due either to lack of time, funds, or force—between what should be mapped and what may be mapped in a country which may be under investigation primarily for determining whether any further expenditure for surveys is justified. The slower hand methods of surveying a vast stretch of country nearly always must be applied so that, should further work be desirable, the original survey may serve as a base for its extension, thus avoiding repetition and waste. Where reconnaissance is made by riding or walking and taking such observations as may be warranted, the investigator nearly always has a feeling of limitation on account of the personal horizon. He feels that he should cover a great deal more ground than he can, in order to eliminate, by positive conclusions due to actual examination, any possibility of errors of omission.

Where reconnaissance is reduced to any form of line or stadia measurement, the investigation of large areas may become expensive. It frequently happens that reconnaissance must be hurriedly made, and only as complete a report rendered as will be permitted by the time and funds available and the character of the territory to be covered.

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* Continued from April, 1923, *Proceedings*.

† Cons. Engr., Portland, Ore.

‡ Received by the Secretary, March 26, 1923.

Time and financial limitations which have hampered work of this kind seem to be lessened or eliminated by the aerial method.

During the winter of 1919-20, a tornado worked havoc in certain timbered areas along the coastal slope of the Olympic Peninsula in Western Washington. This tornado seemingly ricocheted over the vast timber area, recurrently dipping into the timber and then rising above it, along about 100 miles of its course. The earliest attempts to get a preliminary estimate of the damage disclosed that the trails were blocked by fallen timber and that in some places the tangle of timber trunks and tops was so great that experienced woodsmen and cruisers had difficulty in making a mile a day across some of the damaged areas. In determining the approximate damage, a large force of cruisers was required for practically the entire following year.

A large part of this territory is United States National Forest, and the executives of the Forest Service decided at an early stage that aeroplanes equipped for photographing could give the earliest general information on the damage. Arrangements having been made for this service, photographs of a series of strips were taken along the course of the storm. Although some fog was encountered, the photographs showed the location and nature of the damage quite thoroughly. It is hoped that the Forest Service will make public more of the data on the time, result, and cost of this work.

The writer has examined the photographs and has found them peculiarly interesting. The topographic features of the country were thoroughly determinable, and the different kinds of trees could be clearly distinguished. Although the flights were made at a higher altitude than that used in normal mapping, the crowns of the trees could be counted in the photographs.

The more progressive logging companies make their plans from topographic maps prepared mainly from barometric observations taken during the close cruising of the timber, the contour intervals being from 10 to 20 ft., and usually 50 ft. Control levels are generally run along section lines, the contours being sketched as the areas are traversed by the cruisers.

In studying Mr. Matthes' paper, the writer has become impressed by the fact that the aerial method of photographic mapping should offer a wide field in future timber land surveys, particularly those made prior to the opening of new timber land areas for operation. The strip counting of actual trees, which is done in ordinary cruising, may be thus modified to some extent, and the cruising tend more to become an examination of quality and of the average size of the trees, as this cannot be determined from the air. An actual tree count of a territory is thus rendered practicable, and the shape relation and direction of topographic features are determined automatically, and the location of corners and land lines, and their position on the picture is possible in any one of several obvious ways. The necessary surveys can then be best made on photographically pre-determined directions, instead of being run over section lines, as at present.

The present methods of timber cruising, however, will probably be continued without change, because certain practices have been developed, and the language of the industry and the training and special skill of the cruisers

is dependable and understood. The topographic mapping, however, may well be removed from their work, which would then be more that of judging the quality and quantity of timber, in view of the possible adaptation of the better method of base mapping which aeroplane photography seems to offer.

The writer has long contended that base levels for contour work in timbered areas should be run along controlling ridges and streams. This would usually require much less leveling, and the controls would be where they were needed. The contour surveying, thus becoming a separate function, could be done, preferably, in advance of or after the cruising of the timber stand, and thus become a groundwork for the later railway surveys and logging layouts.

The photographic mapping of an area would give a fundamental base, show detail far beyond hand-mapping possibilities, provide the base for the actual surveys necessary in determining elevations, and could be done in much the same manner as Mr. Matthes describes for the contour work on the Tennessee River survey.

The paper gives a cost of about $61 per sq. mile on a territory about 80 lin. miles in extent. Timber cruising generally costs from 30 to 55 cents per acre, under contract in the Northwest, for thorough standard work. Topographic work, as now done, adds to the cost about 20 to 35 cents per acre, thus making the range of present cost 45 to 85 cents per acre. It should be possible on smaller areas to furnish basic photographic maps at $100 to $150 per sq. mile, or at 15 to 20 cents per acre, cruising at 40 cents per acre, with contour survey control at 20 cents per acre. It will not make a much more expensive mapping process, but will furnish a vastly more comprehensive and valuable basic mapping of the territory under study, as judged by standards of visibility, ease of reading, and its actual portrayal of conditions.

Logging involves the application of sound engineering, planning, and execution, by men of sound common sense and experience who have grown up in the work, or by technically trained engineers. The fundamental preknowledge of the problems confronting the operator, as conveyed by its mapping, therefore, requires the best possible standard of informative data and visualization. The writer believes that the application of aerial photographic mapping of timbered areas, properly done, will be of great benefit, as reflected in operating and air survey costs and general results obtainable.

The field for this method, in irrigation and power investigation work, is enormous. The fact that an irrigation project covering a vast area can be quickly and informatively portrayed on a photographic map which will show infinite flat detail and locate the topographical elements of the whole area, will act as a base for planning all the controls and main final survey features which must be determined accurately, and will be justified ordinarily if used for no other purpose. Heretofore, no other method has been available for getting so quickly and accurately a topographic problem in large areas before the engineer and the financier, who must visualize the whole problem. One of the greatest values, in such reconnaissance, is the portraying of the things that are not required to be done, and the better application of effort. Although

it has been well known that aeroplane mapping has been done, and much progress made, the publication of tested and workable methods, as applied on an extensive survey, has marked a starting point for the extension of the methods used.

GERARD H. MATTHES,* M. AM. SOC. C. E. (by letter).†—The discussions on this subject are illuminating, as they show the wide range of adaptability of aerial photography as a tool in the hands of the engineer; also by conveying some conception of the vast field of usefulness of this comparatively new art. To the Engineering Profession, the chief interest in aerial photography is now, and doubtless will continue to be, centered in devising simpler methods of constructing accurate maps, whether of the mosaic or of the hand-drawn type.

Col. Birdseye‡ has set forth clearly the method of utilizing aerial photographs in the mapping operations of the U. S. Geological Survey, and it will be noted that these operations are quite different from those used by the writer. The reason for this is attributable to the fundamentally different principles which underlie the two systems of mapping. In the Geological Survey mapping, accuracy of geographical location is more important than accuracy of detail, whereas in the engineer's map, as exemplified by the survey of the Tennessee River described in the writer's paper, the opposite is true. In the Geological Survey method, it is not practicable to eliminate errors in scale or discrepancies caused by tilt or elevation displacements in individual photographs, because the plan of procedure supplies no information that will reveal the extent of such local errors and discrepancies. Reliance is placed, however, in a rigid ground control that maintains the accuracy of the completed map as regards principal dimensions, and the intervening detail is made to fit in by a process of adjustment that eliminates the errors in scale. The other discrepancies referred to, which are largely local in character, disappear or become negligible, due to the great reduction of the map in being brought to the scale of publication. In the engineer's map, however, such a procedure would not be admissible. The scale is large, and the photographs are taken as nearly as possible to the scale of the finished map. It is imperative, therefore, that each photograph be tested for errors, and corrected before being embodied in the map.

Mr. Hill's remarks§ refer entirely to maps of the mosaic type, that were supplemented by ample horizontal control derived from ground surveys. The writer has been impressed with the high degree of accuracy that was attained in the maps described by Mr. Hill. This accuracy was made possible through the use of improved methods of rectifying photographs, a process in which considerable progress has been made on the Pacific Coast, and which is responsible for the increasing use there of mosaic maps for engineering purposes. It is rapidly becoming recognized as the most efficient way of locating power transmission lines, highways, and railroads.

* U. S. Asst. Engr., U. S. Engr. Office, Chattanooga, Tenn.
† Received by the Secretary, March 19, 1923.
‡ *Proceedings*, Am. Soc. C. E., January, 1923, p. 120.
§ *Loc. cit.*, p. 119.

Mr. Tibbetts'* method of taking oblique views is destined to become popular with engineers, as it requires no special training or equipment. The writer finds that with a rapid lens good work can be done, also with the camera pointed at right angles to the direction of flight. Excellent views have thus been obtained, from an altitude of about 100 ft., of objects about 500 ft. distant. The time of exposure was $\frac{1}{500}$ sec., and the speed of the airplane about 75 miles per hour. As stated by Mr. Tibbetts, plate cameras with focal plane shutters are best adapted to taking oblique views. As regards the indifferent results with ray filters reported by Mr. Tibbetts, it should be noted that haze caused by dust, forest fires, or soft coal smoke, acts as a screen that cannot be combated with filters; but haze in an undefiled atmosphere is capable of being clarified with astonishing results through the use of properly selected filters. Experience is required to decide what filter should be used under given conditions.

Mr. Herington's discussion† of the advantages of aerial photography in the mapping of timbered areas is most timely. There is probably no more exasperating undertaking for an engineer or surveyor than the preparation of a reliable map of a densely wooded region by the older survey methods. The physical obstacles are usually such as to render only an inferior degree of accuracy obtainable, and that at considerable expenditure of time and money. A good photographic mosaic on an adequate scale offers the most elegant means of obtaining an accurate base map of heavily timbered country for any purpose, especially where the land is sectionized and care has been taken to mark the section corners so as to cause them to show plainly in the photographs. Mr. Herington's estimate of 15 to 20 cents per acre for aerial photographic work would make available a first-class mosaic on a scale of 400 ft. to the inch, a scale which would be well adapted for timber valuation work, and would bring out a wealth of information not otherwise obtainable.

Only one aerial timber survey of a large tract has come to the writer's notice. This was made in Canada for a paper manufacturing company, and is of interest in that the photographs made it possible to identify readily the several species of trees.

The writer's experience, in mapping backwoods sections in Tennessee and Alabama, has convinced him that aerial photographs offer by far the most reliable means of mapping extensive timbered areas yet devised. It is not until the stereoscopic properties of the photographs are brought into play, however, that their maximum value can be realized. This brings up the subject on which Mr. Herington requests additional information, namely, the uses of the stereoscope.

A certain amount of skill has to be acquired through practice in order to use the stereoscope effectively. This skill consists in training the muscles of the eye and in learning to interpret the full meaning of what is seen. It must be remembered that through the stereoscope one is enabled to visualize the face of the earth in a manner that is without precedent to the uninitiated. Here, then, is a new aspect of things that must be studied in order to yield

* *Proceedings*, Am. Soc. C. E., January, 1923, p. 117.

† See p. 984.

results. Even the aviator, from his lofty station, can form no conception of what the stereoscope may bring out, for his eye sees only a mere fraction of the detail recorded by a modern camera lens. To the man in the air, the world seems flattened to an absurd degree, and all details appear insignificantly small. The stereoscope, on the other hand, not only restores the relief with surprising fidelity, but brings to the attention an astonishing amount of interesting detail that otherwise would remain meaningless or unnoticeable. For instance, a small path or a logging road through a timber tract, which would escape detection in an ordinary photographic print, in the stereoscope stands out clearly under the trees, and the latter assume an individuality that differentiates them from their shadows on the ground. To the naked eye, the blur caused by deciduous trees without leaves and their own shadows is practically meaningless, but in the stereoscope there is no difficulty in singling out each tree and each shadow. This statement cannot be realized until tested out by actual experience. Again, a fine line appearing in a photograph may be a ditch, a fence, or the shadow of an embankment, and, from a single photograph, it may be quite impracticable to decide which it is. The stereoscope will remove all doubts. Much that is obscure or unintelligible in the average photograph becomes clear and obvious when viewed in the stereoscope. Heretofore, the possibilities of this instrument in connection with aerial photographic mapping have scarcely been realized. The writer has reason to believe that its importance will grow as its uses become better understood. Of late, its value in expediting the mechanical work involved in the Bagley method of radial control, described in the writer's paper, has become manifest. For instance, in selecting control points common to two or more photographs, considerable difficulty was often found in determining the identical location of a point on the several prints, a requirement which is of prime importance in order to secure accuracy. This arose especially where, as in a forested region, there was uniformity of detail that offered no salient features or landmarks. In such cases, it has been found that the location of the point can be quickly duplicated by viewing the prints in the stereoscope and simply using a fine-pointed tool to mark the desired point at the place seen stereoscopically. Surprisingly quick and accurate results can be thus obtained. In the survey of the Tennessee River, the stereoscope was used extensively in both field and office in examining terrain in which the monotony, as viewed from a single photograph, was baffling. In a forest of deciduous trees, for instance, the unbroken array of tree tops rarely furnished any clue as to the topographic features underneath, until viewed in the stereoscope. Then, ridges, gulches, slopes, and plateaus, the existence of which could not be suspected, would at once come into plain view. Similarly, cultivated fields of distressingly uniform appearance would, in the stereoscope, unfold all the minor rises, benches, and drainage courses that usually characterize them in Nature, but which are rarely visible in a vertical photograph.

The system of photographic mapping adopted in the survey of the Tennessee River—which formed the main subject of the writer's paper—is being further developed. Congress has ordered the survey to be resumed, and

has appropriated the sum of $200 000 for the purpose. In Alabama, the Alabama Power Company has adopted the same system in mapping about 500 sq. miles of the Tallapoosa River Valley for the purpose of planning a series of hydro-electric projects. The writer is confident that the continued use of this new system of mapping will operate to develop and improve it until its efficiency and accuracy will be such as to make it one of the most attractive methods of making maps for engineering purposes.

ANALYSIS OF COST OF FREIGHT SERVICE, GRAND TRUNK RAILWAY COMPANY OF CANADA

Discussion*

By J. G. Sullivan, M. Am. Soc. C. E.

J. G. Sullivan,† M. Am. Soc. C. E. (by letter).‡—The author is deserving of the gratitude of railway engineers for bringing this important subject before the public. There is every reason why railroad men should know what transportation is really costing, and, further, the students of the economics of railway location will be well advised in giving this paper very careful study, although they may not agree with all the methods and the indicated conclusions.

In general, the writer's criticisms of the paper resulted in decreasing the ratio of the direct charges to the total.

The reason for this further discussion is that the writer believes that in reading Mr. Newell's paper those who are not familiar with railway operation may arrive at a false conclusion, namely, that, in general, passenger business is more profitable than freight business.

It is not thought that Mr. Newell intended to give that impression, but the writer's experience has been that there are certain classes always ready to grasp at straws to further their arguments as to why passenger rates should be lowered.

In 1894, the writer listened to a member of the "Coxey Army" haranguing a crowd in St. Paul, Minn., whose grievance was high passenger rates. He quoted the cost per 100 lb. weight for carrying a hog from St. Paul to Chicago, and compared this rate with the fare he would have been charged by the railway. One can readily see now such a man would take undue advantage of any figures that might tend to prove his case, especially if he were a member of a legislative body.

The writer cannot agree with the author's conclusion that "passenger service was more profitable than freight", even in the particular case under consideration. The passenger business on the Grand Trunk Railway was the "star boarder", as it is in all railways. The track has to be kept in shape for that particular business. Extra side-tracks of extra lengths must be built so that the freight business can be kept entirely out of the way of its more favored rival. If these facts are not taken into consideration, then, in the writer's opinion, the conclusions are not reliable.

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* This is a discussion of the paper by J. P. Newell, M. Am. Soc. C. E., published in January 1923, *Proceedings*, but not presented at any meeting of the Society.

† Cons. Engr., Winnipeg, Man., Canada.

‡ Received by the Secretary, March 29, 1923.

A few of these adverse factors that increase the cost of freight service may be mentioned in the following questions:

(1) If a road had no passenger service how much could it reduce the first cost and interest of locomotives and cars on account of being able to make better time?

(2) How much could the number and length of passing sidings be reduced if there was no passenger service? (A double-track railway would practically require none.)

(3) It is true that Mr. Newell loads the passenger traffic in computing the cost of maintenance-of-way charges, but how many railway engineers will admit that this loading more than compensates for the actual damage done due to speed, leaving nothing to compensate for keeping the roadway in a condition far better than the economics of freight service would warrant.

(4) How much of the fuel used in stopping and starting freight trains and burned while standing in side-tracks waiting for passenger trains to meet or pass, should be charged to passenger service?

(5) How much of the extra time of train crews and engine crews in freight service, as mentioned in Factor (4), should be charged to passenger service?

(6) Even if it is taken for granted that Mr. Newell's figures for the ratio of direct charges are correct, is he justified in dividing taxes, interest, and general expenses in the same ratio?

Most business concerns will compare the direct charges with the revenue from a certain line and then decide whether or not it is profitable to continue that line. One might get the impression from the author's statement that, "compared with the entire cost, the percentages were 110% for freight and 83% for passenger", that it would be advisable for the Grand Trunk Railway to go out of the freight business; such a conclusion, of course, would be absurd. If Mr. Newell had divided what he calculated as indirect interest, taxes, and general expenses on the basis of revenue received, the result would have been 107% for freight service and 90% for passenger service. If these items had been divided "fifty-fifty", the result would have been 101% for freight service and 100% for passenger service.

The writer does not suggest that these latter divisions are more logical than those used by Mr. Newell, but simply uses them to show what very different results one may obtain, depending on the point of view that suggests the assumptions.

Another feature that supports the writer's views is the fact that although it would have been to the Government's interest to establish the conclusion of Mr. Newell, the Government's solicitors did not feature this conclusion.

The writer wishes in no way to detract from Mr. Newell's excellent paper other than to prevent those who may not be familiar with railway operation making a mistake by drawing false conclusions.

THE DESIGN OF STRUCTURAL SUPPORTS FOR TURBO-GENERATORS

Discussion*

By Messrs. D. A. Allee and Edward H. Cameron†

D. A. Allee,‡ M. Am. Soc. C. E. (by letter).§—This paper enters a field for speculation and discussion not to be lightly dismissed. Without in any way discouraging such speculation, the writer would like to present the situation as it is found to-day.

Three types of foundations have been mentioned, namely, reinforced concrete, structural steel, and a combination of structural steel and reinforced concrete designated "composite". When designs are carefully worked out, giving due consideration to all conditions, it is believed that all types have their fields, and that any type may fairly be expected to give satisfaction; but therein lies the trouble. Which designs in practice turn out to be best? Good foundations rather than good theory are sought.

From long experience in this field, it is found that more steel structures are insufficient in their conceptions and details than is the case with concrete, as applied to turbo-generator foundations. For example, there is a tendency to use a top construction which, in itself, has no ability to maintain alignment in a vertical plane. There is also a tendency to support and grout machine bases on yielding surfaces that, therefore, allow the grout to break. An example of this is a girder with outstanding flange angles. There is, likewise, a tendency to use bracing and stiffening construction, which in themselves are quite effective, but which in practice cannot be consistently carried out, for example, heavy brackets connecting girders to columns giving the appearance of great stiffness. If, after adopting such a type of construction, it is impossible to use such brackets on one side of the foundation because the girder cannot be placed over the column, or if it becomes necessary to move a cross-girder to a position not directly over the column, as is the case with practically all turbines of a certain design, the whole principle of the design has been violated.

In general, it will be found that steel foundations are insufficient of themselves and must be tied to the floor on which they depend for support against swaying and for alignment in a vertical plane.

The composite (framed steel and concrete) design always presents difficult problems, where it is attempted to connect steel work to concrete or to support

* Discussion on the paper by Edward H. Cameron, Assoc. M. Am. Soc. C. E., continued from April, 1923, *Proceedings*.

† Author's closure.

‡ Civ. Engr., Constr. Eng. Dept., General Electric Co., Schenectady, N. Y.

§ Received by the Secretary, March 22, 1923.

steel on concrete. This is particularly true with large machines where loads are excessive. If steel girders are framed to steel columns and simply covered with concrete, protection against total failure of the structure is obtained, but it is still necessary to reinforce the concrete. The function that this concrete cover performs is problematical. If the details are such (they seldom are) that dependence may be placed on the concrete, then the steel core is superfluous. In general, there is no great advantage to be gained in this construction over that of reinforced concrete.

Designs of reinforced concrete and those of all steel should be compared on a common basis. For example, referring to Fig. 1,* it is quite apparent that the steel foundation is not suitable for the conditions shown in the concrete design. It is further apparent that the concrete design may be made much lighter if desired, causing the weight to be less and the clearances to be greater, but at a sacrifice of simplicity, and yet have the design the equivalent of the steel foundation with which it is being compared.

It should be kept in mind that such structures are designed by various grades of talent and, therefore, a design which would prove safe and effective, if made by a competent and careful engineer, might and probably would be unsatisfactory if developed by one less competent. Of the multitude of designs which the writer's company examines, practically none is above criticism in the essentials of good design. It is found that poorly conceived designs in steel construction are so common that the engineers of the company have been very reticent about encouraging their use.

EDWARD H. CAMERON,† ASSOC. M. AM. SOC. C. E. (by letter).‡—Two points brought out by the discussion of this paper are worthy of especial emphasis. The first is the desirability of close co-operation between electrical, mechanical, and civil engineers who are working together in power plant design. A better foundation can be secured if the civil engineer on his part clearly understands the functions of the machine and its component auxiliaries. The mechanical and electrical engineers on their part must recognize the absolute necessity of providing space for the more important bracing elements and should be willing to modify their layout somewhat to make possible the insertion of such members. The second point considered worthy of emphasis is the importance of experience and good judgment on the part of the designer as against an attempt to adhere strictly to fixed unit stresses.

The writer believes that the present custom of fixing a limit to the deflection of members in bending constitutes a very desirable criterion of the rigidity of the pedestal. It has been stated that the entire deflection is taken care of by the grouting. The writer cannot agree on this point. Before grouting, the unit is brought to a level by means of steel wedges. If light girders are used, it is apparent that the relative stiffness of the girder and of the bed-plate will determine whether the driving of the wedge will tend to raise the bed-plate, or merely drive down the girder and allow the bed-plate to deflect with it. Consequently, the girders should be made sufficiently stiff that any load which they

* *Proceedings*, Am. Soc. C. E., January, 1923, p. 40.

† Civ. Engr., Jackson & Moreland, Boston, Mass.

‡ Received by the Secretary, March 28, 1923.

support, other than their own weight, will not cause a deflection greater than the limits ordinarily set.

Mr. Reeves* is a little more positive than the writer regarding the undesirability of separate footings for the individual columns. The writer believes with Mr. Reeves that the continuous mat should be used wherever possible; however, it is not always practical to secure this condition, and several very good designs have been noted in which the columns rest on individual footings. Both Mr. Reeves and Mr. von Fabrice† have emphasized a very important point relative to the delivery and distribution of the column loads and moments into the mat. Following out the analysis given in Fig. 4,‡ Moments M-1, M-3, and M-5, should be cared for at the column bases.

Mr. Beers§ mentions a type of pedestal consisting of two longitudinal concrete walls carrying transverse steel beams which, in turn, support the turbo-generator. The writer has noted several similar installations where a jet condenser was used. This design should be quite simple to build for small units as the deflections would be limited to the comparatively short transverse beams.

Several of the discussors have raised the question as to the desirability of placing reliance on the structural strength of the concrete encasement of a steel pedestal. It is very difficult and often impractical to make a theoretical analysis of the proportions of the load that would be supported by the steel member and the concrete encasement, respectively. The writer believes that the recommendation of Mr. von Fabrice that it is advisable to ignore the structural value of the heavy concrete encasement is justified and should form a part of any conservative specifications for this type of work. The concrete encasement would then be considered only for additional mass and temperature insulation.

Mr. von Fabrice quotes certain standards of shop fabrication which have been found advisable, such as making tension members $\frac{1}{8}$ in. short in 10 ft. and compression members $\frac{1}{8}$ in. long in 10 ft. There are no structures made up of the ordinary rolled steel sections in which especial refinements of common bridge shop practice are more justified.

The writer is in hearty accord with Mr. Allee‖ in that "it is good foundations * * * that are sought". A substantial completed structure is the object of the designer in all civil engineering fields, and there is no question but that good theory based on successful practical application is the best.

In presenting Fig. 1,¶ comparing a structural steel foundation as recommended by one manufacturer with a reinforced concrete type given by another builder, the writer did not intend to imply that the machines would be interchangeable on the foundations, or that they were precisely equivalent. The writer believes, however, that the examples selected are largely comparable and present a striking illustration of the distinct advantage of the structural steel pedestal in regard to space as contrasted with the concrete type.

* *Proceedings*, Am. Soc. C. E., February, 1923, p. 276.
† *Proceedings*, Am. Soc. C. E., April 1923, p. 782.
‡ *Proceedings*, Am. Soc. C. E., January, 1923, p. 46.
§ *Proceedings*, Am. Soc. C. E., February, 1923, p. 278.
‖ See p. 993.
¶ *Proceedings*, Am. Soc. C. E., January, 1923, p. 40.

In the discussion of this paper, there is a very evident preference shown by most of the discussors for the structural steel foundation, perhaps modified by the application of a concrete encasement and slab. If the manufacturers are finding steel designs to be as poorly conceived as intimated by Mr. Allee, it is high time that steps are taken to correct this condition. The writer believes that consideration should be given to the suggestion of Mr. von Fabrice that standards for the design of structural supports for turbo-generators be formulated by the Society.

ENGINEERING EDUCATION

Discussion*

By Howard Constable, M. Am. Soc. C. E.

Howard Constable,† M. Am. Soc. C. E.—Aspiration for a better method of education is evidently abroad among engineers. The engineer starts with mathematics, struggles toward Truth, and neglects a great deal of the broad meaning of Art; the art of understanding and dealing with his fellow man, the art of recreation and inspiration, and, in short, the art of living a rounded out life. How often one finds him ignorant in some fundamental; unfamiliar with parliamentary law; unable to express himself clearly; or awkward in social relations.

The speaker would ask, "What are you going to do about it?". Men, money, methods, and materials, is the order of importance in present-day problems. The human part is generally the most discouraging. Of the men who go into business, 95% fail largely because of inefficient minds, which indicates a rather low standard for the intelligence of the nation. No wonder there is a growing demand for a revision of the educational system.

The speaker has often advised men to try for jury duty. In that work one may learn much of human nature, that what is written is remembered and what is spoken is forgotten; that there is no gratitude in compromises, yet compromise is the general rule; that every transaction may lead to a lawsuit.

Another matter in education is the question of whether the ability and wisdom of men in active practice is utilized as much as might be, for the benefit of the student and the public? In this age, interest in such things is broader than just technical societies. Should not great engineers be more called on by the colleges and the public?

* Discussion of the Technical Papers on Engineering Education continued from April, 1923, *Proceedings*.

† Archt. and Engr., Kingston, Mass.

ENGINEERING RESEARCH

Discussion*

By W. A. Slater, M. Am. Soc. C. E.

W. A. Slater,† M. Am. Soc. C. E.—The speaker is glad to know that the Bureau of Standards has been able to help in some of the work of Engineering Research, and that the Department of Commerce has been able to contribute in a substantial way.

If engineers would make more use of some of the Government institutions for research, the speaker believes that it would make those institutions much more useful than they are at present. Engineers and industries would make the Bureau of Standards, for instance, feel that it was needed, if they would bring pressure from the outside to get done that which is needed and the Bureau would be able to do this work much more effectively.

A great deal of attention has recently been given in the search for laws governing the strength of concrete. The problem has been studied from many angles; the search has been made in as great detail as the facilities available in the different laboratories have permitted. Some investigations have been made which might appear like a misdirection of effort, but all the serious work that has been done is contributing toward the solution of the problem. All the laws have not been established, but it seems that there has been very little really wasteful research on the subject of the strength of concrete.

Another phase of research that has not been discussed is that of making available in English the literature in foreign languages. The speaker knows of no organization in this country that is doing such work. Foreign technical papers discuss in more detail the results of research than American technical papers. No technical paper in this country seems to feel that it can afford the space or the expensive work that a great deal of this kind of publication would involve; and the speaker would inquire whether there is any place for such a publication in the work of Engineering Foundation, or any of the affiliated organizations that have been mentioned. A number of valuable researches published in the German language, for instance, could be cited, that are not available in English, and the speaker believes that their translation would assist greatly in avoiding duplication in the United States of work already done abroad, in helping to interpret work done in this country, and in furnishing data that otherwise may never be obtained.

* Discussion of the Technical Papers on Engineering Research continued from April, 1923, *Proceedings*.

† Engr.-Physicist, U. S. Bureau of Standards, Washington, D. C.

REINFORCED CONCRETE COLUMNS

Discussion*

By Messrs. H. V. Spurr and Austin H. Reeves

H. V. Spurr,† M. Am. Soc. C. E.—The author has performed a valuable service in reviewing the various tests on reinforced concrete columns and in presenting in tabulated form an original interpretation of their meaning.

The problem of the designing engineer is to design safely and economically. The formulas for concrete columns as proposed by various committees, and as stipulated in the building codes of various cities, differ greatly in form and in the allowable safe load on columns of the same make-up. Arbitrary factors have been adopted for the value of longitudinal as well as for that of spiral reinforcement, and different values have been given for the concrete itself.

Designing engineers would welcome a formula for reinforced concrete columns which would meet with general approval, and which would result in both economy and safety.

Economy in column design is largely a question of the design of columns carrying heavy loads in the lower stories of buildings. In such columns large quantities of reinforcement are used, as it is desired to reduce the size to the minimum. In order to design such columns economically, a reasonable stress must be allowed on the concrete, as the percentages of reinforcement are limited for practical reasons. In large columns supporting heavy loads, the cheapest reinforcement is cement; that is, a concrete of high strength and reliability. Strength and reliability are dependent not only on the stone, sand, and cement, but on the control of the proportions and the water content. Columns can never be economically designed until reasonable care is taken to insure a concrete of fair reliability and strength.

Objection is made to the various formulas now in use, in that they do not sufficiently emphasize the importance of securing good concrete, but attempt rather to discount the use of poor concrete. By controlling the water content, and with good aggregate, it is possible to increase the reliability of the concrete and to use higher stresses.

It is reasonable to suppose that spiral reinforcement adds strength to a concrete of a given mix, but this increase should be discounted somewhat in the formula adopted, as cages are not perfectly fabricated and assembled in the field. It would be reasonable to assume that longitudinal reinforcement of large rods well tied by bands or hooping, and encased in concrete, should

* Discussion on the paper by John Tucker, Jr., Esq., continued from April, 1923, *Proceedings*.

† Chf. Engr., Purdy & Henderson Co., New York City.

be capable of taking a safe load approximating that which the same area of steel would sustain in the outstanding legs of a structural column. A reasonable stress for longitudinal reinforcement would seem to be 12 000 lb. per sq. in. For practical or economical limits, 5% of the longitudinal reinforcement should be the maximum.

In building construction in general, repetition of stress can be neglected, due to the large proportion of dead load in the columns. This is especially true in large columns in the lower stories. As the design of columns is based on the strength of concrete at 28 days, it would seem reasonable to assume that the normal increase in strength with age would counteract the effect of time loading within the range of the safe working stresses that would naturally be used without considering this factor.

It would appear, therefore, that the principal elements to be considered in the design of columns, in the order of their importance, are:

1.—The reliability and strength of the concrete.
2.—The age at which it would receive its maximum load.
3.—The increase in strength from a moderate amount of longitudinal reinforcement at 12 000 lb. per sq. in.
4.—The increase in strength due to spiral hooping of mild steel between the limits of three-quarters of 1% and 1½%. This increase in strength should be expressed as a percentage of increase in a safe stress on the concrete in terms of percentage of spiral.
5.—That the ratio of length to diameter would have little effect up to 10, and that above that figure some reduction in the safe stress should be made.

The most important element in the strength of a concrete column, the strength of the concrete itself, can only be determined within certain limits. Tests of a sufficient number of specimens for a given mix, in which the water content had been controlled, have not been made to determine the relation between the strength of the concrete in the columns at 6 months to that of the standard specimens at 28 days. It is reasonable to suppose that the average strength of the concrete in the columns at 6 months would be considerably greater than that at 28 days, and that the range of strength of the columns will not be as wide as that of the specimens. Therefore, if preliminary tests of the concrete mix, proposed to be used for a nominal 3 000-lb. concrete, show that none of the specimens developed less than 2 500 lb., it is reasonable to assume that the concrete in the columns at 6 months will never fall as low as 2 500 lb., provided the materials average well and reasonable care is exercised to control the proportions and the water content. For such a condition, 1 000 lb. per sq. in. on the concrete would not seem to be an excessive safe working stress, which could be increased a certain percentage by the use of spiral reinforcement.

Formulas which are based on a low working stress in the concrete in order to discount poor concrete, but which give a high influence to spiral reinforcement, are unbalanced. Such formulas lead to the use of a maximum quantity of spiral in small columns with a minimum quantity of longitudinal steel,

without any serious attempt to obtain good concrete. With large columns, in which the percentage of spiral is limited by the size of hooping available and practicable, a reduction in the percentage of spiral reinforcement necessitates a reduction of stress in the core, which can only be made up by adding a great quantity of longitudinal reinforcement. Thus, economy of design is impossible for the very columns for which it counts the most in cost. Also, in the large columns in the lower stories of a building, the concrete will be several months old before it receives even its full dead load.

AUSTIN H. REEVES,* ASSOC. M. AM. SOC. C. E.—For more than twenty years practically no advance has been made in the development of a new and proper type of reinforcement for concrete columns. Most thinking engineers who have had to design concrete structures have recognized the fact that the present methods of column design are faulty. It is easy to design a column to pass the building code of any given city or to meet the requirements of the Joint Committee specifications, yet it is difficult (impossible for the speaker) to justify any of the designs thus made with well known principles of mathematics, mechanics of materials, and common sense. This is said with no lack of respect for any one who has contributed a share—by making tests, writing textbooks, or otherwise—to the present knowledge of the subject.

The title of this paper is erroneous. A reinforced concrete column should be able either to support a greater load before incipient failure or to support the same load with greater integrity or at less cost. The speaker will show that the present-day types do none of these things, and, besides, they are unscientific. Therefore, they are not reinforced. (It should be noted that this is true if total volume of concrete is considered—there is no fairness in considering only the core area of one type as against the total area of another, and using the increased ultimate unit stress on this partial area to argue for greater load sustained.)

There are really only three classes of concrete columns: (*a*) plain columns; (*b*) columns containing a mass of steel; and (*c*) reinforced columns. Plain columns need no explanation. The second class includes present-day practice and is a futile attempt to strengthen plain columns. The third class embraces a truly reinforced concrete column on which the speaker has patent rights† pending, and which may be called the mat or "spider" system of lateral reinforcement.

The four headings under which the speaker wishes to develop his remarks are as follows:

1.—A plea for the plain concrete column.

2.—The introduction of a new laterally reinforced system developed by the speaker, which may be called the "spider" system.

3.—Recommendations for different procedure as regards size and test of test specimens.

4.—Proof that the ratio of length to diameter has no place in any concrete column formula.

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* Newark, N. J.

† Patent pending, filed February 16, 1923, Serial No. 619 470.

Mr. Tucker states* that: "As unreinforced concrete columns are never used in practice, tests of specimens of this type are considered of small value." The statement that such columns are never used in practice is in error, but the speaker believes that even if the plain column was never used in practice, the tests should be of great value if properly made and the results diligently studied, because only in that way can the best type of reinforced concrete column be developed.

The author's next sentence is also incorrect, in that it states: "Test results of only eleven such columns can be found, and these results, together with data on the columns, are given in Table 1." The results given in Table 1† do not include test data on any of the following thirty-six plain columns which have also been tested, namely, six plain columns, three of which were 12 by 12 in. by 12 ft., one was 12 by 12 in. by 6 ft., one was 9 by 9 in. by 12 ft., and one was 9 by 9 in. by 6 ft., given in *Bulletin No. 10,* University of Illinois; Column No. 1 583 (12.66 by 12.59 in. by 7 ft. 10 in.), which was tested at the Watertown Arsenal in 1904; twenty-one plain columns tabulated in *Bulletin No. 20,* University of Illinois; five plain columns which were remarkably uniform, reported on page 11 of *Bulletin No. 900,* University of Wisconsin; and three plain columns,‡ tested by H. C. Berry, M. Am. Soc. C. E. The speaker does not mean to infer that this additional number completes the list of available results on tests of plain columns, as he knows of others that have been made.

Mr. Tucker's statement that: "It is evident that the strength of concrete columns will decrease as the length increases", is not supported by tests. Messrs. McKibben and Merrill have stated§: " * * * while the strengths of 10-ft. and 20-ft. columns are approximately equal * * *", other tests show that the length of a column up to, say, 15 diameters, has no marked effect on the strength. Subsequently, the speaker will prove that the ratio of length to diameter has no place in any concrete column formula. This will show that practically everything on page 174‖ of the paper is incorrect, including the curve, Fig. 2, and Equations (3), (4), and (5).

Having discredited practically all the author has said on plain columns, the next step will be to show that, when properly considered, the plain column is the best concrete column produced by any present-day method. Professor Withey has stated:¶

"In the table, however, the areas and percentages are based upon the areas of the cores inside the spiral; these values were used in computing the ultimate strengths and stresses at the yield point * * *. This procedure is permissible since the strength of these columns at the yield point or ultimate was approximately independent of this shattered outside shell."

This is one of many authorities who have found that beyond the yield point (which averaged 74% of the ultimate for the thirty-two columns, the

* *Proceedings,* Am. Soc. C. E., February, 1923, p. 173.
† *Loc. cit.,* p. 173.
‡ *Engineering Record,* February 19, 1910, p. 209.
§ *Proceedings,* Am. Concrete Inst., 1916, p. 209.
‖ *Proceedings,* Am. Soc. C. E., February, 1923.
¶ *Bulletin No. 466,* Univ. of Wisconsin, p. 38.

test results of which are tabulated on page 42 of the same *Bulletin*), the outside shell is useless. It may not be there even when most needed for fireproofing. It has been found* that four spiralled columns began to disrupt the shell at six-tenths the ultimate load and that the four columns containing both spirals and rods began to disintegrate the outside shell at three-quarters the ultimate load. On the contrary, plain concrete columns do not show even warning cracks until the ultimate load is reached. It has been stated* that:

"The four 12 by 12 in. plain columns of this series broke at an average stress of approximately 2 070 lb. per sq. in., the maximum variation of a single result from this mean value being only 40 lb. per sq. in., or 2 per cent."

The remarks† on the six plain concrete columns were: "Failed without warning cracks", "no cracks appeared before maximum load was reached", and the like. This tends to prove that plain columns do not spall off or even crack until the ultimate load is reached, whereas the shell of the so-called reinforced concrete column spalls off, disrupts or disintegrates at from six-tenths to seventy-five one-hundredths of the ultimate load—this is what the speaker calls sustaining the load with greater integrity. For out-to-out dimensions of concrete, other things such as mix, density, age, etc., being equal, it has been repeatedly shown that a plain concrete column will hold approximately the same ultimate load as any rodded, spiralled or spiralled and rodded column. Mr. Berry‡ has recorded the results of tests on nine columns, all 14 in. square and 12 ft. long, three of which were plain, three were rodded, and three were spiralled and rodded. He states:

"The results of the tests impress one with the good showing made by plain concrete and the small increase in strength resulting from the use of reinforcement. The popular notion that the hooped columns are much more desirable because of their greater toughness would seem to be ill-founded because, as can be shown also, by a theoretical analysis, this toughness is not exhibited until after the maximum load is developed, which, if applied by a dead weight instead of a testing machine, would cause almost, if not quite, as sudden a collapse as would occur with either of the other types of columns."

The three plain columns stood an average maximum stress of 2 500 lb. per sq. in. as against 2 580 and 2 810 lb. per sq. in. for the rodded and for the spiralled and rodded types, respectively. Edward Godfrey, M. Am. Soc. C. E., has stated§ that: "Some plain concrete columns have stood much more ultimate load than some so-called reinforced columns in the same series." He has also stated‖ that:

"On the other hand, the results of tests on many varieties and styles of plain and reinforced concrete columns, as I have pointed out on other occasions, shows remarkable agreement in one thing; that is, they practically all begin to fail at nearly the same unit load, no matter what the reinforcement or lack of it." And continues: "As illustrating the negligible effect of steel reinforcement on the initial point of failure of columns, I pointed out,

* *Bulletin No. 300*, Univ. of Wisconsin, p. 19.
† *Bulletin No. 10*, Univ. of Illinois.
‡ *Engineering Record*, February 19, 1910, pp. 206–209.
§ *Proceedings*, Am. Concrete Inst., 1921, p. 176.
‖ *Loc. cit.*, p. 177.

in *Transactions*, Am. Soc. C. E., Vol. LXVIII, 1915, page 141, that the average load at 1st failure of set of tests made at the University of Michigan was as follows:

"Plain concrete.....2 250 lb. per sq. in.
Less than 1 per cent. of hooping.....2 250 lb. per sq. in.
More than 1 per cent. of hooping.....2 230 lb. per sq. in."

The equal strength, the greater integrity, and the lower cost of plain concrete columns as compared with any present-day type of concrete columns argues for one of two things: Either the use of plain columns or the development of a new and truly reinforced concrete column such as the speaker has developed. He never has believed that a given size, out-to-out, column could be increased in strength by attempting to increase its compressive strength or by the use of the spiral to suppress lateral deformation. He has believed that, due to the operation of Poisson's ratio, the weakness of concrete in tension, and the great increase in rate of lateral deformation before failure, the development of a scientific system of lateral reinforcement was the proper solution in order to produce a concrete column of maximum strength.

Concrete is strong in compression, fairly strong in shear, weak in tension, and, as regards reinforcement, has its most valuable asset in its bond strength. Steel bars are best in tension and, because of the bond of the concrete, they are able to relieve the tensile stresses of concrete. By making full use of these properties, it seems conservative to predict that a most scientifically designed and spaced system of horizontal "spiders" would develop twice the strength of the plain column.

The "spider" may be of many forms, one type of which is a cluster of six or eight bars radiating from a central ring to very near the outside of the column, with the outer ends either plain, hooked, or other shape. Light wires may tie the rods at intervals to form a mat to be placed in a horizontal position in the column. These "spiders" may be hung from the top of the column form, supported by a proper central shaft or placed as the column is poured or in any other manner best adapted to an accurate spacing during the pouring of concrete.

On setting, the concrete grips the rods, and as loads are applied the lateral tensile stress is distributed evenly by the bars, the core is restrained in the most rigid manner, and the shell is doing its duty also in sustaining its share of the load until failure. The reinforcement of the column is well protected against heat (in marked contrast to placing the reinforcement within 2 in. of the heat as in the spiral and rodded columns).

The intimation may have seemed excessive that, for a given out-to-out dimension, a properly designed "spider" reinforced concrete column would sustain with equal integrity and with but little more cost, twice as much as an otherwise equal (regarding age, mix, etc.) plain concrete column. It is well known that cubes give higher test results than cylinders, the ultimate unit stress of the cube is generally about one and one-half times that of the standard cylinder. Few engineers seem to realize that the ordinary test specimens do

not give the true maximum ultimate compressive strength of concrete. In order to determine this maximum, the test should be made on very shallow specimens, say, 2 in. high. In specimens of this small height, the restraining effect of the bearing plates of the testing machine is able practically to nullify the effects from the operation of Poisson's ratio. As the average of a sufficient number of tests of cylinders 8 in. in diameter and 16 in. high have been shown to be a good indication of the ultimate strength that a plain concrete column of similar concrete would develop, such tests are alone sufficient as a guide to plain concrete column design.

For the "spider" or for any other truly reinforced concrete column, which can be made to develop the true maximum ultimate compressive stress in the concrete, the ultimate unit compressive stress should be determined by testing specimens not more than 2 in. high. Tensile and modulus of rupture tests should also be made.

The speaker believes it would not be necessary to test prior to each construction job, but that for a concrete of given mix, density, water content, age, class of aggregate, quality of cement, expected character of workmanship, etc., reference might be made to Government tests, Joint Committee tests, or other accepted standard tests which are averages of a sufficient number of specimens to obtain a good forecast of ultimate strength. Such sets of tests would greatly aid the designer and the specification writer.

The speaker will now prove that the ratio of length to diameter has no place in any concrete column formula. Tests indicate that diagonal shear occurs on a plane having an angle of about 35° with the vertical, which shows that when the ratio of length to diameter exceeds 1½, the ratio is no longer important.

A. N. Talbot, Past-President, Am. Soc. C. E., has stated* that:

"It seems hardly necessary to advance the idea here that for concrete columns used in ordinary building construction little attention need be given to the relation between length and lateral dimensions after a length of a few diameters has been reached. * * * Even for 15 diameters we may readily conclude from the calculated results of long column formulas and also from the small lateral deformation found in the columns tested that the difference in strength between a column 15 diameters long and one 5 diameters long is less than the variation among several columns of the same length. * * * For columns eccentrically loaded, the effect of any eccentricity is generally large in comparison with the lateral deflection used in the Euler analysis, and it may be said to be generally independent of the length of the column. Clearly, for conditions of ordinary design a formula for plain columns or for columns reinforced with longitudinal rods need not include the ratio of length to lateral dimensions."

Professor Talbot and Arthur R. Lord, M. Am. Soc. C. E., have also stated† that "In other words, the stress taken by the concrete may be considered to be nearly independent of the slenderness ratio of the column, within the lengths tested, * * *."

* *Bulletin No. 10*, Univ. of Illinois, p. 45.
† *Bulletin No. 56*, Univ. of Illinois, p. 43.

Messrs. McKibben and Merrill* have stated:

"The average compressive strengths of 8-in. by 16-in. cylinders and 5-ft. columns, 14 in. in diameter, are very nearly equal, while the strength of 10-ft. and 20-ft. columns are approximately equal, amounting to 81% of the strength of cylinders."

Further discussion seems unnecessary as sufficient references and quotations have been given to support the speaker's contention, which might be summarized as follows: That plain concrete columns be considered on their merits; that science be used to the fullest extent possible in concrete column design, which would at once eliminate the present-day types of so-called reinforced columns; that testing of specimens be placed on a more scientific basis; and that the ratio of length to diameter be admitted to have no place in any concrete column formula.

* *Proceedings*, Am. Concrete Inst., 1916, p. 209.

LATERAL EARTH PRESSURE: THE ACCURATE EXPERIMENTAL DETERMINATION OF THE LATERAL EARTH PRESSURE, TOGETHER WITH A RESUME OF PREVIOUS EXPERIMENTS

Discussion*

By Messrs. J. C. Meem, E. G. Haines, George Paaswell, D. B. Steinman, M. J. McPike, Edward Godfrey, G. M. Braune, and Thomas H. Wiggin.

J. C. Meem,† M. Am. Soc. C. E.—It would be impossible to speak except in the highest commendation of this master effort, combining as it does research, compilation, analysis, and deduction, and the speaker wishes to express his high appreciation of the author's work, and to state his belief that the publication of this paper indicates that great progress has been made in the solution of problems pertaining to lateral earth pressure. The speaker feels convinced that although he differs from the author—and at times perhaps radically—such differences may not be irreconcilable.

His one and only criticism of the paper and its findings, therefore, is intended to be constructive and to involve a fundamental point, *viz.*, that the testing box or apparatus was not of sufficient width in relation to its height; that is, in order to get results wholly unaffected by lateral arching, horizontal as well as vertical, the box should have been much longer, by at least three or four times the height. That there is such arching action is as uncontrovertible as is the existence of lateral pressure itself, on which it must depend; and there is reason to believe that it exerts itself horizontally, or nearly so, as well as vertically, with certain modifications. The speaker is convinced that the size of the testing apparatus is not so important as its relative proportions. For example: If a box, Fig. 7, is built, having a width, D, equal to its length, L, and its height, H, and if a false bottom, B-C, is put in position, as shown, and the box is filled with sand, the bottom may be supported by bolts through the sand and through a smaller top or washer bearing on the sand. The bolts may be tightened so that the box may be carried about without dislodging the sand. The support of this sand is entirely due to the transmission of its lateral pressure to the sides of the box, and it must undoubtedly be true that there is equal pressure on each of the four sides of the box. It is seen, therefore, that the measurement of the pressure on any one side is not an accurate determination of the lateral pressure, as would be the case if the length of the box, L, was

* This discussion (of the paper by Jacob Feld, Jun. Am. Soc. C. E., published in April, 1923, *Proceedings*, and presented at the meeting of March 7, 1923), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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increased until it very greatly exceeded its height, H. Any measurements should include the whole area, as nothing can be gained by measuring the pressures, of fractional areas, separately.

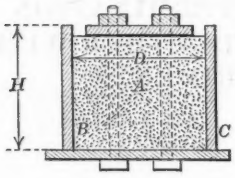

FIG. 7.

In Fig. 7 it can be shown that more satisfactory results can be obtained in a box 12 in. wide, 12 in. deep, and 4 ft. long than in any cubical box of however large a cube.

If, in Fig. 7, it is assumed that the pressure of all the sand or soil above B-A-C is transmitted laterally to the sides, it does not matter whether the measurements of B-A-C, or of H and D are in terms of inches, feet, yards, or rods, for, if the testing material is homogeneous and the length is always sufficient to be greater than 4 H, the relative areas of soil-producing pressures may always be encompassed by the same geometric figures. For instance, if the area producing pressure against the sides is measured in terms of one-half of all the sand above B-A-C, then it does not matter whether the area is expressed in inches, feet, or rods. For the casual reader, it should be noted that this does not mean that the pressure per area is the same in all cases, but that the pressure per area can always be measured by the same relative formula in terms of the height.

In the case of small models for testing, objection is often made to the use of perfectly dry sand; but the reason for this is apparent when it is realized that cohesion is relatively a much greater factor in small than in large models; that is, a cubic foot of moist sand may be cut vertically or even moulded or tunneled, whereas a deep cut or a large tunnel through sand of the same consistency would break, or tend to break down. Thus, the dry sand gives more nearly the same relative pressure in terms of the height.

After all, it is not difficult to realize that the results obtained by small models with dry sand are applicable (with slight modifications for changes in factors of moisture and cohesion) to operations of any size, even including those of a tunnel through solid rock, when the width of the tunnel is so great that the rock tends to break or become a mass of aggregates equivalent to large boulders. In ordinary soil, with which nearly all engineers in the East are familiar, there is usually an overlay of clay or hard compacted soil for varying depths; below that, and usually within the prism of the operation, sand is often found. The overlay of compacted soil does not usually tend to break down until the excavating operators have disturbed, or caused movement of, the sand, and then there occurs the phenomenon, referred to by Mr. Haines, and elsewhere by Mr. Moulton, of an almost vertical crack, caused by a section of clay which is pulled away by its own weight and the slight lateral movement of the sand at its base; and although the crack may seem to be continuous and to separate absolutely the two masses of soil, there must undoubtedly be spawls, cobbles, etc., projecting here and there and acting as braces, which prevent the soil back of them from breaking down and filling the crack. Of course, this applies as well to all more or less cohesive soils which are deeply cut or tunneled, even though there is no stratum of sand.

The measurement of the lateral pressure of this mass against the sheathing or structure is equivalent to that of a boulder resting on a background of other boulders, the pressure of which on the side of a small model would be slightly different, of course, from the resultant pressure of a sliding aggregate of small particles, such as dry sand; but it should not be difficult to correlate the pressure by proper assumption as to a mean angle of repose, or, more correctly perhaps, to an angle of internal friction, as noted by the author, and probably dependent on the angle of repose.

Bridges and foundations—in fact all structures—are designed for maximum loads or conditions which they are rarely if ever required to meet. Why, then, in the determination of soil pressure, should not a relatively safe angle of repose be assumed for all conditions, modified, if necessary, by the angle of internal friction or as determined. For almost all conditions of soft ground, other data being unavailable, the speaker uses an angle of repose of 45°, modified by an angle of rupture half way between that and the vertical. This angle of rupture, thus far, has been only an assumption, and if the author may show that an angle of internal friction should be used instead of it, and give and determine its properties more definitely, the speaker feels that this alone will be a long step in advance.

The speaker, at one time in his crude experimentation, constructed a box of 5 ft. cube with four movable boards at the front (Fig. 8), each of these boards being tied over a pulley to a pail carrying water or sand. No special results were obtained, however, because of the collapse of the boards with the initial movement. Later, he constructed arms on which the boards might slide, but discontinued the experiment, realizing that the lateral arching, both horizontal and vertical, prevented accurate determinations. He believes, however, that if a box is built, 10 ft. wide, 6 ft. high, and 24 ft. long (to prevent lateral arching), and the front boards are attached to heavy full-length I-beams rolling on bearings on arms attached to the box (Fig. 9), and if these boards are held back by cords passing to spring balances or to adjustable weights, that the results obtained will be reasonably conclusive as to the determination of lateral earth pressure.

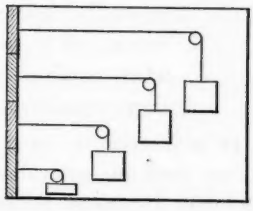

FIG. 8.

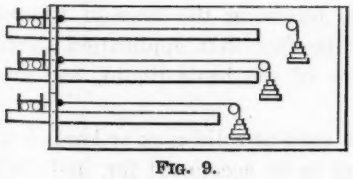

FIG. 9.

Again, it may be suggested that such a determination, even if made accurately and conclusively for dry sand, would not be applicable to soils of other conditions. The speaker, however, believes firmly that from such a determination the pressures with all kinds of soils can be formulated with reasonable accuracy.

Relative to this, the speaker believes that, in the ordinary or practical determination of soil pressures, five classifications of soil are all that are necessary, as follows:

(1) Cohesive.—Hard rock or clay, which does not break down or tend to break down under the operation.

(2) Semi-cohesive.—Normally dry sand, clay, gravel, etc., in fact, all soils ordinarily encountered which break or tend to break down under the operation.

(3) Non-cohesive.—Dry hot sand, such as beach sand, etc. (Dry grain may be included under this classification.)

(4) Semi-aqueous.—Water-bearing sand, gravel, etc., where the introduction of water does not affect the stability of the soil materially. (For instance, if a box is filled with dry sand reasonably free from finer particles such as clay, bog, etc., and if this sand is loaded, no radical change will be noted if water is introduced, except that there may be some slight settlement due to the compacting of the soil and a change in its cohesive properties. If, however, an excess of clay, bog, or other so-called non-granular soil is incorporated with the sand, the introduction of water causes these particles to be displaced by pressure, almost as if the mass was pure clay or bog, and this gives a fifth class called aqueous.)

(5) Aqueous.—Plastic clays, quicksand, bog (also mercury, grout, pure water, etc.)—soils which are displaced by ordinary pressure and, when confined under pressure, transmit that pressure in all directions equally.

It is interesting to note that plastic clay, containing as much as 40% of fine or medium granular sand, may be jacked into existing voids for considerable distances through specially constructed hydraulic rams.

The speaker believes it to be not only possible but probable that simple and practical formulas for the lateral pressure of earth will finally be established, modified only as required for each of the foregoing classes, but not by the necessity for determining separately the pressure of each type of soil, as clay, sand, gravel, etc., to analyze this.

Aqueous soils Class (5), may be eliminated, because the determination of the pressures in such soils is simple, following the laws of hydrostatic pressure, with due regard, of course, to the fact that application should be made on the basis of the specific gravity of the basic liquid, and not the weight of the mass per volume.

Class (1), or wholly cohesive materials, such as solid rock or hard clay, can also be eliminated, as there is no pressure to be accounted for, and the only necessity for timbering or lining, if required, is to prevent disintegration or accident by spawling, etc. When the operation is large enough to cause the rock, clay, etc., to tend to break down, it automatically enters Class (2), and thus it is necessary to deal with only Classes (2), (3), and (4).

In Class (4), or semi-aqueous materials, in which the water is contained in or seeps through the voids without causing erosion, there are two distinct or different types of pressure, acting in unison, first, the lateral pressure of the soil, modified only slightly by a change in its cohesive property,

due to the presence of water, but otherwise conforming to the laws of lateral pressure of normally dry or semi-cohesive soils. The water exerts pressure through the voids of the soil, and finds necessarily reduced areas on which to exert its full head or pressure. Gauges, for instance, through the walls of submerged structures buried in soil, almost always show pressure due to the full hydrostatic head; and although it is true that these reduced areas have full-head water pressure, there are between them larger areas in which the soil pressure acts alone. If this was not the case, the structure would be wholly surrounded by water pressure, and in all cases buoyant.

Here it should be noted that, when the cover is very shallow, or the operation calls for a constant churning up of the material, it will always be safer to count on full or net hydrostatic pressure; that is, in all cases of submerged structures with shallow cover, the maximum pressure should be considered, whether aqueous or combined. Analysis of Class (4), therefore, results as follows: The total pressure, as noted in Classes (2) or (3), is equal to the earth pressure plus the pressure of water over a reduced area due to the hydrostatic head. The speaker believes that 30%—or at most 40%—of the total area of water pressure is all that acts ordinarily on a submerged structure, but, for safety, he uses 50%, that is, P = total pressure; $P\,X$ = earth pressure; $P\,W$ = water pressure; therefore, $P = P\,X + \frac{P\,W}{2}$.

It should be noted, especially by the casual reader, that this determination applies only to static or approximately static conditions, and that where there is sufficient flow of water to cause erosion or collapse, hydraulic considerations must govern.

Finally, as to Classes (2) and (3), to which it is seen that the other classes may be directly or indirectly referred, it has already been noted that Class (3) should give the same formulas as Class (2), modified by the factor of cohesion, which changes the angle of repose more or less materially, but does not otherwise cause any changes in a formula applicable to either case.

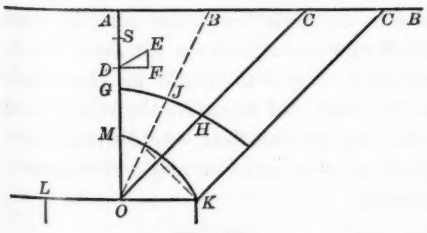

FIG. 10.

Assuming a trench, of depth A-O (Fig. 10), which is sheeted and braced as it is excavated, continuous arching action is set up by virtue of the lateral pressure between the face, A-O, and the undisturbed ground, O-C, and when the full depth is reached, there is —to express it somewhat crudely —set up a sand arch, A-G-H-C, which undoubtedly possesses the three necessary fundamentals of a true arch, that is, lateral pressure at the key, B-J, thrust against the haunches, A-G and H-C, and internal cohesion supplied by the soil below G-H, which holds its so-called centering in place. The part of the area that causes pressure in A-G must, of course, be determined by experiment, but the speaker has assumed it as being the whole of the

area above *G-H*, to the left of the plane *B-O*, bisecting the angle *A-O-C*, or $\frac{90° - \phi}{2} = 45° - \frac{\phi}{2}$, ϕ being the angle of repose.

He has also assumed that the thrust is due to the weight divided by the tangent of ϕ. The formula thus developed results with modifications as follows:

P = lateral pressure in any area;
A = Area (volume) of soil covering P;
W = Weight per volume of soil;
ϕ = Angle of repose; thus, $P = A \times \frac{W}{\tan \phi}$.

The center of pressure of the whole mass, *H-B-O*, is taken to be where a line or plane drawn through the center of gravity of the mass parallel to the angle of repose intersects the face of the sheeting or wall, and ordinarily should be found slightly above a point measuring half its height.

From the foregoing the determination of the pressure in a tunnel may be readily made.

If *L-O-K* is the roof of a tunnel (Fig. 10), in soil of which the depth is *A-O*, the pressure on *O-K* will be (but probably little more than) that due to the area, *O-M-K*.

The speaker assumes, however—and always with a factor of safety in view—that this pressure is due to the weight of soil below an area measured by $\frac{O\text{-}K \times O\text{-}K \tan S}{2}$, where $S = \phi + \frac{1}{2}(90° - \phi) = \frac{\phi}{2} + 45°$.

It is thus seen that by this determination full pressure of all the soil results when the depth of soil, *A-O*, is less than *S-O*, *S-O* being equal to $O\text{-}K \times \tan\left(\frac{\phi}{2} + 45°\right)$.

In this presentation of the principles as he finds them, the speaker hopes he has shown it to be possible to establish one basic formula for every condition of lateral earth pressure, one variable of which should be the height of trench, trench structure, or wall (or the width of tunnel), the other being based solely on the cohesive element of the soil in question, establishing therefor an angle of repose—and finding expression in terms of an angle of rupture—or an angle of internal friction as determined.

E. G. Haines,* M. Am. Soc. C. E.—The speaker has had the pleasure of reading the complete thesis presented by Mr. Feld, which is a very interesting document, as in it the author retraces the whole theory of lateral pressures from an early date to the present. The persistence with which the theory of the angle of repose has continued since once it was expressed, whether right or wrong, as long as it agrees fairly well with observed facts,

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is of great interest. The angle of repose for soils was assumed by most of the early writers to be about 45°, and is now ascribed various angles corresponding to the characteristics of different soils. The author has re-stated the limits of such angles as 0° for water and 90° for rock. Although both are doubtless true within the limits in which the engineer usually is interested, the latter is not strictly true as a vertical cut in solid rock, unsupported, will fail at some depth, as has been shown,* and the failure will not be vertical throughout, as has been substantiated by H. G. Moulton, M. Am. Soc. C. E.†

The speaker has no particular fault to find with most of the theories for lateral pressure, or with tests made for ascertaining such pressures, except that, when they are applied to soils in the natural state in large masses, it is found that the soils do not abide by the rules. Most of the tests have been made with reasonably dry sand, which is rarely found in practice. Sand is practically all silica; is fairly uniform in texture; has a low cohesive value; has a high frictional value; has normally a low water content; and, in short, is not only a granular but a granulated material.

The soils found in practice are composed of practically all the mineral elements derived from all classes of rocks, from the hard volcanic rocks and the limestones and sandstones to the softer clays and fine silts. Natural soils range from a dense, quasi-elastic mass, through the granulated stage, to an almost hydrostatic condition, and the degree of moisture or water content is variable. It is not reasonable to suppose that these soils will act in practice the same as sand under test conditions.

In arranging any device for measuring pressures, one must make some assumptions as to how the material will act, and when a machine is designed on those assumptions, dry sand readily accommodates itself to them. In making tests, using vertical shutter boards, it is invariably found that as sand is added behind the shutter boards and measuring instruments, a certain amount of pressure is obtained for a certain depth of sand. If a second similar depth is added, there is a similar increment, but it is also added to the first, etc. The resulting pressure diagram resembles a triangle standing on its base, at least at the time the test is conducted.

With soil in excavation, the process is reversed. As excavation is carried down step by step there is a certain amount of pressure with each step, but it is also added to each one above. That is not theory, it is established fact, and may be observed on any deep timbered excavation in dry ground. The pressure diagram for excavation, therefore, also resembles a triangle, but its position is reversed, and it has its base at the top, at least at first. The speaker believes that Mr. Meem was the first to call attention to that fact.

The pressure diagram may not remain a true triangle in either the case of an embankment, or an excavation; if there is a change in the water content or other conditions, the pressure diagrams change, but with an excavation, at no time, unless considerable water is present at the bottom, does the center of pressure ever transfer to a point much below the mid-height and, with

* *Transactions*, Am. Soc. C. E., Vol. LX (1908), p. 27.

† *Transactions*, Am. Inst. of Min. and Metallurgical Engrs., February, 1920.

embankments, there is evidence in some cases that the center of pressure is as high as the mid-height.

If material from behind a retaining wall is carried away by the drainage through weep-holes, or if the tide is carrying out or settling the material, the center of pressure becomes higher, and the upper material behind the wall will act as an arch. The reverse is true of an excavation. In the beginning there is no pressure, but the timbering as it is placed and keyed tight, transmits pressure into the bank, and the reaction of the bank which is a solid and quasi-elastic, converts the whole system into an arch, regardless of the direction of the stresses. Such pressure increases with depth of excavation. The contact between the timbering and bank is less perfect as the depth increases and, at some depth, the bank will fail, because of the shear from its own weight. When this occurs, there is a breaking up of the material and as it approaches the granulated state there is also a corresponding readjustment of the pressure, and it approaches, but never fully reaches, that of an embankment.

About three years ago, the speaker had occasion to investigate some cracks in the soil parallel to an excavation about 85 ft. deep. The crack was beneath a crushed stone or gravel sidewalk and was not apparent on the surface. Although the excavation had been made two years before, the crack was found by measuring outward a distance equal to one-half the depth of excavation. Measurements were made at a number of points for a distance of 1 000 ft., with the same success. That crack was so wide that one could stick his hand into it up to the wrist and had been there for more than two years in material that one might class as sand. The surface of the crack was partly coated with the fine loam and stone dust brought down by rain water and melting snow. By means of a small shaft the crack was traced downward for 26 ft. until it was lost in a vein of loose gravel. It is also safe to state that the crack could be traced at other points to-day, five years after the excavation was made. The point of interest is that, if, in this case, there was any arching action of the material from the side, why was not that crack closed? It should also be noted that the crack, instead of inclining at any angle—the so-called angle of repose—was practically vertical for the 26 ft. of depth; in fact, it inclined slightly outward, with increase of depth, although the material was one to which is usually ascribed an angle of repose of $1\frac{1}{2}$:1.

There are many ideas and theories in regard to the pressures developed by soils. The speaker has contributed some of the theory himself, but as time goes on there is an inclination to discard one theory after another. Engineers have about reached a time when they will have to develop a new soils physics and a new theory of soil pressure, because most of the current ideas are wrong.

Soil is both the most common and the most complex construction material with which the engineer has to deal, and until it is subjected to analysis like other materials, and until its properties are much better known,

there can be only little hope for its successful use, and failures may be safely predicted.

One of the things that adds to the trouble is that engineers have borrowed from the layman names for broad general classes of soils and are attempting to use them to define certain definite materials. It does not follow that a so-called clay from the vicinity of the Great Lakes will possess the same characteristics as one from the Lower Mississippi Valley or elsewhere.

One of the most important new developments that has been brought out in the soils investigation recently, was presented to the Society in a Progress Report* in 1921 by the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations, etc., and elaborated somewhat by the presentation before the Society at its Annual Meeting in 1923 of samples of the colloidal element in clay. It is not known how far-reaching these discoveries will be, but the Committee hopes to study it with the help of the Society, and it is believed it will pay for the research, as there is little doubt that the presence or absence of that element constitutes a large measure of the stability of the soil, and its resistance to stress, either vertical or lateral.

GEORGE PAASWELL,† M. AM. SOC. C. E.—This paper is probably the best yet presented on the subject, from the standpoint of both the experimental data it contains and its thorough historical review. That it is consistent with well founded theory, makes one hope that it marks the end of the search for the proper method of determining the thrust on a retaining wall. A great many statistics have been accumulated, covering both experimental and theoretical research, and the author, in studying the accumulated data and assembling it, deserves more than the usual remarks of appreciation.

The speaker believes that there are two distinct problems connected with the determination of lateral earth pressure: (1) The retaining wall problem; and (2) the soil mechanics as evinced by the lateral pressure action. Experiment has been confined, to the present time, almost exclusively to Problem (1). What is the retaining wall problem? Does it vitally depend on the proper method of thrust determination, or are other factors of more importance? The experiments show conclusively that the Coulomb or a modified Rankine method gives the maximum pressure that can occur on a wall, taking a simple slope angle corresponding to that of a dry sand, namely, 30 degrees. With this value of the slope, and with the introduction of other approximations, the thrust expressions can be placed in very simple form. The speaker has shown‡ that the following two formulas give the thrust on a wall when the fill is supporting a surcharge which has been reduced to an equivalent earth fill the ratio of depth of which to the total depth of fill behind the wall is:

$$T = (1 + 2c)\ J\ (\tfrac{1}{2} w\, h^2)$$

$$T = (1 + 2c)\ K\ (\tfrac{1}{2} w\, h^2)$$

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* *Proceedings*, Am. Soc. C. E., February, 1921, Papers and Discussions, p. 9.

† Section Engr., Public Service Comm., First Dist., New York City.

‡ "Retaining Walls: Their Design and Construction."

Table 36 gives the values of the thrust coefficients, K and J, for a number of slope values of the back of the wall. These coefficients represent the maximum thrust that can occur on a wall. The engineer, therefore, is justified in using these values to determine the thrust just as in the design of other structures maximum loads are used even though the actual loads are considerably less. There appears to be no justification for determining the exact values of the slope angle as these values are minute modifications that do not materially affect the formulas, and rarely occur in actual practice. Thus, the usual experimental material of dry sand is one most rarely found in practice, the usual fills consisting of loam, clayey materials, various undetermined mixtures, rock, and the like, for none of which a reasonably accurate slope value can be predicted. For this reason, the speaker feels that it is better to assume a thrust value based on a theory in reasonable accord with experiment, such as the values given previously, and then to proceed with the more important business of the design of the wall itself. That is the main problem; too often the designer deems the problem of the retaining wall solved when the thrust has been found. That is the easiest part of the problem. Examine the records of the failures of retaining walls, and note how few collapses are due to overturning moments. These remarks apply, of course, to the retaining wall built to carry an ordinary fill.

It seems reasonable to conclude that as far as the design of a retaining wall is affected, the exact determination of the earth pressure is an interesting but not necessarily a vital problem, and the speaker feels that, in view of this conclusion, experimenters on lateral earth pressures should take more care to distinguish between the retaining-wall problem and the so-called soil-mechanics problem.

TABLE 36.—PRESSURE COEFFICIENTS.

β, degrees.	COULOMB COEFFICIENT, K.			Rankine coefficient, J.
	ϕ'			
	0°	15°	30°	
0	0.33	0.30	0.29	0.33
3	0.36	0.32	0.32	0.34
6	0.38	0.34	0.34	0.35
9	0.40	0.37	0.37	0.37
12	0.43	0.40	0.39	0.40
15	0.45	0.42	0.43	0.43
18	0.48	0.45	0.47	0.47
21	0.51	0.48	0.50	0.51
24	0.54	0.52	0.57	0.56

The author is correct in stating that a granular material is neither a fluid nor an elastic solid, but as it is obviously in a condition intermediate between these two, it should have some of the properties of its limiting conditions. Thus, it has been pointed out that elastic failure of solids may be explained on the theory of a constant angle of internal friction; that is, when materials start to fail under simple stress, failure surfaces will be found,

that may be anticipated by assigning to the elastic solid an angle of internal friction of about 20 degrees. It is interesting to note that this introduction of an internal friction component into ordinary elastic theory seems to reconcile the granular and elastic solids, and, therefore, it seems reasonable to assume that some data on lateral earth pressure may be furnished by the theory of elasticity.

In conclusion, the speaker would state that, with no reflection on the value of the data presented, the paper adds nothing to the needed data on soil mechanics, and little more than is already known on retaining wall statistics. The rational bracing of a cut, and the proper design of a subsurface structure, whether it is a large sewer, a subway, a tunnel, etc., awaits the development of a soil mechanics. The homogeneous granular materials of experiments are of little service in solving these problems.

D. B. STEINMAN,* M. AM. SOC. C. E.—The speaker desires to record his sincere appreciation of Mr. Feld's paper as an authentic contribution to the knowledge and understanding of lateral earth pressure phenomena. This paper helps to clarify a subject in which there has been much confusion and unscientific vagueness. A mass of literature has been published on the subject, with an abundance of mathematical derivations and ingenious graphic applications; but there has been a dearth of reliable experimental data on which to test the correctness or incorrectness of these basic assumptions. Most of the experimental work previously done in this field has been too crude or unscientific to yield any conclusive results. By examining the vast literature on soil mechanics, including the digesting and collating of more than 650 references, by designing and constructing an ingenious large-scale experimental apparatus for measuring earth pressures, by conducting an extensive and laborious series of tests scientifically planned and carefully executed, and by analyzing the results of those tests and reporting them in this excellent paper, Mr. Feld has rendered a real service to the Engineering Profession.

The author's experimental findings may be regarded as conclusive on a number of questions of fundamental importance, one of which is the direction of the resultant pressure on the back of a retaining wall. His experimental results appear to establish beyond further question the fact that this pressure is inclined above the normal to the back of the wall at an angle ϕ', which ϕ' is the angle of friction between the wall and the retained material. The Coulomb theory, which assumes that the resultant pressure is normal to the back of the wall, and the Rankine theory, which assumes that the resultant pressure is parallel to the free surface of the fill, are thus overthrown. The author's results are also valuable in establishing the height at which the resultant pressure acts.

In his presentation of the distinctions between the angle of repose, the angle of internal friction, and the angle of internal resistance, Mr. Feld has contributed to a much needed clarification of some of the fundamental conceptions in soil mechanics; and in demonstrating that the angle of internal resistance is the proper angle to use in earth-pressure formulas, he has made a

* Cons. Engr., New York City.

valuable new contribution to the application of earth-pressure theories. His findings on the effects of time, temperature, and surcharge on lateral earth pressure open many new questions for further study and experiment.

There are a few points in connection with the paper on which the author may be able to throw further light, one of which is the feature in the construction of the testing apparatus, whereby the side-wall friction was relieved. This feature is open to possible adverse criticism on the ground that it may have influenced the results to agree with a preconceived assumption. The detail consists of movable steel plates bearing on rollers against the side-walls and conforming in outline to the wedge of rupture. It may be contended, with some justification, that this feature of the design assumes the theory of the wedge of rupture in advance, and that, consequently, any results obtained with this apparatus confirming that theory would be inconclusive. This construction may help to produce the condition assumed. The tests would have been more conclusive if some method had been projected to establish the validity of the wedge theory without resorting to a device which assumes that validity in advance.

In the conclusions from Mr. Feld's tests, there is one anomaly which deserves further study and investigation. In the tests on vertical walls, the measured pressures appear to confirm the Coulomb theory despite the fact that the same tests disprove the fundamental assumption on which that theory is based. The Coulomb theory assumes that the resultant pressure is normal to the wall, whereas the tests have proved this assumption to be wrong.

It appears paradoxical that the Coulomb theory which is based on the incorrect assumption of normal pressure, should fit the results of the tests better than the general wedge theory which embodies the correct assumption of inclined pressure. The author may have some explanation of this inconsistency, or further tests might help to eliminate it from the conclusions.

M. J. McPIKE,* ASSOC. M. AM. SOC. C. E.—In presenting his paper, the author showed that the pressures obtained on the scale-beams varied directly with the temperature. He stated that a movement of only $\frac{1}{40\,000}$ in. was the maximum movement of the gate for maximum pressure.

With such a small movement of the gate, it would seem that any slight variation in the length of the bell-cranks and connections of the apparatus itself would indicate pressures chargeable to expansion in the apparatus rather than to the soil, and this expansion would be a factor for consideration in the final results. It would be of interest to know whether any attempt was made to eliminate this factor.

It is to be hoped that the experiments will be carried to a further conclusion and that reliable data will be furnished for the proper design of walls with uniform back-fill as well as those with a combination of fills, and especially those with a water-bearing soil overlaid with a dry fill.

EDWARD GODFREY,† M. AM. SOC. C. E. (by letter).‡—Several things that

* Asst. Chf. of Design Section, Public Works Dept., U. S. Navy Yard, Brooklyn, N. Y.
† Structural Engr. (Robert W. Hunt & Co.), Pittsburgh, Pa.
‡ Received by the Secretary, March 7, 1923.

stand out in any investigation of retaining wall theories may be epitomized as follows:

1.—A vast amount of intricate theory and theoretical investigation is expended on this subject.

2.—All earth pressure theories are based on a granular mass, generally without any assumption of cohesion.

3.—Grain and dry sand are about the only materials that are ever collected in sufficient quantities to require a container of large size, the nature of which materials fits the assumption of the theory.

4.—Grain and dry sand are kept in bins, but never as the fill behind a retaining wall.

5.—Moist sand is seldom used as a fill, although it does not differ much from the material assumed by the theory.

6.—Loose garden earth has about the same properties as moist sand, but garden earth or soil is generally too valuable to be deposited behind a retaining wall, and if so placed, it would be compacted not loose.

7.—The sides of excavations in ordinary compact earth will stand alone. Unless much moisture is present, excavations are shored usually only as a precaution. The writer has seen pits for deep and large piers, largely undercut, excavated many feet into the soil without shoring.

8.—Coal which is perhaps the next material in order of importance to earth that requires walls or bins for its confinement, can be made to stand several feet high in a vertical plane with a little care.

9.—The facts stated in Paragraphs 7 and 8 give legitimate grounds for the question, Why delve in fine theory in deriving a formula for a great force that, in fact and every-day practice, may be zero? Does not this kind of theorizing discredit the theory?

10.—A theory must be had for design, because it is known that any granular material will exert a great horizontal force on a wall, and that earth under certain conditions may also exert a great force.

11.—The real test of a retaining wall supporting earth fill occurs when the fill is saturated with water or is expanded by so-called frost action.

12.—No theory will codify the effect of these two conditions on a retaining wall, for the reason that no two walls are subjected to the same conditions in this respect.

13.—Experience with walls that have been built is the safest guide, in view of the impossibility of obtaining an exact theory. If this experience can be connected with a theory that is not too complex, that theory is a good guide for the extension of the rules of design.

14.—Tests on the lateral pressure of sand are valuable as a general guide in using a theory of earth pressures, but such tests should not be taken too seriously.

The natural corollary to these fourteen points is, that the only consistent theory for the design of retaining walls is one that approximates the results of tests and at the same time produces a wall corresponding in stability with

successful walls. The simplest method of design is the equivalent fluid pressure method.

It is much simpler to determine the horizontal pressure against a retaining wall by assuming that the back-fill is a liquid the weight of which is some fraction of that of the soil. Such calculations are much easier to make and to check than those that necessitate the solving of a tedious formula, and when they are made, one may readily assure himself that they are correct.

The coefficient for the equivalent fluid pressure is all that is required in the solution of a problem in design, where there is no surcharge. Tests are very valuable for cases of surcharge or superimposed load.

The equivalent fluid pressure method of design is not restricted to any theory, as it can be made to give results identical with the theories mentioned by Mr. Feld, merely by using a suitable coefficient. Furthermore, this method of design does not mean that its user is under the hallucination that granular masses or earth act as liquids.

It is true that a fluid does not exert a vertical force parallel to the back face of a wall (the friction), and that strict adherence to the fluid pressure theory must neglect that force. It is true, also, that a vertical force is seldom considered in a design, and for safety should be neglected.

On page 607*, Mr. Feld states that clay banks with overhanging tops and ditches with vertical sides are often seen, and that they are examples where cohesion is acting. Cohesion of earth makes the pressure of earth less than that of sand. Tests in which sand is used as a fill, therefore, are not proper criterions for judging the effect of earth fill against a retaining wall, unless those tests are interpreted with these facts in view.

In next to the last column of Table 5,† values are given that could be used as a coefficient for an equivalent fluid pressure. Neglecting the values for very small depths, it is seen that these coefficients are very regular, showing nearly constant results. The average is 0.264. This would be high for earth pressure, because if it is assumed that a stone wall of width one-third its height is stable as a retaining wall, the coefficient for equivalent liquid pressure is about 0.17. Walls of these proportions are found to be stable. The conclusion, therefore, is that in the design of retaining walls to sustain earth fill the pressure of sand is greater than need be provided; and this is what is to be expected because of the cohesion of the earth.

It is significant to note that exactness in the lateral pressure theory, even for the regular sand used in these tests, is somewhat discredited by the results in Table 3.‡ The horizontal pressures in Columns (13) and (14) should be alike, but there are large differences in these values. These differences amount to as much as 27 per cent.

On page 649,§ under Conclusion 7, it is stated that it is better to use the wedge theory, disregarding the wall friction, and a complex formula is given for determining values of H and E. Why should E be found in this case,

* *Proceedings*, Am. Soc. C. E., April, 1923.
† *Loc. cit.*, p. 623.
‡ *Loc. cit.*, p. 620.
§ *Proceedings*, Am. Soc. C. E., April, 1923.

and why not at once reduce the problem to the equivalent fluid method and simplify all calculations?

One of the conclusions from Mr. Feld's tests given on page 648* is that loads on the fill beyond the wedge of rupture have practically no effect on the lateral pressure. On page 645,* under "Conclusions", he states, "There is appreciable increase in pressure due to loads not on the wedge of rupture." This seems to be a direct contradiction of the statement on page 648. This is an important matter, because the designer wants to know at what distance from a retaining wall a railroad track will exert lateral pressure against the wall.

On page 646,* Mr. Feld states, "Rolling or moving loads on a tamped fill have no effect on the pressure." Perhaps this is only the case where the load that rolls over the tamped sand is equal to or less than that which was exerted to tamp it. It would seem that a railroad load on a tamped earth fill would have a considerable effect on the pressure. Certainly, it would be poor engineering to neglect the superimposed load of a railroad track in a retaining wall to support such a track. It would be helpful to the designer if tests could discover what effect should be computed for a load on the embankment such as a railroad track? The writer would take the load per foot, say, 10 000 lb. for a locomotive, and divide it by 13, the width of the track, and use this 770 lb. as 7.7 ft. of surcharged earth, and would find the equivalent fluid pressure for this 7.7 ft. of earth. How would Mr. Feld solve the problem?

The author has worked out the coefficient to be used in the equivalent fluid method, although he does not so state it, and it would take some searching to find it. It is suggested that Mr. Feld, in his closure, discover whether one-half the vertical depth of the triangle of surcharge added to H will not give the horizontal pressure of his tests with about the same coefficient as that found without surcharge.

G. M. BRAUNE,† M. AM. SOC. C. E. (by letter).‡—As the writer was responsible for the conception and part-time direction of the experiments discussed by Mr. Feld, and in collaboration with Professor C. C. Myers, for the design of the apparatus used therein, it would probably add to the completeness of the paper if a fuller statement concerning the origin of the problem was given.

The validity of the theoretical earth pressure formulas had always been looked on with doubt by many engineers, as no experiments on a large scale had ever been undertaken prior to those made in Cincinnati. Several years ago, when the writer was engaged on the construction of some abutment wing walls, 37 ft. high, he recalls the discussion regarding the proper dimensions for such walls. One member of the engineering force who was considered an expert in earth pressure formulas, after devoting several weeks of study to the proper design, submitted his computations to the resident engineer, who, like a number of others, did not have much confidence in earth pressure formulas. After much deliberation, he finally told the writer, who was associated with

* *Proceedings*, Am. Soc. C. E., April, 1923.

† Dean, School of Eng., and Prof. of Civ. Eng., Univ. of North Carolina, Chapel Hill, N. C.

‡ Received by the Secretary, March 19, 1923.

the contractor and was urgently requesting plans so that the work could proceed, that he would discard the theoretical computations and make the base four-tenths the height. Another point in the design of walls which was, and still is, under discussion, relates to the direction of the earth pressure; in other words, the question of whether there will be any frictional resistance between the filling material and the back of the wall. The late Professor Mohr, of Dresden, Saxony, contended that no such frictional resistance should be considered or would exist, whereas Dr. Müller-Breslau held just as firmly that the resultant earth pressure was not normal to the back of the wall. Dr. Müller-Breslau contended that as the wall was acted on by a lateral force, some slight movement would occur; and that however slight this movement might be, frictional resistance would come into action. Rankine's theory* assumes,

"* * * an incompressible, homogeneous, granular mass, without cohesion, the particles being held together by friction on each other; the mass being of indefinite extent, having a plane top surface, resting on a homogeneous foundation, and being subject to its own weight. These assumptions lead to the ellipse of stress and make the resultant pressure on a vertical wall parallel to the top surface."

When the wall is considered absolutely rigid, the Rankine theory holds true; and the resultant will be parallel to the top surface. As the wall is under stress, however, a slight movement will occur; therefore, on account of this movement, the conditions mentioned will be changed, and the resultant will no longer remain parallel to the top surface. In his paper entitled "Earth Pressures: A Practical Comparison of Theories and Experiments."† L. D. Cornish, M. Am. Soc. C. E., questioned the assumption made in Rankine's theory that the direction of the resultant earth pressure is parallel to the top surface of the filling material. In discussing that paper, the writer alluded‡ to the experiments which he hoped to undertake at the University of Cincinnati, the results of which are now presented to the Society in Mr. Feld's paper. These, together with a number of similar, experiences convinced the writer that some extensive experiments should be conducted, and he decided that if an opportunity was presented, he would undertake such experiments on a large scale.

The writer became connected with the University of Cincinnati in 1912 and commenced the study of the proper design of apparatus. After reviewing a number of experiments that had been made, he finally adopted an apparatus similar to that used by Dr. Müller-Breslau in his experiments at Charlottenburg in 1906. Dr. Müller-Breslau used a free wall in front of the bin and determined the magnitude of the forces by measuring the deformation of supporting struts. His results—given in the discussion of the paper by Mr. Cornish referred to previously—show that the resultant earth pressure makes an angle of about 27° with the normal to the back of the wall and its point of application is about 0.33 h above the base of the wall. His experiments included various slopes of surcharge, and his results showed

* "Design of Walls, Bins, and Grain Elevators," by Milo S. Ketchum, M. Am. Soc. C. E.
† *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 191.
‡ *Loc. cit.*, p. 218.

that the direction of the earth pressure was not controlled by the slope. The Cincinnati experiments made on a much larger apparatus fully confirmed these results of Dr. Müller-Breslau. The writer preferred a more direct method of measuring the forces than those adopted by Dr. Müller-Breslau, and enlisted the services of Professor Myers in the design of the Cincinnati machine. Together, they designed an apparatus substituting platform scales for measuring struts. Complete details of this apparatus were worked out by Mr. Wulfkoetter, a Fellow of the Civil Engineering Department, under the writer's direction, and were completed in 1916. Further work on this problem was delayed on account of the World War. In 1919, Mr. Feld came to the writer as Fellow in the Civil Engineering Department, and the suggestion was made to him that he complete this problem and submit it as a Doctor's thesis. Under the writer's direction, Mr. Feld made a new set of detailed drawings of the apparatus which embodied, however, few fundamental changes in the original design. Owing principally to the high price of steel, and also with the idea of obtaining a more rigid structure, the writer made the bin of concrete, and for the same reason, made the gate of wood, instead of steel. In order to eliminate the frictional resistance between the filling material and the sides of the bin, the writer conceived the idea of permitting the sides of the bin to move with the filling material. This idea was an error and in making the detailed drawings of the apparatus as finally built, only that part of the side walls was made movable that was in direct contact with the breaking prism.

The writer received valuable aid and suggestions from William Cain, M. Am. Soc. C. E., when he first undertook the design of the apparatus, and at his suggestion the front wall was made 6 ft. high, as this height was considered the minimum to adopt in order to obtain practical data. In discussing this problem with Dean Ketchum, the writer's attention was called to an important point that probably might have been overlooked in making these experiments; that is, whether the scales would record active or passive pressures. In order to eliminate any uncertainty, Dean Ketchum suggested that if the riders on the scale-beams were moved out and the scales over-balanced the pressures that would be recorded when the scale-beams were permitted to float, would be the active, and not the passive, pressures.

The writer wishes to take this opportunity to express his appreciation of the perseverance and conscientious work that was done by Mr. Feld in conducting these experiments, and he believes that the publication of this paper in the *Transactions* of the Society will be extremely valuable to the Engineering Profession.

THOMAS H. WIGGIN,* M. AM. SOC. C. E. (by letter).†—This paper has much merit, not only for its store of well digested experimental data, but also for its painstaking correlation of the new data with the results of previous researches on the same subject.

* Cons. Engr., New York City.

† Received by the Secretary, April 2, 1923.

The writer would suggest the following explanations, many of which are not new:

1.—The direction and point of application of resultant earth pressure depend on the kind of support, that is, its degree of rigidity; also on the state of stress in the earth at the time equilibrium is reached between the pressure of the earth and the resistance of the wall.. The author's experimental walls, of course, received only the "active" pressure of newly placed sand. Trench bracing may be so strong and so wedged as to develop part of Rankine's "passive" pressure, or may be so weak and slack as to compel the earth to support itself in part by cohesion, such cohesion being greater than that assumed possible in earth by any of the theories under consideration.

2.—With rigid support the pressure on a vertical surface from a level field of earth will be exactly horizontal. This seems to be evident from a consideration of the symmetry that must exist in undisturbed earth supported by itself. Manifestly, conditions are the same on each side of any vertical plane, so that there is no reason for the earth pressure to slope more in one direction than in another.

3.—If, in a level field, one could substitute a rigid wall for the earth on one side of a vertical plane, without disturbing in the least the state of stress in the earth, the pressure on that wall would be exactly horizontal.

4.—In earth pressure experiments like those by the author, the wall is placed first and the earth is then shoveled in behind it. The wall is practically incompressible vertically, and, through the agency of the friction of the earth on the wall, retains the adjacent earth in suspended condition. This tends to produce Mr. Meem's arching effect, with the wall for one abutment; there is, of course, a vertical component from the arch thrust, and this component is registered on the scales provided by the author in his experiments. In other words, the resultant in this case is inclined, as the author proves.

5.—The fill against a retaining wall is generally placed by the same method as that used by the author; hence, the experiments give correctly the nature of the thrust against the ordinary retaining wall. A retaining wall must rock forward, however, to sustain its load (although in rare cases it may settle more at the heel than at the toe); hence, the author's wall, supported rigidly at top and bottom, cannot entirely represent the ordinary wall. The Goldbeck cell for measuring pressures in earth offers a better means than was formerly available for observing pressures and changes of pressure with lapse of time in earth back of actual structures. It would be extremely valuable to have long-time observations on actual structures, and a parallel set of the author's tests, using the same earth.

6.—In a sheeted trench, the pressure will depend on the method of sheeting and the care taken to prevent earth movement. A trench in moderately cohesive earth may be braced by longitudinal planks and cross-struts. In this case, the sheeting cannot carry a vertical component and the earth has to continue to do so, as it did before the trench was dug. In other words, pressure on the support is horizontal. With vertical sheeting, on the other

hand, particularly when not wedged tightly, vertical earth loads may be thrown on the sheeting; that is, the resultant earth pressure may be inclined.

7.—Mr. Meem has called attention to the fact that for certain kinds of earth, trench support may be designed successfully on the assumption that most of the earth pressure comes near the top of the trench. If the earth has some cohesion, as is usual, the section of earth which tends to slip down, when the support of the earth which was in the trench is removed, may be held up by various combinations of braces. Manifestly, a support anywhere in the depth of the trench would hold up a wedge of cohesive material. The brace, *A*, in Fig. 11, might do all the work, if cohesion at and below *cd* were sufficient; or the brace, *B*, might do all the work or only a small part of it. As earth is of many degrees of cohesion and plasticity, trench bracing must be adapted to the material encountered, and will never be solved by any universal rule except one that would be extravagant in most cases.

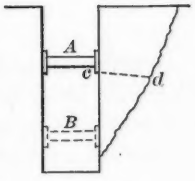

FIG. 11.

8.—In connection with the slides of the Panama Canal, the formation of vertical cracks has been explained mathematically by Professor Slichter, if the writer remembers correctly. A high bank of cohesive earth or weak rock, Fig. 12, fails by shear along surfaces which are vertical at the top and curve to a nearly horizontal direction near the bottom. All deep trenches show this tendency, which, of course, is more noticeable when the support is inadequate or the braces are not tight enough. It is not a wedge, but a semi-segment, that is to be supported in these and perhaps in most cases.

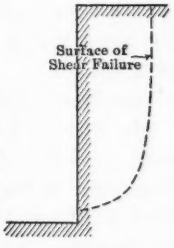

FIG. 12.

9.—After the pressure of various kinds of earth has been measured satisfactorily, it remains—in colder climates—to determine the effect of freezing at the tops of retaining walls. The retaining wall will continue to be a structure requiring local experience as well as sound judgment, but that does not diminish the value of the author's completed and prospective work.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

WILLIAM AUGUSTUS HOWELL, M. Am. Soc. C. E.*

DIED MARCH 12, 1923.

William Augustus Howell, the son of George W. Howell and Mary Elizabeth (Ward) Howell, was born in Newark, N. J., on June 16, 1859. His father, who was of Welsh descent, was for many years City Treasurer of Newark. His mother was descended from one of the original founders of Newark.

Mr. Howell was graduated from New York University in 1878, receiving the degrees of B. S. and C. E. Three years later, he was awarded the degree of Master of Science.

He began his professional career in 1879 as Rodman on the Pennsylvania Railroad, rising by successive steps to Supervisor in 1889. A year later, he resigned his position with the Pennsylvania Railroad to become Chief Engineer of the New York, Susquehanna and Western Railroad.

In 1890 and 1891, Mr. Howell was Engineer of Construction of a ten-mile section of the Jamaica Railroad in the West Indies. A year later, he was in charge of the construction of an electric road between Newark and Jersey City, N. J., and, in 1893, he entered the employ of the City of Newark, where he remained until his death on March 12, 1923.

In the thirty years during which Mr. Howell served in the Engineering Department of Newark, he had charge of the grade elimination on the Pennsylvania, Delaware, Lackawanna and Western, and New Jersey Central Railroads, and for the past sixteen years he was the responsible head of the Street Paving Bureau of Newark.

In 1884, Mr. Howell was married to Katherine Brokaw MacDonald, of New Brunswick, N. J., who, with two married daughters and six grandchildren, survives him. His death came as a great shock to his many friends, as only three days before it occurred he had performed his usual duties in the City Hall.

He was a member of the American Society for Municipal Improvements, having served as President in 1915, and on many of its important committees, and was also the author of numerous reports and papers relating to street pavements. He was also a member of the American Road Builders' Association and the Newark Chapter of the American Association of Engineers.

Mr. Howell was a Republican in politics, and a member of the Presbyterian Church. He was a member of St. John's Lodge, No. 1, F. and A. M., the New Jersey Society, Sons of the American Revolution, the Delta Upsilon Fraternity, and Corinthian Council, Royal Arcanum.

Mr. Howell was elected an Associate Member of the American Society of Civil Engineers on April 3, 1907, and a Member on June 6, 1911.

* Memoir prepared by Morris R. Sherrerd and Edward S. Rankin, Members, Am. Soc. C. E.

KARL DE WITT SCHWENDENER, M. Am. Soc. C. E.*

DIED JANUARY 22, 1922

Karl De Witt Schwendener was born at Cleveland, Ohio, on April 25, 1885. He was educated at Harvard University, from which he was graduated in 1908 with the degree of Bachelor of Science.

During the summer vacation of 1907 he was employed as Levelman on the Buffalo, Rochester and Pittsburg Railway. After his graduation from Harvard, Mr. Schwendener went to Los Angeles, Calif., and from December, 1908, to May, 1910, was engaged as Draftsman, and, later, as Chief Engineer of the California Building and Land Syndicate, in charge of engineering work relating to building construction, surveys, and sub-division of properties.

From June to December, 1910, he served as Designing Engineer for the Los Angeles Railway Corporation, for which Corporation he designed, among other structures, reinforced concrete carshops and two carhouses.

In January, 1911, Mr. Schwendener was appointed Assistant Inspector of Buildings of the City of Los Angeles and had charge of the inspection of steel frame and brick construction.

In May, 1912, he was appointed Chief Engineer of the Department of Buildings of the City of Los Angeles, in charge of all the inspection and engineering work of the Department. In addition to ten fire-engine houses and various other city buildings which were constructed under his supervision, he assisted in the preparation of revised building ordinances and State laws. He also prepared a treatise on the design of wood beams and columns, containing a series of tables, which has become a textbook in the Department, and among the architects and builders of Los Angeles.

During the World War, Mr. Schwendener was commissioned a Captain in the Engineer Corps, and served overseas where he was in charge of construction work. He was on the firing line in the Argonne and, for three months, with the Army of Occupation in Germany.

He died of pneumonia on January 22, 1922, after a brief illness, and is survived by his wife and three children.

Mr. Schwendener was an active member of the Los Angeles Chamber of Commerce, the City Club, and American Legion Post 110.

The high esteem in which he was held by his fellow engineers of the Los Angeles Section of the Society, in which he took an active part, is shown in the following resolution which was adopted at the meeting of the Section on April 12, 1922:

"*Whereas,* it has pleased the Almighty to take from us our beloved fellow-member and associate, Karl D. Schwendener; and

"*Whereas,* through his going away we have sustained the loss of an able brother engineer, adviser, and a friend whose place will never be filled. Possessing those strong sterling qualities of manhood and professional integrity that won for him the high place in the counsels of this Society and those of

* Memoir prepared by John E. Rockhold, Assoc. M. Am. Soc. C. E., and F. G. Dessery, M. Am. Soc. C. E., and from information on file at the Headquarters of the Society.

us who knew him best, we feel an irreparable loss of a loyal comrade and associate we had learned to love, respect, and honor;

"Therefore, Be It Resolved, that these resolutions be entered upon the permanent records of the Los Angeles Section, American Society of Civil Engineers, and a copy be sent to his bereaved family, with our heartfelt sympathy in their affliction."

Mr. Schwendener was elected an Associate Member of the American Society of Civil Engineers on April 1, 1914, and a Member on November 25, 1919.